

Sensor fusion for accurate computation of yaw rate and absolute velocity

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ABSTRACT

In the presented sensor fusion approach, centralized filtering of related sensor signals is used to improve and correct low price sensor measurements. From this, we compute high-quality state information as drift-free yaw rate and exact velocity (accounting for unknown tire radius and slipping wheels on 4WD vehicles). The basic tool here is a Kalman filter supported by change detection for sensor diagnosis. Results and experience of real-time implementations are presented.

1 INTRODUCTION

During the last decade, a number of automotive control systems for motion control has appeared and become standard in high-end vehicles. These systems may benefit from more accurate state information, as the vehicle's speed and yaw rate, as listed below. The longitudinal slip is here defined as the relative difference of the wheel's peripheral speed and its absolute speed, while the slip angle is defined as the difference in angle between the steering wheel and the wheel's velocity vector.

- ABS needs absolute velocity information to compute the slip.
- The anti-spin system has problems to compute the optimal slip on four-wheel driven (4WD) vehicles.
- A dynamic stability system basically controls

the slip angle. A car that under-steers will have a positive angle while over-steering causes a negative angle. The basic idea is to brake the rear inside wheel during under-steering and the front outside wheel during over-steering. To compute the slip angle, the yaw angle taken from a gyro is needed. Since this is corrupted with noise, a dead-zone is needed. Because of drift in the yaw angle, a D-controller is used, since only the derivative of the slip angle is known accurately enough.

- An adaptive cruise controller including a radar needs accurate yaw rate information for its situation awareness.

Instrumental for improving such systems using accurate state informations is knowledge of the offsets in Table 1. Adaptive estimation of these is the core of the approach.

NIRA Dynamics AB is together with Linköping University developing adaptive filters for automotive applications. The department of electrical engineering at Linköping University has long experience in sensor fusion in airborne navigation systems, which have similar problems. Bringing over this competence from aircraft to cars was originally the motivation for this work. The approach herein is based on Kalman filtering, change detection and sensor fusion theory, which is thoroughly described in Gustafsson (2000).

Figure 1 shows the structure of the signal processing. Only existing sensors in modern, high-end cars are used. The result is a yaw rate with drift less than

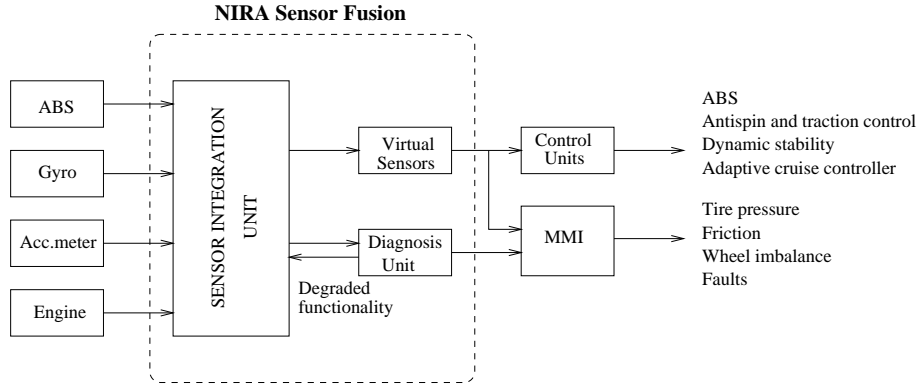


Figure 1: Overview of the sensor fusion system.

δ_r	Absolute difference in average and nominal wheel radius [mm]
δ_{12}	Relative difference between front left and right wheel radius
δ_{34}	Relative difference between rear left and right wheel radius
$\delta_{o,acc}$	Accelerometer offset [m/s^2]
$\delta_{o,gyro}$	Gyro offset [rad/s]
$\delta_{sc,gyro}$	Gyro scale factor relative error

Table 1: Offset parameters for high-precision filtering.

0.2 degrees per second, and absolute speed with an error in the order of centimeters per second, without any prior knowledge of tire radius.

Section 2 describes estimation of yaw rate, while Section 3 describes absolute velocity estimation. As a related project with many cross-couplings, virtual sensors for estimating tire-road friction and tire pressure are described in an accompanying paper. The approach is patent pending Gustafsson and Ahlqvist (2000).

2 ACCURATE YAW RATE COMPUTATION

Accurate yaw rate computation is considered. The first possibility of using only a yaw rate sensor is suffering from an unavoidable measurement offset which varies in time.

Another possibility is to compute the curve radius from wheel angular velocities taken from the wheel speed sensors in the ABS system (hereafter simply called ABS sensors), which can be converted to yaw rate information. Again, there will be an offset caused by imperfect knowledge of the tire radii. By integrating the information from both sensors, these

two offsets can be estimated accurately in an adaptive filter or Kalman filter, as illustrated in Figure 2. Furthermore, the Kalman filter has the advantage of attenuating measurement noise, implying a high accuracy *virtual* yaw rate sensor.

As applications for an accurate yaw rate, we have lateral slip computation, used in vehicle stability systems and friction estimation.

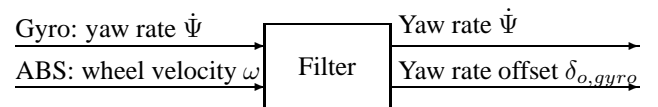


Figure 2: Basic structure of high precision yaw rate computation.

Basic relations

Figure 3 defines the notation used in this section. The well-known relations between yaw rate $\dot{\psi}$, lateral acceleration a_y , longitudinal velocity v_x and curve radius R are (see any text book on vehicle dynamics as

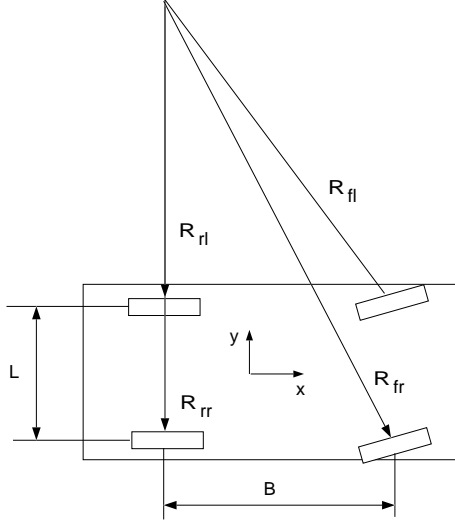


Figure 3:

Adler (1993), Gillespie (1992) and Wong (1993))

$$\begin{aligned}\dot{\Psi} &= \frac{v_x}{R} = v_x R^{-1} \\ a_y &= \frac{v_x^2}{R} = v_x^2 R^{-1} = v_x \dot{\Psi}\end{aligned}$$

The ABS sensors measure rotational wheel velocities ω , where the index convention is that *rl* means rear left, *fr* means forward right and so on.

A geometrical relation from Figure 3 is used to compute the curve radius, where R is defined as the distance to the center of the rear wheel axle,

$$\frac{v_{rr}}{v_{rl}} = \frac{R_{rr}}{R_{rl}} = \frac{R + L/2}{R - L/2}$$

Solving for R^{-1} (the inverse to avoid numerical problems when driving straight ahead) gives

$$\begin{aligned}R^{-1} &= \frac{2 \frac{v_{rl}}{v_{rr}} - 1}{L \frac{v_{rl}}{v_{rr}} + 1} \\ &= \frac{2 \frac{\omega_{rl} r_{rl}}{\omega_{rr} r_{rr}} - 1}{L \frac{\omega_{rl} r_{rl}}{\omega_{rr} r_{rr}} + 1}\end{aligned}$$

The wheel radius is denoted r . The wheel radii ratio is subject to an offset

$$\frac{r_{rl}}{r_{rr}} \triangleq 1 + \delta_{34}$$

The offset's influence on the denominator is negligible, so we will use the following expression for inverse curve radius:

$$\begin{aligned}R^{-1} &= \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \left(\frac{\omega_{rl}}{\omega_{rr}} (1 + \delta_{34}) - 1 \right) \\ &= R_m^{-1} + \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \frac{\omega_{rl}}{\omega_{rr}} \delta_{34}\end{aligned}$$

Here we have introduced the computable quantity

$$R_m^{-1} \triangleq \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \left(\frac{\omega_{rl}}{\omega_{rr}} - 1 \right)$$

for the inverse curve radius.

Finally, the velocity at the center of the rear wheel axle is

$$\begin{aligned}v_x &= \frac{\omega_{rl} + \omega_{rr} r}{2} \\ &= \frac{\omega_{rl} + \omega_{rr} (r_m - \delta_r)}{2} \\ &= v_{x,m} - \frac{\omega_{rl} + \omega_{rr}}{2} \delta_r\end{aligned}$$

where r_m is the nominal wheel radius, and δ_r the absolute error in this value. Again, index m indicates a computable value.

Measurements

The measurements under consideration are

- y_t^1 from gyro (yaw rate sensor).
- $y_t^2 = v_{x,m} R_m^{-1}$ from ABS sensors.
- Possibly y_t^3 from lateral acceleration sensor.¹

The gyro signal is subject to an offset and scale factor error

$$y_t^1 = (1 + \delta_{sc}) \dot{\Psi}_t + \delta_{o,gyro} + e_t^1$$

Here $\delta_{gyro,sc}$ is the scale factor error in the gyro, which enters the measurement non-linearly. A good working approximation might be to use

$$y_t^1 = \dot{\psi} + \hat{\psi} \delta_{sc,gyro} + \delta_{o,gyro}. \quad (1)$$

¹ Everywhere when an accelerometer is mentioned, this may, and should, be supported by a vertical accelerometer to compensate for a non-horizontal position of the car.

The nominal velocity $v_{x,m}$ differs from the true one because of unknown absolute wheel radius according to (1). The measurement is thus related to known and unknown quantities as

$$\begin{aligned} y_t^2 &= v_{x,m} R_m^{-1} \\ &= \left(v_x + \frac{\omega_{rl} + \omega_{rr}}{2} \delta_r \right) \left(R^{-1} + \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \frac{\omega_{rl}}{\omega_{rr}} \delta_{34} \right) \\ &= \underbrace{v_x R^{-1}}_{\dot{\psi}} + R_m^{-1} \frac{\omega_{rl} + \omega_{rr}}{2} \delta_r + v_{x,m} \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \frac{\omega_{rl}}{\omega_{rr}} \delta_{34} \\ &\quad - \frac{\omega_{rl} + \omega_{rr}}{2} \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \frac{\omega_{rl}}{\omega_{rr}} \underbrace{\delta_{34} \delta_r}_{\approx 0} \\ &\approx \dot{\psi}_t + R_m^{-1} \frac{\omega_{rl} + \omega_{rr}}{2} \delta_r + v_{x,m} \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \frac{\omega_{rl}}{\omega_{rr}} \delta_{34} \end{aligned}$$

For the accelerometer, we have

$$\begin{aligned} y_t^3 &= v_x \dot{\psi}_t + \delta_{o,acc} \\ &= \left(v_{x,m} - \frac{\omega_{rl} + \omega_{rr}}{2} \delta_r \right) \dot{\psi}_t + \delta_{o,acc}. \end{aligned}$$

Again, there is a non-linear scaling factor error due to absolute wheel radius. A linearization as above is necessary. Note that the two scale factors are linearly independent when the velocity is changing.

In summary, the slowly time-varying parameters in Table 1 must be estimated (in order of relative importance):

Offset estimation by least squares

Here we neglect the scale factor errors and only use wheel velocities and the gyro. Eliminating the yaw rate from the first two measurements yields a linear regression in the two offsets:

$$\bar{y}_t = \varphi_t^T \delta + \bar{e}_t$$

where

$$\begin{aligned} \bar{y}_t &= y_t^1 - y_t^2 \\ \varphi_t &= \left(1, v_{x,m} \frac{1}{L} \frac{2}{\frac{\omega_{rl}}{\omega_{rr}} + 1} \frac{\omega_{rl}}{\omega_{rr}} \right)^T \\ \delta &= (\delta_{o,gyro}, \delta_{34})^T \\ \bar{e}_t &= e_t^1 - e_t^2 \end{aligned}$$

With an accelerometer, the regression quantities are

$$\begin{aligned} \bar{y}_t &= y_t^1 - \frac{y_t^3}{v_x} \\ \varphi_t &= \left(1, \frac{1}{v_x} \right)^T \\ \delta &= (\delta_{o,gyro}, \delta_{ACC})^T \\ \bar{e}_t &= e_t^1 - \frac{e_t^2}{v_x} \end{aligned}$$

The least squares estimate is computed by

$$\hat{\delta} = \left(\frac{1}{N} \sum_{t=1}^N \varphi_t^T \varphi_t \right)^{-1} \frac{1}{N} \sum_{t=1}^N \varphi_t^T y_t$$

The important question of *identifiability*, that is, under what conditions are the offsets possible to estimate, is answered by studying the rank of the matrix to be inverted in the LS solution. For the accelerometer sensor, the matrix is given by

$$\frac{1}{N} \sum_{t=1}^N \varphi_t^T \varphi_t = \begin{pmatrix} 1 & \frac{1}{N} \sum_{t=1}^N \frac{1}{v_{x,t}} \\ \frac{1}{N} \sum_{t=1}^N \frac{1}{v_{x,t}} & \frac{1}{N} \sum_{t=1}^N \frac{1}{v_{x,t}^2} \end{pmatrix}$$

In short, this matrix has full rank if and only if the velocity changes during the time horizon. Furthermore, the more variation, the better estimate.

Similarly, the offsets are identifiable from yaw rate and ABS sensors if the velocity *or* the curve radius changes anytime.

The offsets can be estimated *adaptively* in a standard way by recursive least squares (RLS) algorithm, or least mean square (LMS) or a Kalman filter.

Kalman filter

The Kalman filter is completely specified by a state space equation of the form

$$\begin{aligned} x_{t+1} &= Ax_t + Bv_t \\ y_t &= Cx_t + e_t \end{aligned}$$

where the covariance matrices of v_t and e_t are denoted Q and R , respectively. The unknown quantities in the *state vector* x_t are estimated by a recursion

$$\hat{x}_{t+1} = A\hat{x}_t + K_t(A, B, C, Q, R)(y_t - C\hat{x}_t),$$

where the filter gain $K_t(A, B, C, Q, R)$ is given by the Kalman filter equations. Thus, the design problem is to setup the state space model.

Using the state vector

$$x_t = \begin{pmatrix} \dot{\Psi}_t \\ \ddot{\Psi}_t \\ \delta_{o,gyro} \\ \delta_{34} \\ \delta_{sc,gyro} \\ \delta_r \end{pmatrix}$$

a continuous time state space model is

$$\dot{x}_t = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} x_t + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} v_t$$

$$y_t = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & v_{x,m} \frac{1}{L} \frac{2}{\omega_{rl} + 1} \frac{\omega_{rl}}{\omega_{rr}} \end{pmatrix} \begin{pmatrix} \hat{\psi}_t \\ 0 \end{pmatrix} + e_t$$

$$R_m^{-1} \frac{\omega_{rl} + \omega_{rr}}{2}$$

It is here assumed that there is an unknown input v_t that affects the yaw acceleration, which is a common model for motion models, basically motivated by Newton's law $F = ma$.

A discrete time state space model can be derived

$$x_{t+1} = \begin{pmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x_t + \begin{pmatrix} T_s^2/2 \\ T_s \\ 0 \\ 0 \end{pmatrix} v_t$$

$$y_t = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & v_x \frac{1}{L} \frac{2}{\omega_{rl} + 1} \frac{\omega_{rl}}{\omega_{rr}} \end{pmatrix} x_t + e_t$$

which is used by the Kalman filter.

Experimental results Figure 4 illustrates a test drive with four laps in a large slightly elliptic roundabout (radius approximately 90 meters). The marked paths are obtained by dead-reckoning. Due to the real offset in the gyro, the straightforward attempt of dead-reckoning leaves the roundabout with a phase error of 180 degrees. After compensation, the plot shows perfect resemblance with the map and reveals exactly which lane was followed.

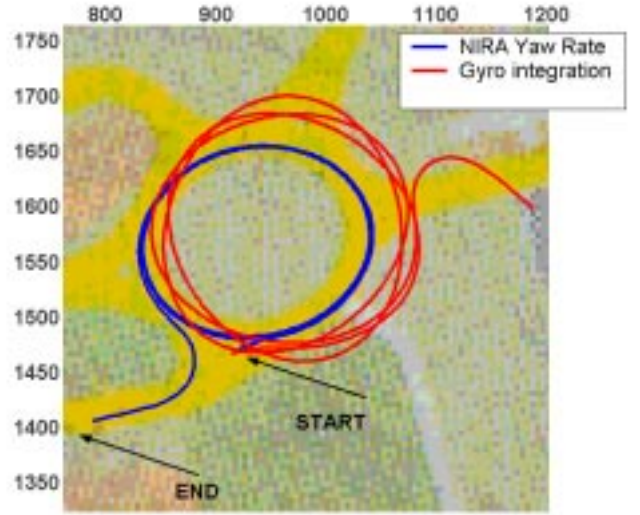


Figure 4: Path obtained by dead-reckoning the gyro signal and the estimated yaw rate, respectively.

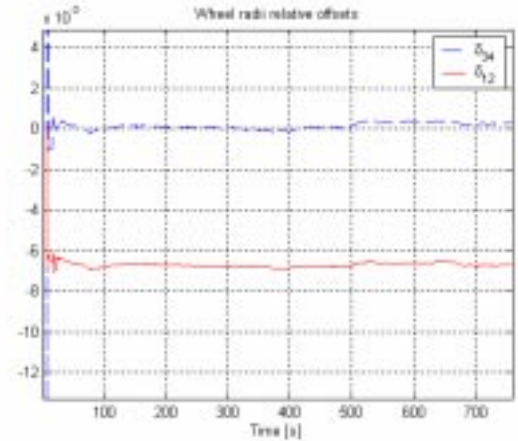


Figure 5: Wheel offset estimation. The true offsets are not known here.

Related publications

Related material is the article Hac and Simpson (2000), where sensor fusion is used for wheel speeds and lateral accelerometer, the patent Shivashankar *et al.* (1996), where two accelerometers and steering angle are used, and the patent Williams (1991), which adapts the offsets when the steering wheel angle and lateral acceleration are both close to zero. None of these include the gyro signal.

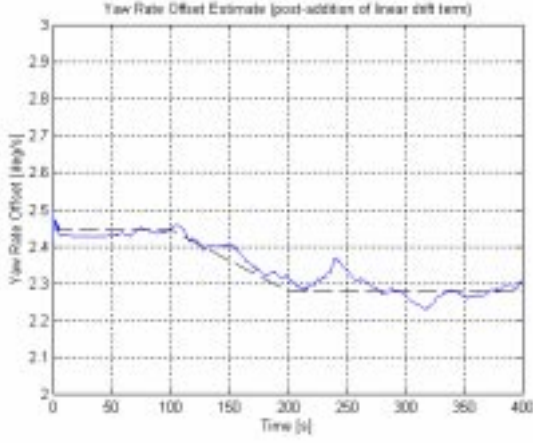


Figure 6: Gyro offset estimation, where a linear drift has been added to the sensor signal afterwards. The Kalman filter tracks the time-varying offset with an error not exceeding 0.1 deg/s.

3 ACCURATE SPEED COMPUTATION

The standard approach to compute velocity is to use the wheel speed signals, possibly averaging over right and left wheels and preferably using non-driven wheels to avoid wheel slip. This approach obviously has shortcomings during braking when the wheels are locked and during wheel spin on 4WD vehicles. For 4WD vehicles an additional problem is that there will even during normal driving be a small positive offset in velocity caused by the wheel slip.

This approach uses an accelerometer as a complement to the wheel speed signals, as illustrated in Figure 7. In this way, the velocity can be computed after locking the wheels when braking. For 4WD vehicles and otherwise when non-driven wheel speed signals are not available, the system compensates for wheel slip and gives accurate velocity and accelerometer information.



Figure 7: Basic structure of high precision speed computation.

Measurements

The sensor signals to be fused and their characteristics are here:

- ABS sensors provide wheel rotational speed ω that can be transformed to a scaled velocity at any position in the car:

$$\begin{aligned} y_t^1 &= \omega_t r_m + e_t^1 = \omega_t (r + \delta_r) + e_t^1 \\ &= v_{x,t} + \omega_t \delta_r + e_t^1. \end{aligned}$$

This holds for a non-driven wheel. Fusion of the driven wheels is also possible, but then the wheel slip must be modeled. This will show up as a scale factor error.

- An accelerometer in longitudinal direction a_x

$$y_t^2 = \dot{v}_{x,t} + \delta_{o,acc} + e_t^2.$$

Summing up to time t gives

$$\bar{y}_t^2 = \sum_{k=0}^t y_k^2 = v_{x,t} - v_{x,0} + \delta_{o,acc} t + \bar{e}_t^2.$$

The offset factor here (1 or t) is linearly independent of the one from the ABS sensor (a small variation in angular speed ω is needed), so the offsets $\delta_{o,acc}$, δ_r are observable.

Kalman filter

Basically, the same estimation approaches as for yaw rate are possible: least squares (using y_t^1 and \bar{y}_t^2) or Kalman filtering (using y_t^1 and \bar{y}_t^2). The on-line implementation uses a Kalman filter. With the state vector

$$x_t = \begin{pmatrix} v_{x,t} \\ \dot{v}_{x,t} \\ \delta_r \\ \delta_{o,acc} \end{pmatrix}$$

the state space model becomes

$$\begin{aligned} x_{t+1} &= \begin{pmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x_t + \begin{pmatrix} T_s^2/2 \\ T_s \\ 0 \\ 0 \end{pmatrix} v_t \\ y_t &= \begin{pmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} x_t + e_t. \end{aligned}$$

Everything else is similar as to the previous section.

Experimental results

The numerical illustration is based on a test drive modified in the following way. First, all offsets are manually tuned such that the path obtained by dead-reckoning of the sensor signals fits a road map perfectly. Then the offsets in Table 1 are added to the measurements, and the algorithm tries to estimate them. Figure 8 shows how the individual tire radii are estimated. It takes less than a minute to find a value with less than half a millimeter error. As a consequence, the velocity error decreases significantly, see Figure 9.

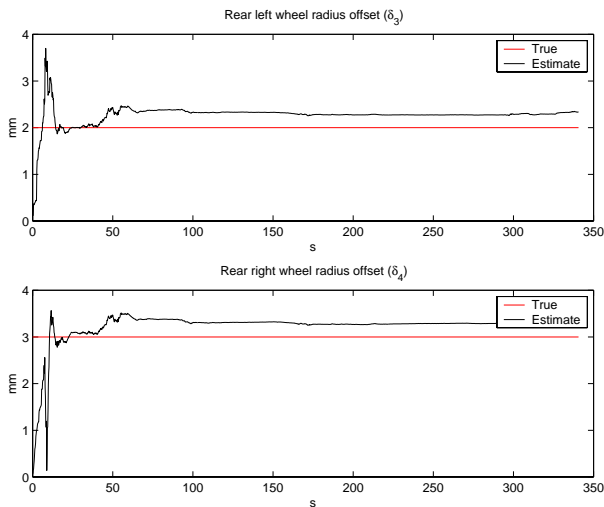


Figure 8: Estimation of error in nominal tire radius as a function of time. The added offsets on 3 and 4 mm, respectively, are after 60 seconds estimated with an error less than 0.5 mm.

4 CONCLUSIONS

The technique of sensor fusion, which is standard in avionic navigation systems, has been brought over to the automotive problems of yaw rate and absolute velocity estimation. By simultaneously estimating sensor offsets with the state variables of interest and by using both temporal and spatial (multi-sensor) correlation, the motion states of the car are obtained with an accuracy far better than by using each sensor individually.

The results using standard sensors in a Volvo S80 are a yaw rate value with drift less than 0.2 degrees

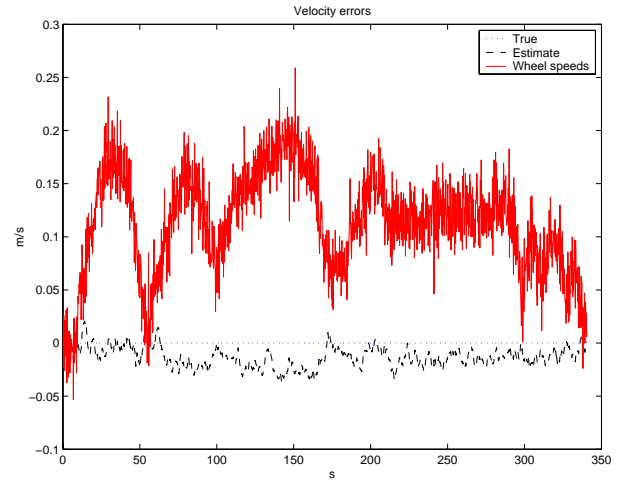


Figure 9: Accurate speed estimation follows from the knowledge of tire radius offsets. Here the velocity error is shown (true is zero) using nominal wheel radius (upper curve) and estimated wheel radius (the curve close to zero).

per second, and absolute speed value with an error in the order of centimeters per second, respectively. No calibration is needed, and the system adapts to temperature and aging drifts in sensors and wheel radii.

The wheel radius estimates (here δ_{34} and δ_{12}) are useful for tire pressure estimation, which is described in an accompanying paper.

The question of order of excitation and degree of observability is not addressed here. Basically, the offsets are much easier to estimate than scale factor errors. Also, relative difference in tire radius is easier to estimate than absolute value (the average error). The current implementation switches the adaptivity depending on the current excitation, as is coupled to accelerations and turning of the vehicle.

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