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EE236B homework assignment #7

Due: 3/6/02.

Write a Matlab code that solves the unconstrained minimization problem

$$\text{minimize } x^T x + \log \sum_{i=1}^{m} \exp(a_i^T x + b_i), \tag{1}$$

using Newton's method with backtracking line search.

Experiment with the backtracking parameters α and β to see their effect on the total number of iterations. For a few values of α and β, plot $f(x^{(k)}) - f^\star$ and the steplength versus iteration number. Include a typical set of plots with your solution. Apply your code to the data generated by the m-file `hw7prob1.m` on the class website. The calling sequence is `[A,b] = hw7prob1` (where $A = \begin{bmatrix} a_1 & a_2 & \cdots & a_m \end{bmatrix}^T \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$).

Remarks

- The gradient and Hessian of the function $g(y) = \log \sum_{i=1}^m \exp(y_i)$ are given by

  $$\nabla g(y) = w, \quad \nabla^2 g(y) = \mathbf{diag}(w) - ww^T$$

  where

  $$w = \frac{1}{\sum_{i=1}^m e^{y_i}} \begin{bmatrix} e^{y_1} \\ \vdots \\ e^{y_m} \end{bmatrix}.$$

  The gradient and Hessian of a function f defined as $f(x) = g(Ax + b)$ are given by

  $$\nabla f(x) = A^T \nabla g(Ax + b), \quad \nabla^2 f(x) = A^T \nabla^2 g(Ax + b) A.$$

- To solve the Newton equation $\nabla^2 f(x) v = -\nabla f(x)$, use Matlab's backslash operator, *i.e.*, `v = -hess\grad`.

- A good stopping criterion is

  $$\sqrt{-v^T \nabla f(x)} \le \epsilon,$$

  where v is the Newton step at x, and ϵ is a small tolerance (*e.g.*, $\epsilon = 10^{-6}$). (The advantage of this criterion is that it is affinely invariant, as opposed to a stopping criterion $\|\nabla f(x)\|_2 \le \epsilon$.)