MODELLING MAGNETIC FIELDS USING GAUSSIAN PROCESSES

AUTOMATIC CONTROL
REGLERTEKNIK
LINKÖPINGS UNIVERSITET

Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson

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REGLERTEKNIK
LINKÖPINGS UNIVERSITET

Contribution

We derive a **Bayesian nonparametric model** allowing for joint estimation of the **magnetic field** and the **magnetic sources** in complex environments. The model is a Gaussian process which exploits the divergence- and curl-free properties of the magnetic field by combining well-known model components in a novel manner.

Problem formulation

Consider a magnetic environment. We want to find a magnetic map of this environment representing both the magnetic sources as well as their induced magnetic field. Such maps could be used in indoor navigation systems.

Magnetic Fields

There are two different, but closely related ways to describe the magnetic field, denoted with the symbols $\mathbf{B}$ and $\mathbf{H}$. These fields have to obey the magnetostatic equations

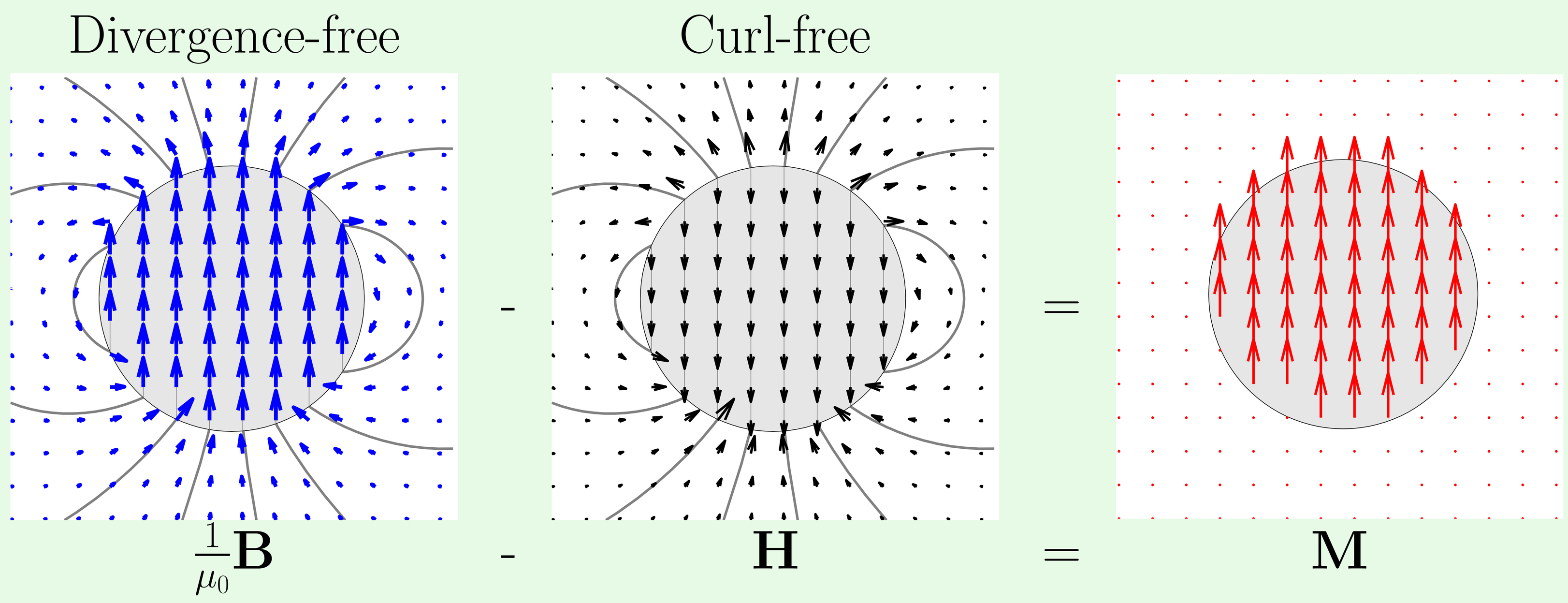
$$\nabla \cdot \mathbf{B} = 0 \quad \leftarrow \text{divergence-free property,}$$
$$\nabla \times \mathbf{H} = \mathbf{0} \quad \leftarrow \text{curl-free property.}$$

Further, these two fields are coupled as

$$\frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M},$$

where $\mathbf{M}$ is the magnetization describing our magnetic environment.

Example - Uniformly magnetized sphere



Modeling

We collect noisy three-dimensional magnetic field measurements $\mathbf{y}_{\mathbf{B},k}$ at known positions $\mathbf{x}_k$. At these positions we also know that the magnetization is zero $\mathbf{y}_{\mathbf{M},k} = \mathbf{0}$. This can be modeled as

$$\mathbf{y}_{\mathbf{B},k} = \mathbf{f}_{\mathbf{B}}(\mathbf{x}_k) + \mathbf{e}_{\mathbf{B},k}, \qquad \mathbf{e}_{\mathbf{B},k} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 \mathrm{I}_3), \tag{1a}$$
$$\mathbf{y}_{\mathbf{M},k} = \mathbf{f}_{\mathbf{M}}(\mathbf{x}_k) = \mathbf{f}_{\mathbf{B}}(\mathbf{x}_k) - \mathbf{f}_{\mathbf{H}}(\mathbf{x}_k), \tag{1b}$$

where $\mathbf{f}_{\mathbf{B}}(\mathbf{x}_k)$, $\mathbf{f}_{\mathbf{H}}(\mathbf{x}_k)$ and $\mathbf{f}_{\mathbf{M}}(\mathbf{x}_k)$ represent the $\mathbf{B}$-, $\mathbf{M}$- and $\mathbf{H}$-field respectively.

We place this in a statistical setting by putting Gaussian process priors on $\mathbf{f}_{\mathbf{B}}$ and $\mathbf{f}_{\mathbf{H}}$ (and consequently also implicitly on $\mathbf{f}_{\mathbf{M}}$). Also consider these two priors to have a common constant mean function (corresponding to the earth magnetic field)

$$\mathbf{f}_{\mathbf{B}} \sim \mathcal{GP}\big(\boldsymbol{\beta}, \mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}')\big), \quad \mathbf{f}_{\mathbf{H}} \sim \mathcal{GP}\big(\boldsymbol{\beta}, \mathrm{K}_{\mathbf{H}}(\mathbf{x},\mathbf{x}')\big), \tag{1c}$$
$$\boldsymbol{\beta} \sim \mathcal{N}(\mathbf{0}, \sigma_\beta^2 \mathrm{I}_3), \tag{1d}$$

where $\mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}')$ and $\mathrm{K}_{\mathbf{H}}(\mathbf{x},\mathbf{x}')$ are covariance functions ensuring the divergence- and curl-free properties of $\mathbf{f}_{\mathbf{B}}$ and $\mathbf{f}_{\mathbf{H}}$ having the form

$$\mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}') = \sigma_f^2 \exp\left\{-\frac{\|\mathbf{x}-\mathbf{x}'\|^2}{2l^2}\right\} \cdot \left(\left(\frac{\mathbf{x}-\mathbf{x}'}{l}\right)\left(\frac{\mathbf{x}-\mathbf{x}'}{l}\right)^{\mathsf{T}} + \left(2 - \frac{\|\mathbf{x}-\mathbf{x}'\|^2}{l^2}\right)\mathrm{I}_3\right), \tag{1e}$$

$$\mathrm{K}_{\mathbf{H}}(\mathbf{x},\mathbf{x}') = \sigma_f^2 \exp\left\{-\frac{\|\mathbf{x}-\mathbf{x}'\|^2}{2l^2}\right\} \cdot \left(\mathrm{I}_3 - \left(\frac{\mathbf{x}-\mathbf{x}'}{l}\right)\left(\frac{\mathbf{x}-\mathbf{x}'}{l}\right)^{\mathsf{T}}\right). \tag{1f}$$

Combined model

By combining the measurements $\mathbf{y}_k = \left[\mathbf{y}_{\mathbf{B},k}^{\mathsf{T}}\ \mathbf{y}_{\mathbf{M},k}^{\mathsf{T}}\right]^{\mathsf{T}}$, the model (1) can be reformulated as one function $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^6$ with one zero-mean Gaussian process prior

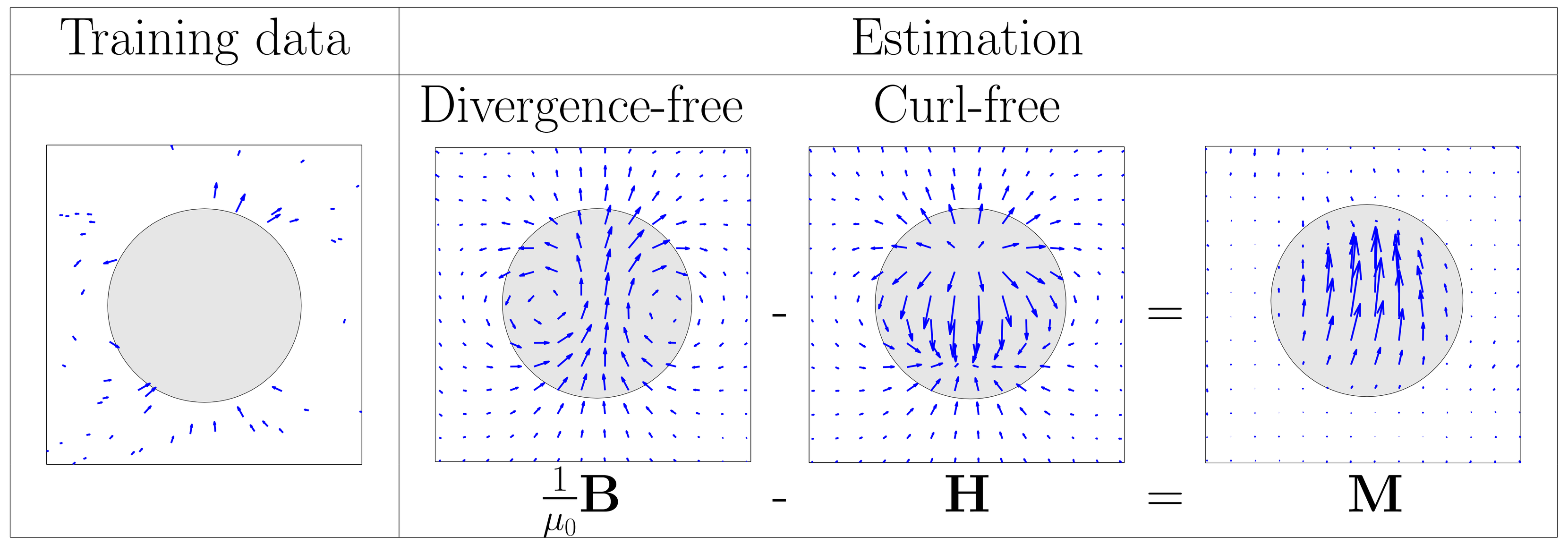
$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix}\mathbf{f}_{\mathbf{B}}(\mathbf{x}) \\ \mathbf{f}_{\mathbf{M}}(\mathbf{x})\end{bmatrix} = \begin{bmatrix}\mathrm{I}_3 & 0_3 \\ \mathrm{I}_3 & -\mathrm{I}_3\end{bmatrix}\begin{bmatrix}\mathbf{f}_{\mathbf{B}}(\mathbf{x}) \\ \mathbf{f}_{\mathbf{H}}(\mathbf{x})\end{bmatrix} \sim \mathcal{GP}(\mathbf{0}, \mathrm{K}(\mathbf{x},\mathbf{x}')), \tag{2a}$$

where

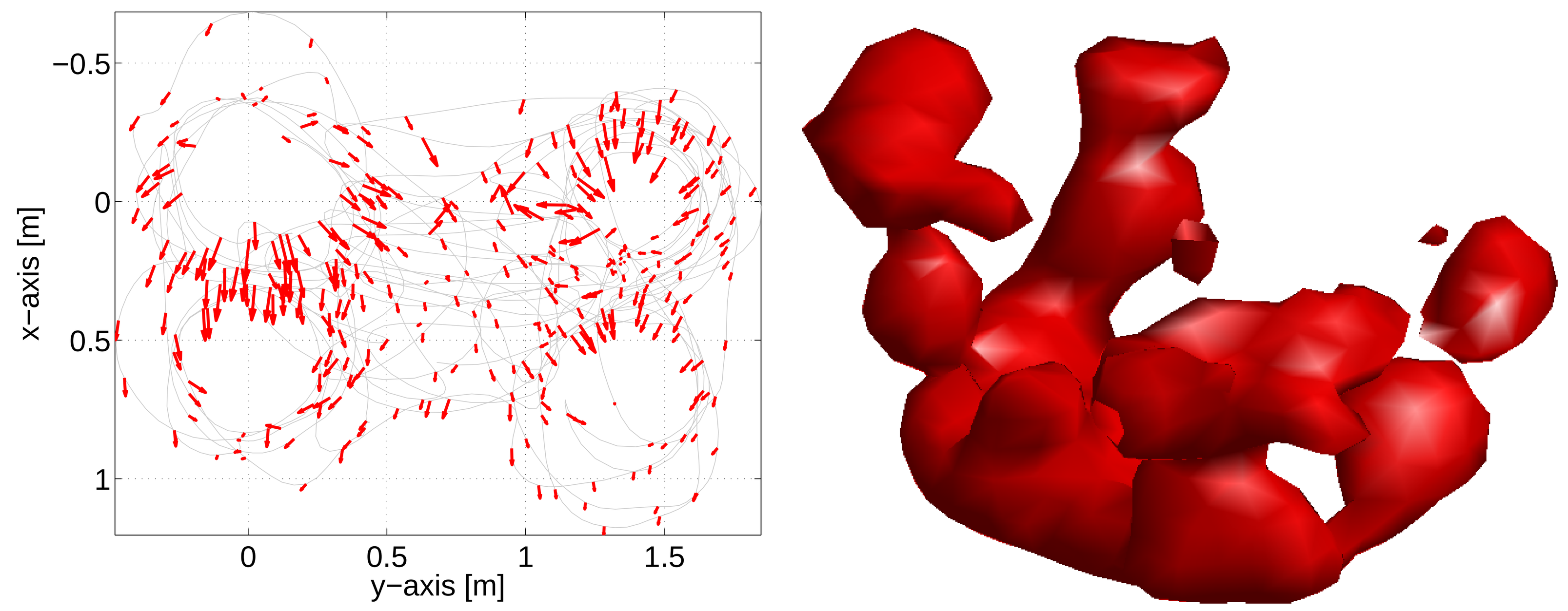
$$\mathrm{K}(\mathbf{x},\mathbf{x}') = \begin{bmatrix}\mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}') + \sigma_\beta^2 \mathrm{I}_3 & \mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}') \\ \mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}') & \mathrm{K}_{\mathbf{B}}(\mathbf{x},\mathbf{x}') + \mathrm{K}_{\mathbf{H}}(\mathbf{x},\mathbf{x}')\end{bmatrix}. \tag{2b}$$

Results

Simulation: Consider the example with a uniformly magnetized sphere. In total 50 training inputs are chosen from a region outside the sphere. Using these measurements, the model enables estimation of the $\mathbf{B}$-, $\mathbf{H}$- and $\mathbf{M}$-field, which resemble the true fields.



Real world data: A three-axis magnetometer has been used to measure the magnetic field at various locations around a metallic table. The position and the orientation of the magnetometer unit were measured using an optical reference system (Vicon). As a conclusion, the geometrical shape of the table could be reconstructed using these measurements.



(a) Training data (seen from above) (b) Estimated shape of the $\mathbf{M}$-field of the table

Conclusion

The advantage with the proposed model is its ability to describe both the magnetic field and the magnetic sources in a nonparametric manner. This has been verified using both simulated and real world data.

Future work

- Include prior information about geometry.
- Do simultaneous localization and mapping.

{nikwa, manko, schon, fredrik}@isy.liu.se

http://www.control.isy.liu.se/