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EE236B homework assignment #8

Due: Wednesday 3/13/02.

You are asked to implement an interior-point algorithm for the SDP

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^n x_i \\ \text{subject to} & W + \textbf{diag}(x) \succeq 0. \end{array} \qquad (1)$$

The variable is $x \in \mathbf{R}^n$. The matrix $W = W^T \in \mathbf{R}^{n\times n}$ is given.

1. *Dual problem.* Form the Lagrangian, and derive an explicit expression for the dual function g. Show that the Lagrange dual can be simplified as

$$\begin{array}{ll} \text{maximize} & -\textbf{Tr}\, WZ \\ \text{subject to} & Z = Z^T \succeq 0 \\ & Z_{ii} = 1,\ i = 1, \ldots, n. \end{array} \qquad (2)$$

   Note that $Z = I$ is dual feasible. What is the corresponding bound on the optimal value of (1)? Can you derive this bound directly, without using any duality theory?

2. *Central path.* We will use the standard logarithmic barrier for the positive semidefinite cone, *i.e.*, $\log\det X^{-1}$ for $X = X^T \succ 0$. Derive and simplify the conditions under which a strictly feasible x is on the central path, *i.e.*, minimizes

$$t \sum_{i=1}^n x_i - \log\det(W + \textbf{diag}(x)).$$

   Show explicitly how to find a matrix Z that is feasible for the dual (2), given a central point $x^*(t)$. What is the corresponding duality gap?

3. *SUMT.* Write a Matlab function that solves problem (1) with a guaranteed relative accuracy of 0.1%, or a guaranteed absolute accuraccy of 10^{-5}. By this we mean that on exit your solution must satisfy

$$\sum_{i=1}^n x_i - p^\star \le \max\{0.001|p^\star|, 10^{-5}\},$$

   where $p^\star$ is the optimal value of (1).

   Along with your m-file, give a description of your code, which includes the formulas for the gradient and Hessian of barrier and related functions, how you find an initial strictly feasible x, what starting value you use for t, and how your stopping criterion guarantees the required accuracy.

Test your code on a variety of simple instances (diagonal, 2x2, ...). Then test it on some larger problems, with random (symmetric) W, *e.g.*, `A=randn(10,10); W=A+A';` Experiment with the effect of μ on the total number of Newton steps required to solve the problem.

Give the solution for the matrix W defined in `hw8prob1.m` on the class website. The calling sequence to generate W is is `W = hw8prob1`.

Remark. The gradient and Hessian of the logarithmic barrier $\phi(x) = \log\det F(x)^{-1}$ for the linear matrix inequality

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i \succeq 0, \quad F_i = F_i^T \in \mathbf{R}^{n\times n},\ i = 0, \ldots, m$$

are given by

$$\nabla\phi(x)_i = -\,\mathbf{Tr}\, F_i F(x)^{-1}, \quad \nabla^2\phi(x)_{ij} = \mathbf{Tr}\, F_i F(x)^{-1} F_j F(x)^{-1}, \quad i, j = 1, \ldots, m.$$