Identification, Model Validation and Control

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Acknowledgment: Useful discussions with
U Forssell and H Hjalmarsson

Outline

1. Introduction
2. System Identification (in closed loop)
3. “Identification for Control”
4. Model validation
5. Illustration

To Think About

. . .

give us

grace to accept with serenity the things that cannot be changed,

courage to change the things that should be changed,

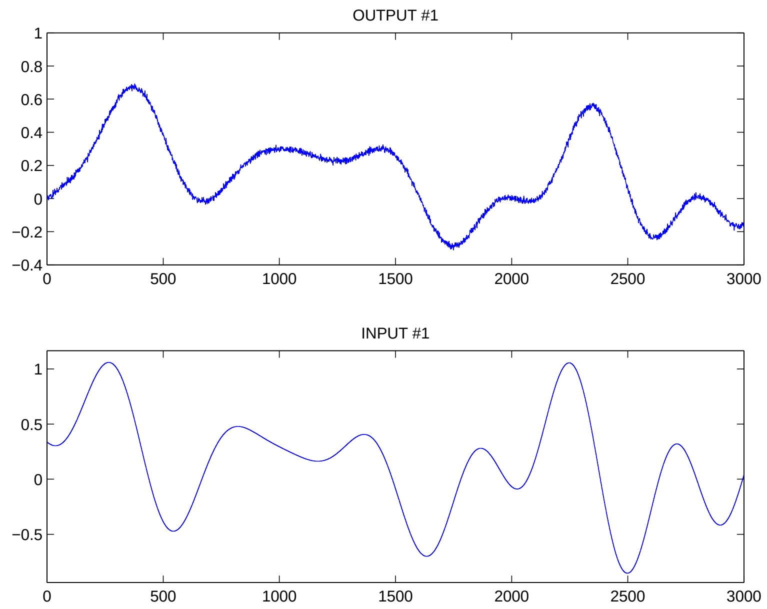
and wisdom to distinguish the one from the other.

R. Niebuhr, 1934.

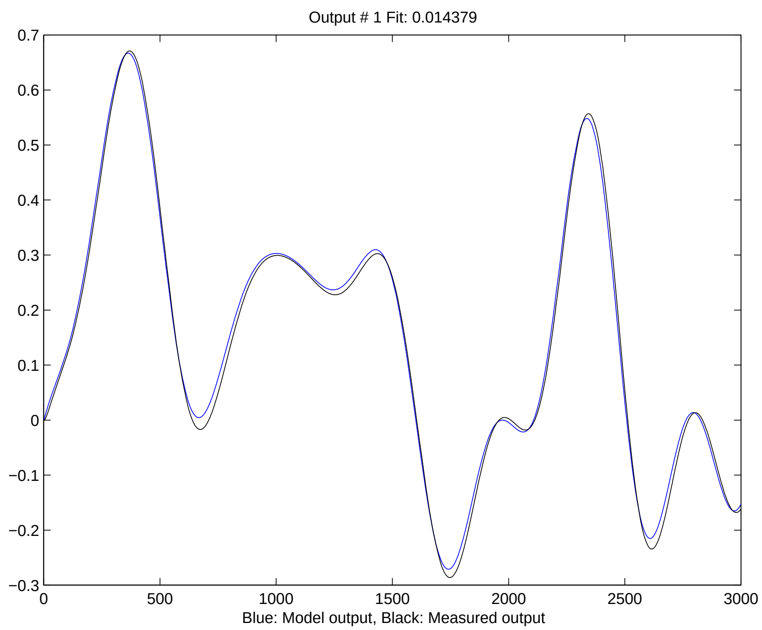
Example

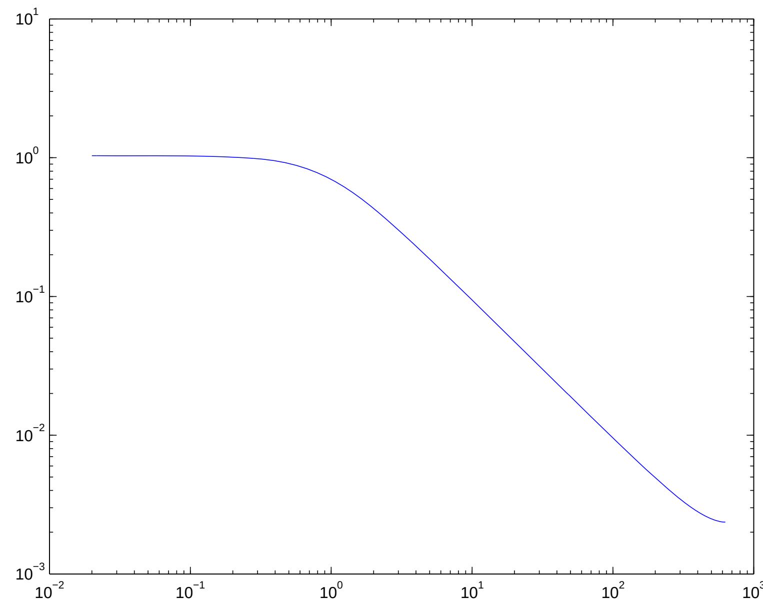
$$G(s) = \frac{1}{(s+1)(0.05s+1)} \qquad \text{(Skelton)}$$

Sample at $T = 5$ ms and perform an identification experiment.



Find that a first order model will give a nice fit and pass validation tests:

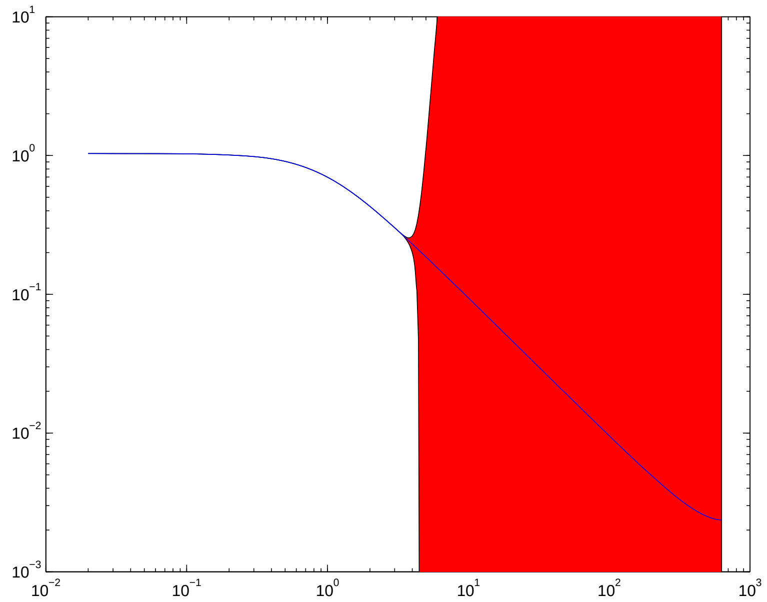




Construct a high gain regulator (aiming at a bandwidth of 10 Hz) and get surprised by the fact that the closed loop system is unstable. The nice and well fitting first order model did not predict this!

Is this a shortcoming of system identification?

Let's ask the model that was estimated from the data, how it assesses its uncertainty (in an unprejudiced way):



Respect the Uncertain!

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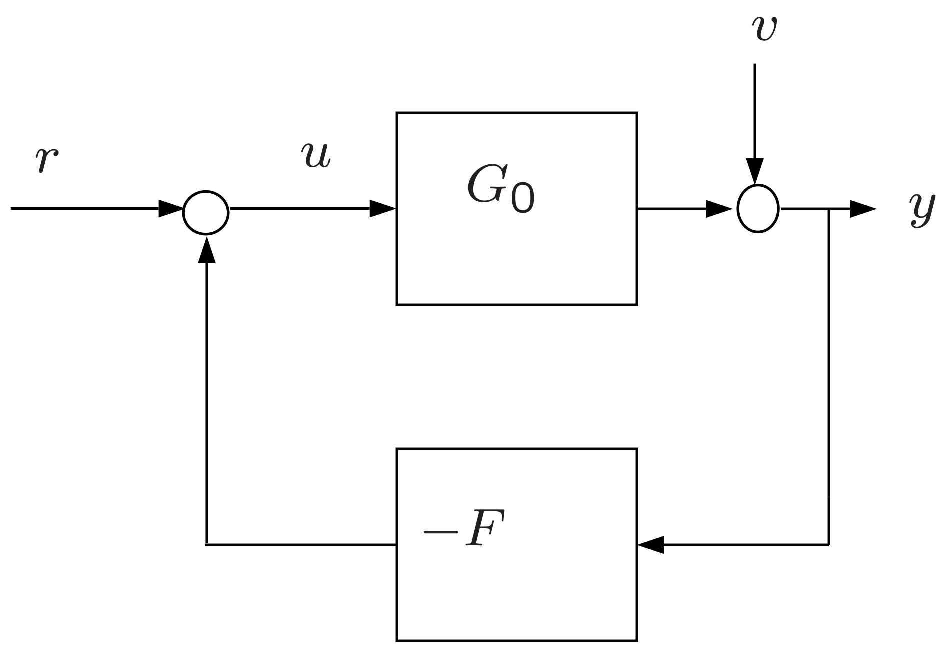
Identification
Possibly Closed Loop Experiments



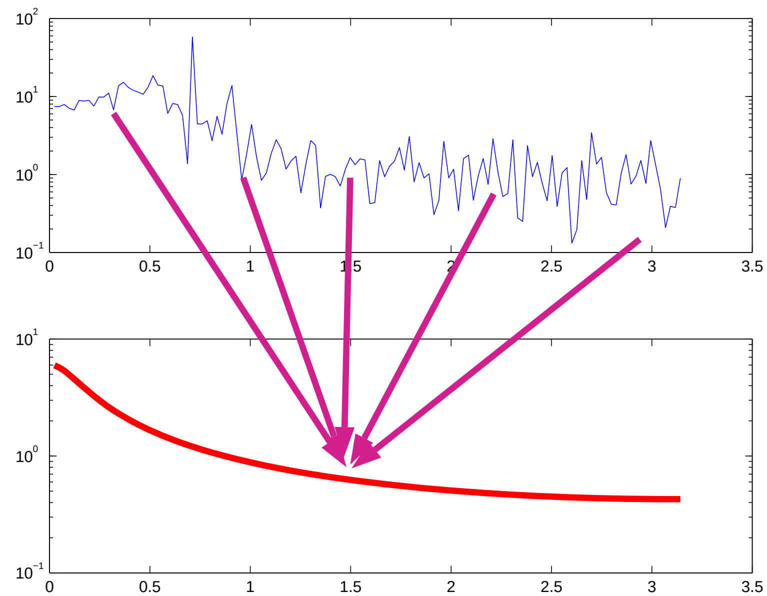
- True system: $y(t) = G_0(q)u(t) + v(t)$

Models and Methods

- Model: $\hat{y}(t|\theta) = G(q,\theta)u(t)$
- Method (direct method): $\hat{\theta} = \arg\min_\theta V(\theta)$
- $V(\theta) = \sum |L(q)[y(t) - \hat{y}(t|\theta)]|^2$
- $V(\theta) \approx \int |G(e^{i\omega},\theta) - \hat{\hat{G}}(e^{i\omega})|^2 |L(e^{i\omega})|^2 |U_N(\omega)|^2 d\omega$
- $\hat{\hat{G}}(e^{i\omega}) = \frac{Y_N(\omega)}{U_N(\omega)}$ (ETFE)
- U_N, Y_N The DFT's

Smoothing the ETFE

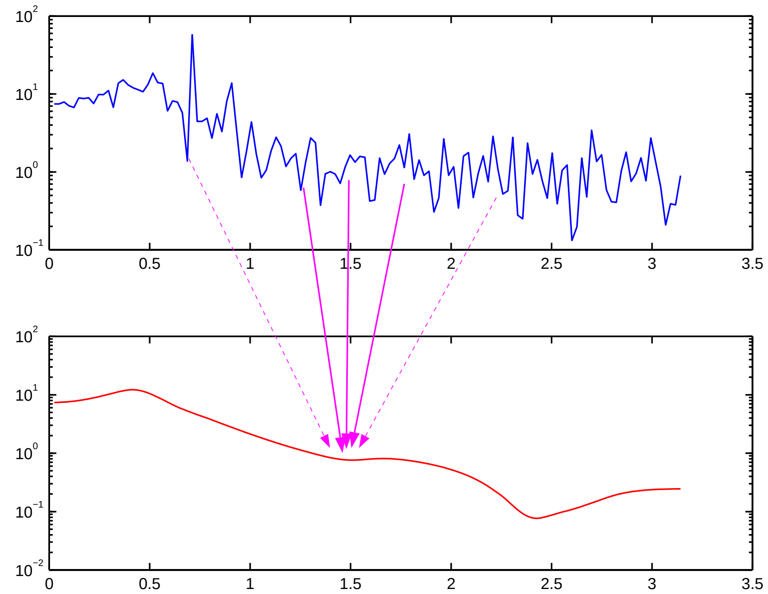
Low order model; Firm Bode plot



The actual weighting depends on L and U.

Smoothing the ETFE

High order model; Malleable Bode plot



Properties

Two error sources:

- Bias error: "Too firm Bode plot"; wrong interpolation rules for the frequencies
- Variance error (random error): Model errors that can be traced back to the disturbance v

As model order increases, Bias $\downarrow$ and Variance $\uparrow$ (less averaging taking place).

Total error (MSE) $=$ Bias $+$ variance.

Objective, choose model order so that MSE is minimized.

Bias Error Characterization

Input and disturbance spectra:

$$\Phi_u = \Phi_u^v + \Phi_u^r, \quad \Phi_v(\omega) = \lambda_0 |H_0(e^{i\omega})|^2$$

Limiting model as more data is used:

$$G_* = \arg\min_G \int |(G_0 + B) - G|^2 |L|^2 \Phi_u(\omega) d\omega$$

$$|B|^2 = \frac{\lambda_0}{\Phi_u} \cdot \frac{\Phi_u^v}{\Phi_u} \cdot |H_0 - 1/L|^2$$

Bias Error Notes

$$G_* = \arg\min_G \int |(G_0 + B) - G|^2 |L|^2 \Phi_u(\omega) d\omega$$

$$|B|^2 = \frac{\lambda_0}{\Phi_u} \cdot \frac{\Phi_u^v}{\Phi_u} \cdot |H_0 - 1/L|^2$$

- Best fit in a weighted frequency norm.
- Open loop $\Rightarrow \quad \Phi_u^v = 0 \quad \Rightarrow \quad B = 0$
- Prefilter Lv and input spectrum Φ_u interchangeable. Can focus on special frequency ranges. (Extreme case: Frequency analysis)
- $1/L$ acts like a disturbance model.

The Variance Error

Asymptotically:

$$\operatorname{Cov} \hat{G}(e^{i\omega}) \approx \frac{n}{N} \frac{\Phi_v(\omega)}{\Phi_u^r(\omega)}$$

- n: Model order
- N: number of observations,
- Φ_v: Noise spectrum.
- $\Phi_u^r = |S|^2\Phi_r$: that part of the input spectrum that originates from r.

This is basically a measure of the information contents in the data (like the Cramèr-Rao bound).

You can manipulate the bias by prefiltering but not really the variance

Indirect Identification of Closed Loop Systems

Closed loop system: $y(t) = G_c(q)r(t) +$ disturbance

$$G_c = \frac{G_0}{1 + FG_0}$$

Choose a parameterization of the closed loop $G_c(q) = G_c(q, \theta)$, identify the closed loop system, and solve for the open loop dynamics, using knowledge of the regulator.

(The indirect model error $= e_c(t) = y(t) - \frac{G}{1+FG}r$. Use the model dependent prefilter $L = 1+FG$. Then $L(q)e_c(t) = y+FGy-Gr = y-Gu$. So direct identification = indirect identification with $L = 1 + FG$.)

Parameterizations for Indirect Identification

- $G_c(q,\theta)$
- $G_c(q,\theta) = \frac{G(q,\theta)}{1+F(q)G(q,\theta)}$
- Dual Youla

$$G_c(q,\theta) = L(q)Y(q)(N(q) + S(q,\theta)Y(q))$$
$$G(q,\theta) = \frac{N(q) + Y(q)S(q,\theta)}{D(q) - X(q)S(q,\theta)}$$

- All the same theory

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Indirect Identification: Bias

$$G_* = \arg\min_G \int \left| \frac{G}{1+FG} - \frac{G_0}{1+FG_0} \right|^2 |L|^2 \Phi_r d\omega$$
$$= \arg\min_G \int \left| \frac{G_0 - G}{1+FG} \right|^2 |L|^2 \Phi_u^r(\omega) d\omega$$
$$= \arg\min_G \int |G_0 - G|\, |L|^2 |S(G)|^2 |S_0|^2 \Phi_r d\omega$$

Beware: Requires exact knowledge of regulator!

Asymptotic variance

Same as for direct identification:

$$\operatorname{Cov} \widehat{G}_N \approx \frac{n}{N} \frac{\Phi_v(\omega)}{\Phi_u^r(\omega)}$$

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Many contributors:

BDO Anderson, KJ Åström, R Bitmead, J Doyle, M Gevers, S Gunnarsson, H Hjalmarsson, R Kosut, R Schrama, R Skelton, P van den Hof, Z Zang, . . .

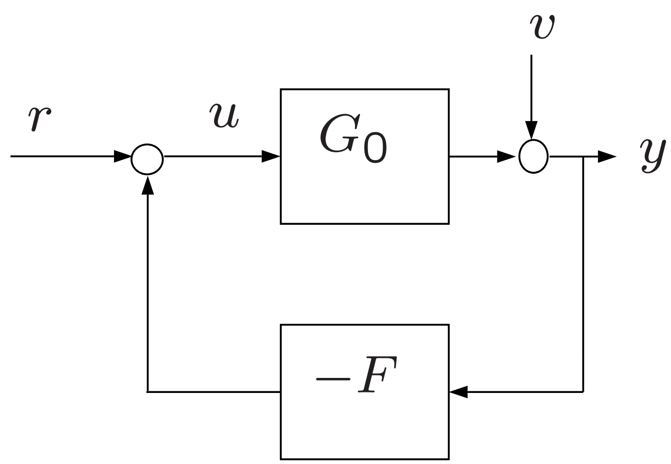
Control

Feedback is:

- Forgiving ...
- Demanding ...

Models may be bad, but need to be reliable in certain frequency regions

Experiment Design for Control



We affect the model quality by the experiment design:

- Open/closed loop
- Regulator F, Spectrum of r (u)
- Prefilter L

"Identification for control" often also means that the model structure used is quite simple.

A Design Problem

Look for a model $\widehat{G}$ that minimizes the weighted distance to the true system within a certain model structure

$$J(\mathcal{D}) = \int \mathsf{E}|\widehat{G} - G_0|^2 W(\omega) d\omega$$

for a given weighting function W.

The constraint is $\mathsf{E}u^2(t) \leq C$

The Solution

Then the solution is

- Open loop, $F = 0$
- Input spectrum $\Phi_u \sim \sqrt{W \Phi_v}$
- Prefilter $|L|^2 \sim \sqrt{\frac{W}{\Phi_v}}$

Identification for Control

OK, so what W to choose? What frequency regions to emphasize?

If G_d is the desired closed loop, the typical weighting function is $W = |G_d/G_0|^2$, i.e. emphasize the frequency range where a gain increase is required.

Not Known!(?)

Iterative approach?

Iterative Approaches to Identification for Control

Variants on the theme:

1. Fix a low order model structure
2. Make an identification experiment and fit a model
3. Compute a regulator for this model and try it out
4. If result not satisfactory, design a new experiment and go to step 2.

Leading idea: New experiment = closed loop with current controller

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Why New Experiment = = Current Closed Loop?

The bias expressions for closed loop experiments have Φ_u (Φ_u^r) as weighting functions. The input power will be large where a gain increase is achieved in closed loop.

The fit will be made for inputs that should resemble the ones to be used in the desired controller.

$$\lim_{\text{Expmnt length}\to 0} \text{Iterative design} = \text{Adaptive Control}$$

Need for Iterative Experiments?

Bias considerations: Want the weighting (prefilter) $|G_d/G_0|^2$. G_0 unknown. Iterate:

1. Infer knowledge about G_0 from measured data
2. Change prefilter accordingly

No need for new experiments, just new prefilters

MSE: Bias and variance considerations: Optimal solution requires a new experiment; new input spectrum, not just a new prefilter.

Iterative experiments required only if variance aspects taken into account!

But why throw away old experiments??

Is There a Need for Closed Loop Experiments?

- If output power constrained
  - Note that

$$\Phi_y = |G_0|^2 \Phi_u^r + |S_0|^2 \Phi_v$$
$$\Phi_y^{\text{open}} = |G_0|^2 \Phi_u + \Phi_v$$

- If regulator uses both $\widehat{G}$ and $\widehat{H}$.
- Weighting (prefilter) with S_0 (input power contains S_0) "for free". Well, ...

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Iterations well posed?

Trying to obtain a certain closed loop system G_d: Regulator $F(G)$ determined from model so that $\frac{F(G)G}{1+F(G)G} = G_d$.

Iterations. Fix regulator F. Estimate

$$G(F) = \arg\min_G \underbrace{\int \left| \frac{FG}{1+GF} - \frac{FG_0}{1+G_0F} \right|^2 \Phi_r d\omega}_{J(F,G)}$$

If this converges to G^* we must have

$$J'_G(F(G^*), G^*) = 0$$

(This is also the right hand side of the ODE associated with the corresponding adaptive control scheme)

Iterations Well Posed? cont'd

Is this the right point? The distance of interest is

$$J(F(G),G) = \int \left| \frac{F(G)G}{1+F(G)G} - \frac{F(G)G_0}{1+F(G)G_0} \right|^2 \Phi_r d\omega$$
$$= \int \left| G_d - \frac{F(G)G_0}{1+F(G)G_0} \right|^2 \Phi_r d\omega$$

Best model: $\arg\min_G J(F(G),G)$

Min at $J'_F(F(G^*),G^*)F'_G(G^*) + J'_G(F(G^*),G^*) = 0$

The iterative/adaptive schemes have a fixpoint at $J'_G(F(G^*),G^*) = 0$
OK iff $J'_F = 0$. $(\Leftrightarrow G_0 = G)$

That is Model has to be good for the iterations to be meaningful.

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Model Validation

Not sufficient to know that you have done your best, it must be good enough too!

Identification: Find the best model, in a given class and according to a given criterion.

Model validation: Find out if this model is good enough (for control design).

Residual analysis

The residuals:

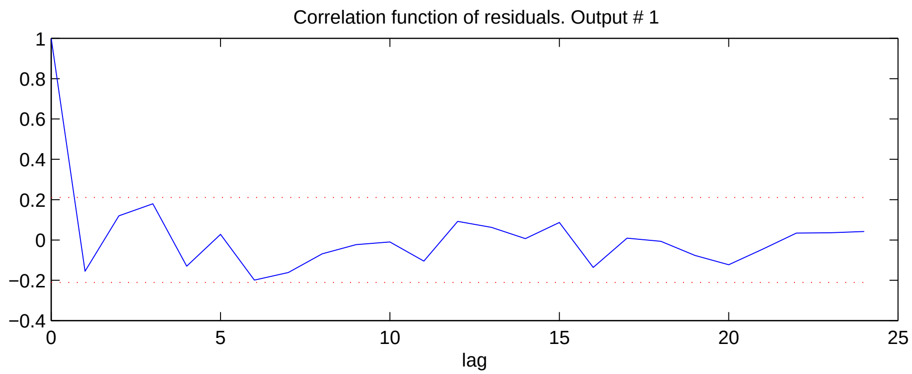
$$\varepsilon(t) = y(t) - \widehat{G}(q)u(t)$$
$$\varepsilon_F(t) = L(q)\varepsilon(t)$$

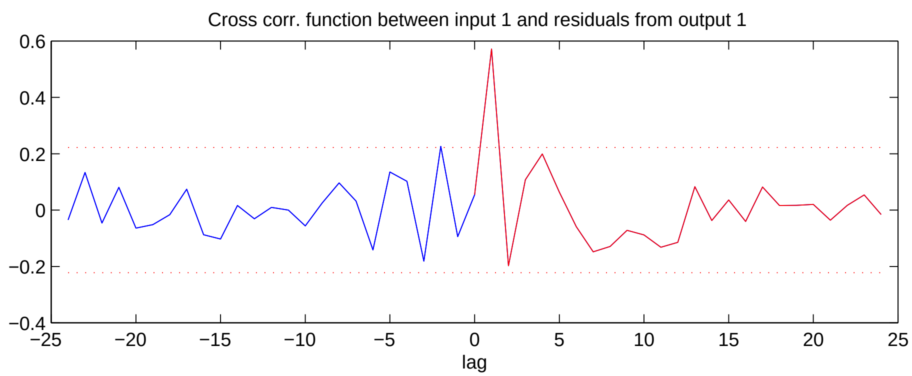
Questions:

- Are the residuals unpredictable (white)?
- Are there clear traces of the input in the residuals?

Residual analysis: Variants on the theme of checking cross correlation between u and ε_F.

Typical Residual Analysis





Form P =sum of squares of red values. Increase model order until $P \leq P_0$ for some given level P_0.

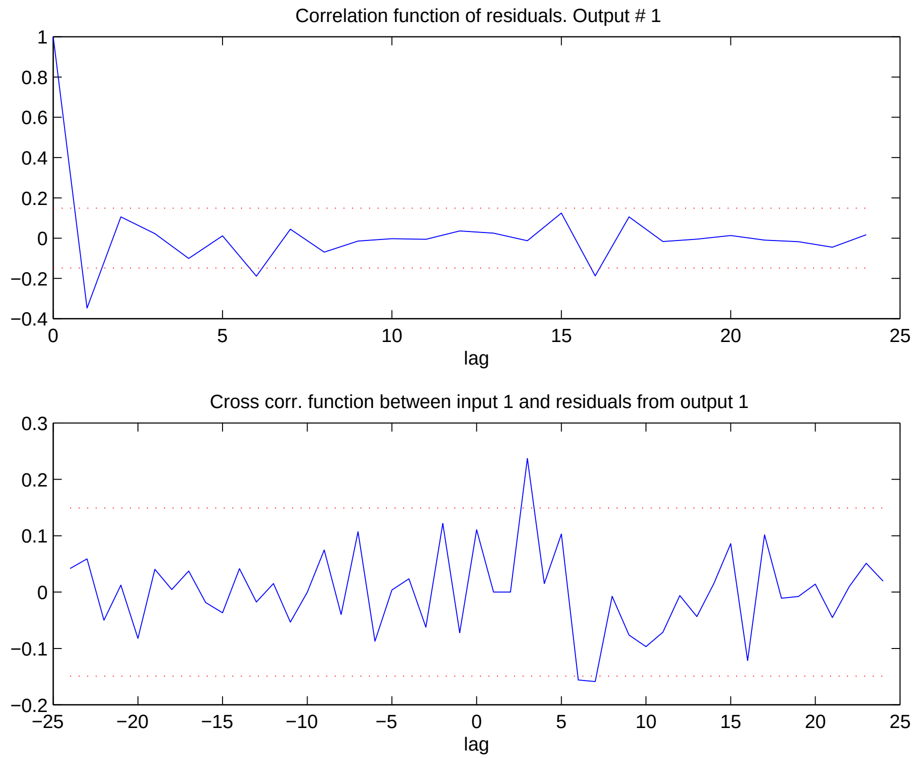
Assessing the MSE
(abridged edition)

$\hat{G}$ any model, Data subject to $y = G_0 u + v$. P as on previous slide. Then

$$\int \left|\hat{G} - G_0\right|^2 |U_N|^2 |L|^2 d\omega$$
$$\leq \underbrace{P}_{\text{Known}} + \underbrace{\underbrace{\text{corr}^2(u,v) \cdot \|v\|^2}_{\text{if "independent"} \sim 1/N}}_{\max |v|^2 \text{else}} + \underbrace{\text{impulse response tail}}_{\text{Prior smoothness}}$$

Quite general.

Conventional Presentation



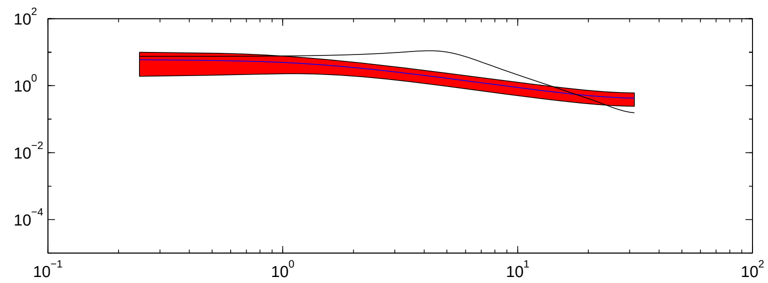
Control Oriented Presentation

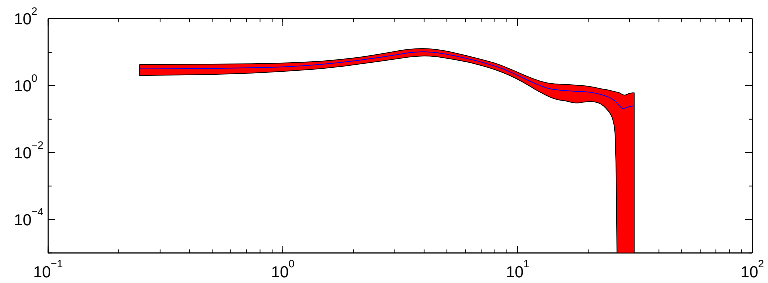
- Model G's Bode Plot with uncertainty region.

- The Model Error Model: High order ARX model from u to $y - Gu$, presented as Bode plot with uncertainty.

Example: `IDDATA1`

`G=arx(z1,[1 1 1])`

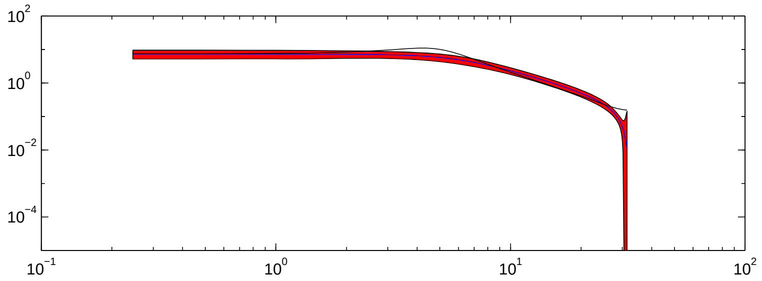


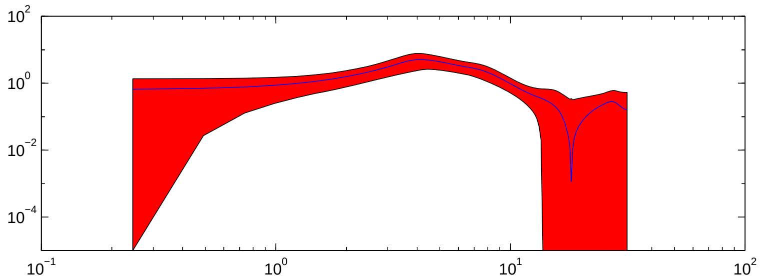


Model Not Falsified When Model Error Confidence Bound Contains Zero

"Model and Sidekick"

`IDDATA1, G=arx(z1,[2 2 1])`





Model Not Falsified

```
IDDATA1, G=armax(z1,[2 2 2 1])
```

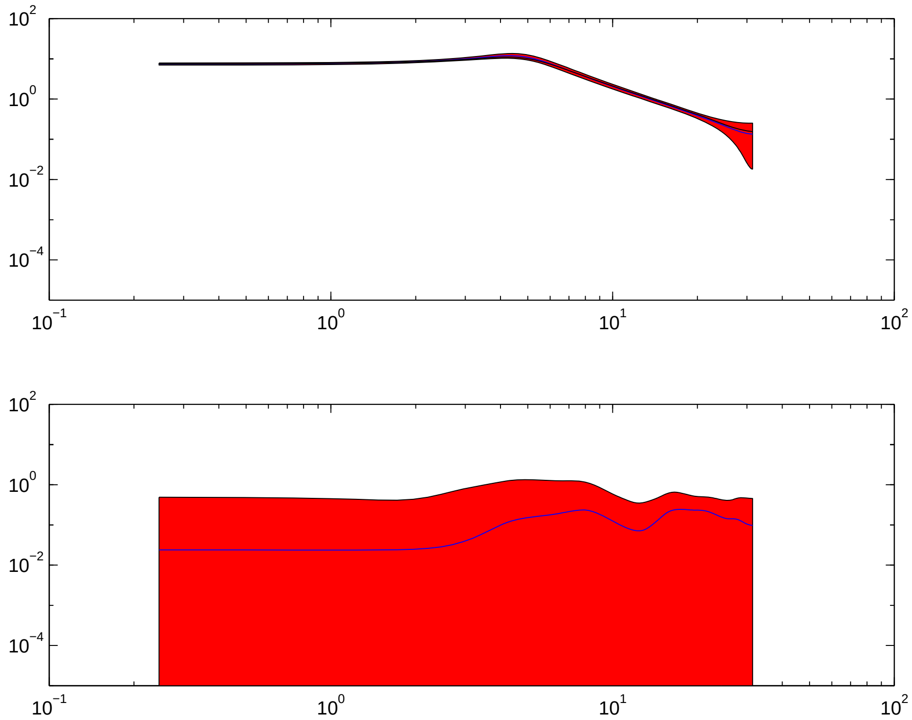


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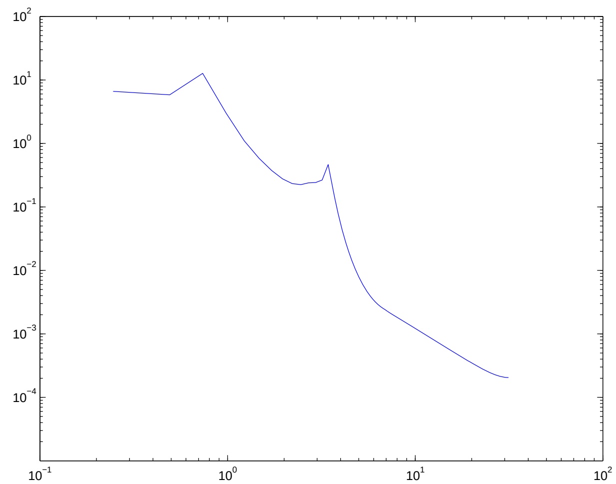
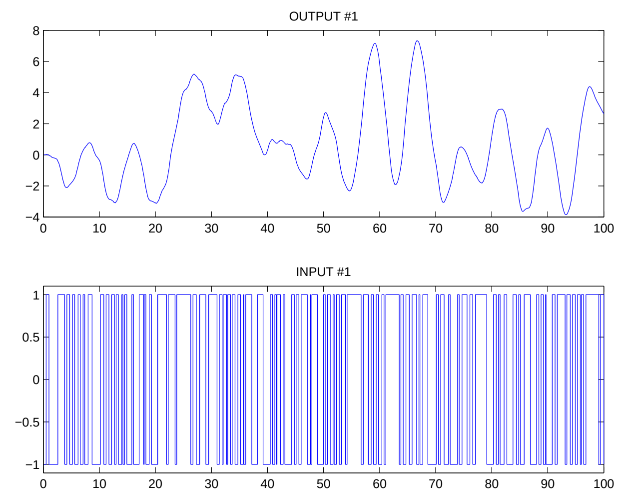
Illustration (Schrama)

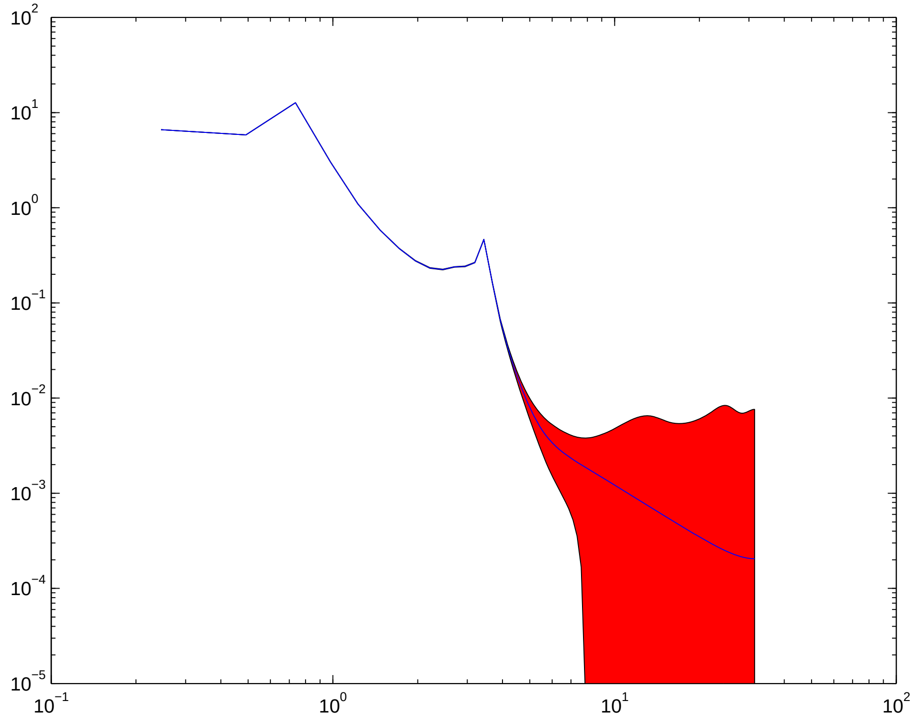


Bode plot



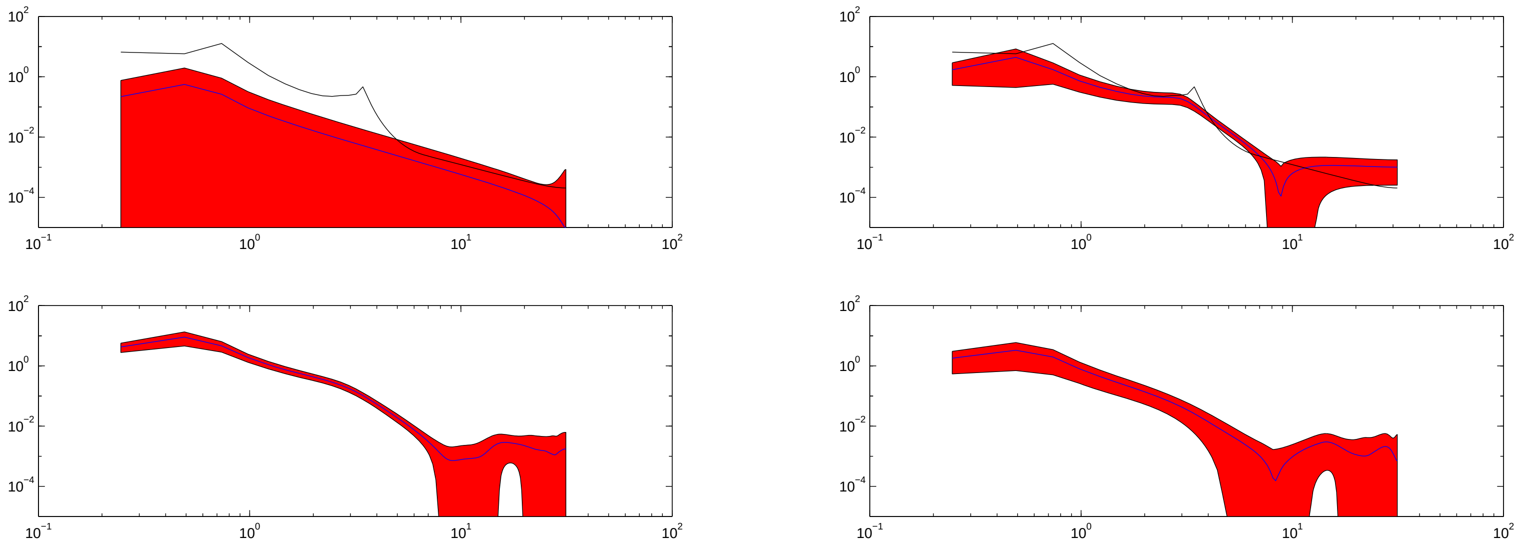
Simulated data with additive white noise (standard deviation $= 0.01$)

Information Contents
Unprejudiced CR-bounds marked



Theoretical uncertainty region for noise level 0.01 (as in data shown)

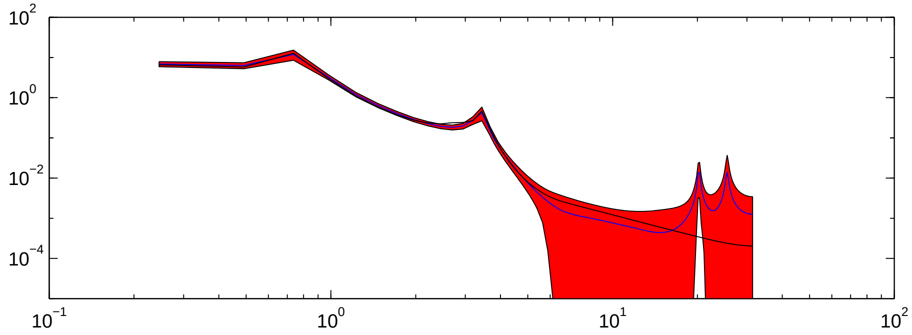
ARMAX models

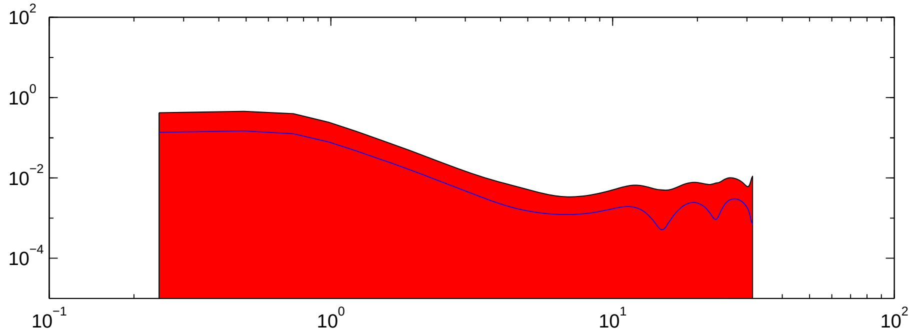


2nd order model. 4th order model.

First Unfalsified Model

9th order ARMAX

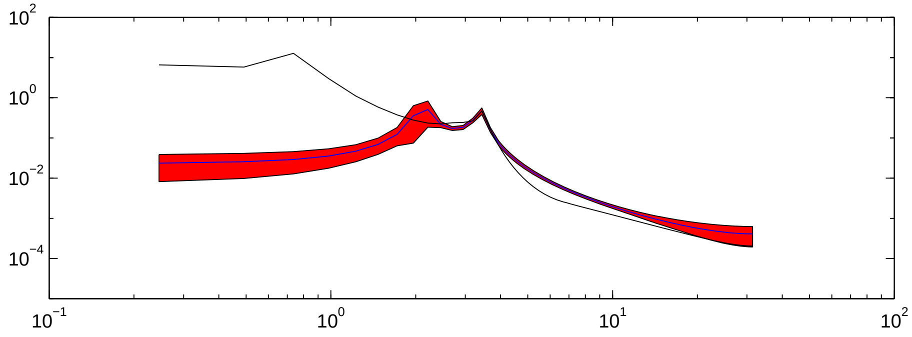


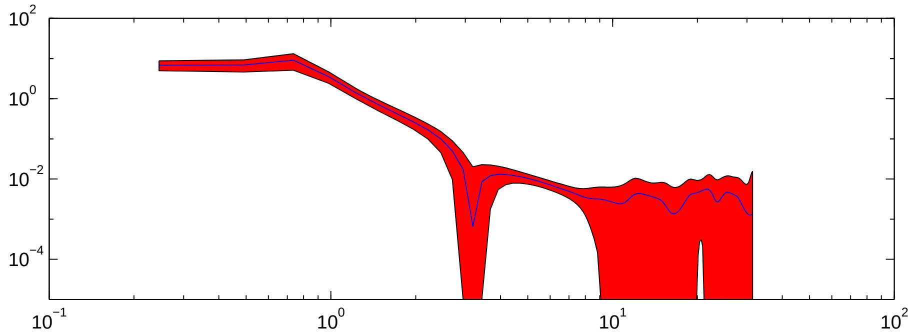


Filter Out a Band

(Inside Reasonable Region, around second peak)

First validated model (using filtered data) 4th order ARMAX





Statements

Rather Sweeping and Rather True

- The first model that passes the model validation minimizes the mean square error.
- For this model the bias error is dominated by the variance error.
- The variance error can be estimated from the observed data for a validated model
- You cannot improve the mean square error by prefiltering – you "just" get a good fit over a smaller frequency range by a lower order model.

The Know's and No-Know's

You don't know:

- Frequency details finer than the uncertainty principle (No hard bounds without prior knowledge)
- Anything outside the excited frequency range. Need to invoke the unprejudiced variance bound.

You do know

- A reasonable frequency domain error bound in excited regions

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Conclusions

- Whenever possible, no reason to be without a validated model.
- No reason not to use all collected data, even if extra experiments turn out to be necessary. (Unless essential time-variation in system)
- Even when full validation is not desired (adaptive control/automated design), there should be important work for the model's sidekick, the model error model.

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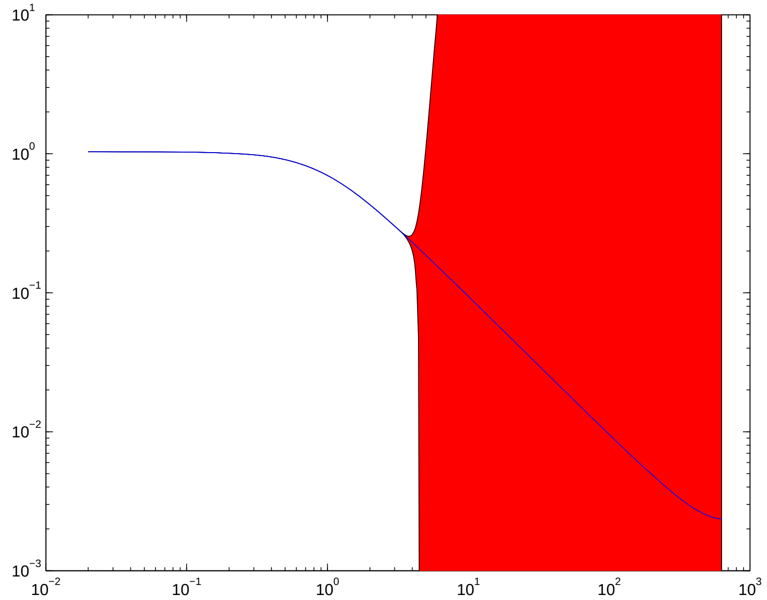
Conclusions, cont'd

- Perfectly OK to use simple model for control design. The simple model can be a reduced order version of the validated one, or estimated with constrained frequency prefilter/experiment. Validated model then helps in the control design by suggesting necessary robustness and sensitivity.
- You can manipulate bias by prefiltering, but not really variance. This is the reason why iterative experiments may be necessary.



Respect the Uncertain!