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# Ancestor Sampling for Particle Gibbs: Supplemental material

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**Fredrik Lindsten**  
 Div. of Automatic Control  
 Linköping University  
 lindsten@isy.liu.se

**Michael I. Jordan**  
 Dept. of EECS and Statistics  
 University of California, Berkeley  
 jordan@cs.berkeley.edu

**Thomas B. Schön**  
 Div. of Automatic Control  
 Linköping University  
 schon@isy.liu.se

## Abstract

This supplemental material contains the proof of [1, Proposition 1].

## 1 Proof of Proposition 1

With  $M = T - t$  and  $w(k) = w_t^k$ , the distributions of interest are given by

$$P(k) = \frac{w(k) \prod_{s=1}^M h_s(k)}{\sum_l w(l) \prod_{s=1}^M h_s(l)} \quad \text{and} \quad \tilde{P}_p(k) = \frac{w(k) \prod_{s=1}^p h_s(k)}{\sum_l w(l) \prod_{s=1}^p h_s(l)},$$

respectively. Let  $\varepsilon_s \triangleq \max_{k,l} (h_s(k)/h_s(l) - 1) \leq A \exp(-cs)$  and consider

$$\begin{aligned} \left( \sum_l w(l) \prod_{s=1}^p h_s(l) \right) \prod_{s=p+1}^M h_s(k) &\leq \sum_l \left( w(l) \prod_{s=1}^p h_s(l) \prod_{s=p+1}^M h_s(l) (1 + \varepsilon_s) \right) \\ &= \left( \sum_l w(l) \prod_{s=1}^M h_s(l) \right) \prod_{s=p+1}^M (1 + \varepsilon_s). \end{aligned}$$

It follows that the KLD is bounded according to,

$$\begin{aligned} D_{\text{KLD}}(P \parallel \tilde{P}_p) &= \sum_k P(k) \log \frac{P(k)}{\tilde{P}_p(k)} \\ &= \sum_k P(k) \log \left( \frac{\prod_{s=p+1}^M h_s(k) (\sum_l w(l) \prod_{s=1}^p h_s(l))}{\sum_l w(l) \prod_{s=1}^M h_s(l)} \right) \\ &\leq \sum_k P(k) \sum_{s=p+1}^M \log(1 + \varepsilon_s) \leq \sum_{s=p+1}^M \varepsilon_s \leq A \sum_{s=p+1}^M \exp(-cs) \\ &= A \frac{e^{-c(p+1)} - e^{-c(M+1)}}{1 - e^{-c}}. \end{aligned} \quad \square$$

## References

- [1] F. Lindsten, M. I. Jordan, and T. B. Schön, “Ancestor sampling for particle Gibbs,” in *Proceedings of the 2012 Conference on Neural Information Processing Systems (NIPS)*, Lake Tahoe, NV, USA, Dec. 2012.