Localization in Sensor Networks Based on Log Range Observations

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***Abstract*— This contribution presents a unified framework for localization and tracking in sensor networks based on fusing a variety of signal energy measurements as provided by for instance acoustic, seismic, magnetic, radio, microwave and infrared sensors. The received energy from such sensors generally decays exponentially, and a log range model is introduced for the sensor observations in logarithmic scale, which is linear in transmitted power and the path loss exponent. Field trial sensor data confirms the validity of the log range model. The novelty in this contribution lies in a systematic least squares approach to eliminate these nuisance parameters and also the sensor noise variances. Details on how to solve the resulting low-dimensional non-linear least squares criterion are given, and how to extend the algorithms to target tracking. Explicit formulas for the Cramér-Rao lower bound are given for both localization and tracking.**

Keywords: sensor networks, Kalman filtering, estimation.

I. Introduction

A sensor network scenario is considered, where each sensor unit has a multitude of sensors measuring received signal strength (RSS) from one target. The considered problem focuses on target localization, but the reverse problem of navigation of one sensor from several beacons ('targets') with known position is also covered by reversing the role of transmitters and receivers. An underlying assumption is that communication constraints between the sensor units make any algorithm based on the signal waveform (like coherent detection) infeasible. Communication only allows for sending RSS measurements to other sensor units.

This contribution derives explicit algorithms and performance bounds for energy-based measurements, including sensors of radio, acoustic, seismic, infra-red (IR) or microwave energy. Localization from RSS is of course a fairly well studied problem, see the surveys [1]–[3] and the papers [4], [5], though the major part of literature addresses the related problem of localizaton from time of arrival (TOA) and time-difference of arrival (TDOA) measurements. While TOA measures range and TDOA range differences computed from propagation time, energy based localization utilizes the exponential power decay of the involved signals. Measuring the received power in decibels, the measurements are proportional to the logarithm of distance, and this is the main difference to time-based localization approaches. Dedicated approaches to this problem assume that the path loss exponent is known [4], [5], or include the RSS measurements as a general non-linear relation [3]. Several *ad-hoc* methods to eliminate nuisance parameters have been proposed in this context, including taking pairwise differences or ratios of observations.

The contribution here extends the theory of RSS based localization using an approach where the path loss exponent and transmitted power are explicitly removed from a set of RSS measurements using the separable least squares principle, after which the resulting problem is non-linear in target state parameters only, and a standard low-dimension non-linear least squares (NLS) problem, where efficient numerical algorithms exist. Algorithms of different complexity and performance are outlined for this framework. Tracking algorithms are also described, which are based on stating the localization NLS problem formulation as the measurement relation in an extended Kalman filter.

The fundamental performance bound implied by the Cramér-Rao lower bound enables efficient analysis of sensor network architecture, management and resource allocation. This bound has been analyzed thoroughly in the sensor network literature, primarily for TOA, TDOA and angle-of-arrival (AOA), [1]–[3], but also for RSS [6], [7] and with specific attention to the impact from non-line-of-sight [8], [9]. Numerical explicit algorithms and Cramér-Rao lower bounds (CRLB) for both stationary and moving target are derived for the NLS problem formulation.

The near term plans for the project is to conduct a measurement campaign in a sensor network field trial. Some measurements, that have been gathered in order to illustrate the validity of the sensor model, will be presented.

II. Sensor Model

This section formulates the exponential *received signal strength (RSS)* decay as a model of received signal strength as a function of log range which is linear in the environmental nuisance parameters. This model will be referred to as the *log range linear model (LRLM)*.

A. The Log Range Linear Model

Consider a sensor network, where each sensor unit is located at position p_k measuring a variety of RSS using different sensors. The received power from each sensor type i at unit k is assumed to *in average* follow an exponential decay rate

$$\bar{P}_{k,i} = \bar{P}_{0,i} \|x - p_k\|^{n_{p,i}}. \tag{1}$$

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Both the transmitted energy $P_{0,i}$ and path loss constant $n_{p,i}$ are assumed unknown. Further, these are different for each sensor type, but spatially constant in the local environment where the sensor network operates.

We assume that the power is measured in decibels with additive noise $e_{k,i}$ as

$$P_{k,i} = P_{0,i} + n_{p,i} \underbrace{\log\left(\|x - p_k\|\right)}_{\triangleq c_k(x)}, \tag{2}$$

$$y_{k,i} = P_{k,i} + e_{k,i}. \tag{3}$$

The fundamental log range (LR) term $c_k(x)$ is here introduced. The sensor error has zero mean (otherwise, the mean can be incorporated in the nuisance parameter $P_{0,i}$) and variance $\mathrm{Var}(e_{k,i}) = \sigma^2_{P,i}$. This model implies that the energy-based measurements get worse quality with increasing distance [1]. Both mean and variance are assumed constant for each sensor unit, but different for different sensor types. In this section, $\sigma^2_{P,i}$ is assumed known or accurately estimated from measurements when no target is present. Later on, more details on how this can be done is given.

The model (2) is linear in the nuisance parameters $P_{0,i} = \log \bar{P}_{0,i}$ and $n_{p,i}$,

$$y_{k,i} = P_{0,i} + n_{p,i} c_k(x) + e_{k,i}, \tag{4}$$

a fact that will be utilized in the next section.

Non-line of sight (NLOS) is a major issue in radio based localization. Basically, NLOS invalidates the exponential model (1). NLOS is less of an issue for seismic, magnetic and acoustic waves, partly because of their different nature and partly because of the rather limited range of operation.

B. Fusion of LRLM

Collecting all relations of the kind (4) for different sensor types $i = 1, 2, \ldots, M$ and sensor units $k = 1, 2, \ldots, N$ yields a non-linear equation system

$$\mathbf{y} = \mathbf{h}(x) + \mathbf{e}, \tag{5a}$$

$$\mathbf{y} = \begin{pmatrix} y_{1,1} \\ y_{2,1} \\ y_{3,1} \\ \vdots \\ y_{N,M} \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} e_{1,1} \\ e_{2,1} \\ e_{3,1} \\ \vdots \\ e_{N,M} \end{pmatrix}, \tag{5b}$$

$$\mathbf{h} = \begin{pmatrix} P_{0,1} + n_{p,1} c_1(x) \\ P_{0,2} + n_{p,1} c_2(x) \\ P_{0,2} + n_{p,1} c_3(x) \\ \vdots \\ P_{0,M} + n_{p,M} c_N(x) \end{pmatrix}, \tag{5c}$$

$$\mathrm{Cov}(\mathbf{e}) = \mathbf{R} = \mathrm{diag}\left(\sigma^2_{P,1}(x) I_N, \ldots, \sigma^2_{P,M}(x) I_N\right). \tag{5d}$$

Solving this in the least squares sense is the subject of the next section.

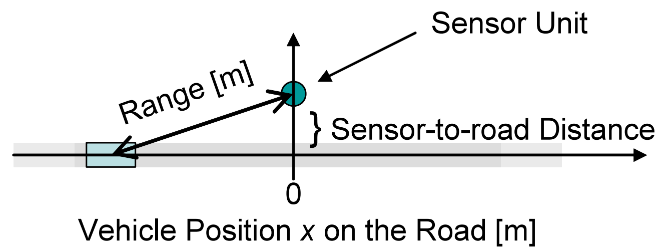

Fig. 1. Field trial setup for sensor model validation.

C. Sensor Model Validation

Sensor measurements from field trials are used to illustrate the validity of the log range linear model. The sensor unit, equipped with an acoustic and a seismic sensor, is located a few meters from a road, see Figure 1. The positions of the vehicle and sensor are known in this model validation case, that is, $c_k(x)$ is known, and (4) becomes a standard least squares problem in $P_{0,1}$ and $n_{p,1}$.

Figure 2 visualizes the received signal energy as a function of the vehicle position x along the road, where the origin is defined as the closest point to the sensor.

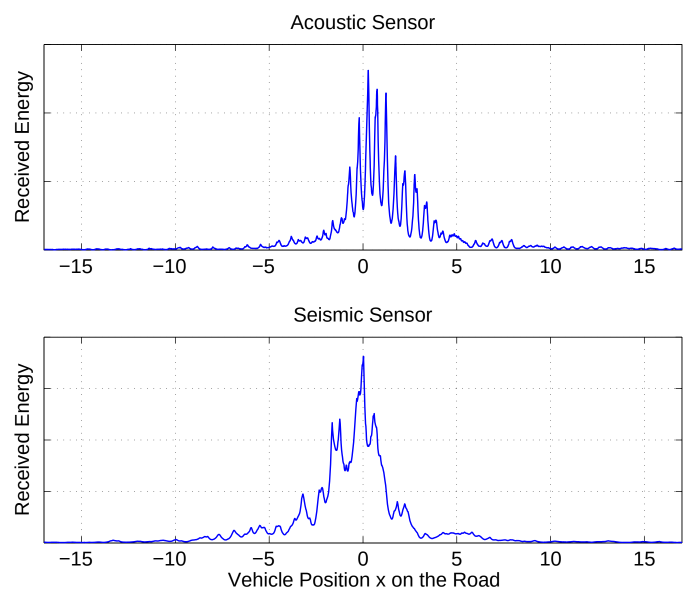

Fig. 2. Received sensor energy in linear scale at the acoustic and seismic sensor, respectively.

Figure 3 illustrates that the log range linear model is very reasonable, and that the data fits the model in (4). For example, the exponent estimates at the specific field trial environment are $n_{1,1} = -2.3$ for the acoustic sensor, and $n_{1,2} = -2.6$ for the seismic sensor.

D. Sensor Network Example

For illustrative purposes, we consider the simple example of ten sensor units randomly positioned and each one equipped with a sensor of the same type. The received power over all possible sensor locations is depicted in Figure 4, together

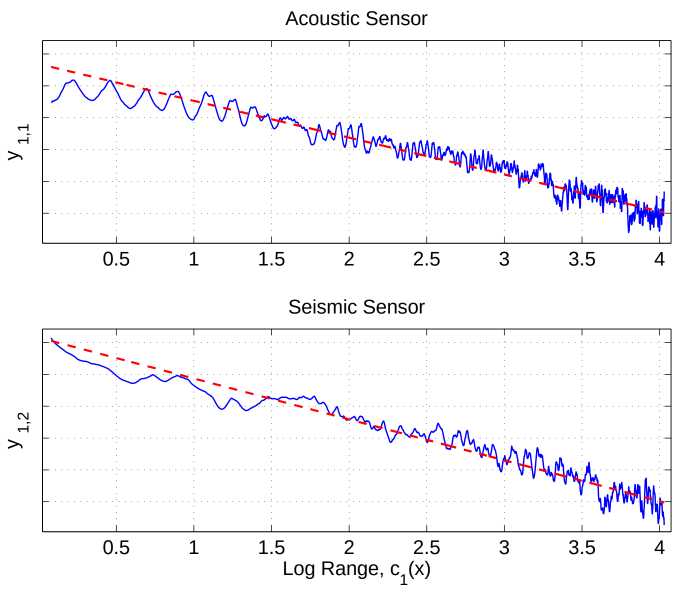

Fig. 3. Received sensor energy in log scale, together with a fitted linear relation as modeled in (4).

with the actual sensor locations and the unknown position '+' to be estimated. In this case, the transmission power $P_0 =$

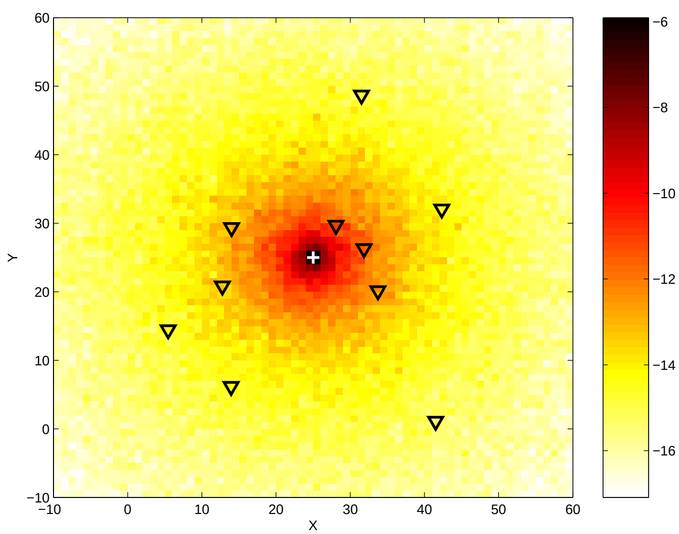

Fig. 4. Received power per position for the simple example with four sensor units equipped with the sensor type.

$\log(0.001)$, and the environmental parameter $n_p = -2.5$. The measurements are corrupted by noise with standard deviation $\sigma_P = 0.3$.

III. NLS Estimation Using the LRLM

Let $\theta_i = (n_{p,i}, P_{0,i})^T$ denote the unknown parameters in the LRLM for sensor type i, and $\theta = (\theta_1, \theta_2, \ldots, \theta_M)^T$ the vector of unknowns for all sensor types. The non-linear least squares approach aims at minimizing the sum of squared errors between observations and the model with respect to target location x and the nuisance parameters in θ. This can be stated as

$$\widehat{(x,\theta)} = \arg\min_{x,\theta} V(x,\theta), \tag{6a}$$

$$V(x,\theta) = \sum_{i=1}^{M}\sum_{k=1}^{N} \frac{\big(y_{k,i} - h(c_k(x),\theta_i)\big)^2}{\sigma_{P,i}^2}, \tag{6b}$$

$$h(c_k(x),\theta_i) = \theta_{i,1} + \theta_{i,2} c_k(x), \tag{6c}$$

$$c_k(x) = \log\big(\|x - p_k\|\big). \tag{6d}$$

The goal in this section is to eliminate the nuisance parameters θ_i, including the path loss constant $n_{p,i}$ and transmission power $P_{0,i}$, and the unknown noise variances $\sigma_{P,i}$ for $i = 1, 2, \ldots, M$.

A. Elimination of Nuisance Parameters by Separable Least Squares

Using the separable least squares (LS) principle, the environmental parameter, $n_{p,i}$, and transmission power, $P_{0,i}$, can be eliminated explicitly from (6) for each sensor type i. The algebraic minimimizing argument of (6b) is given by

$$\hat{\theta}_i(x) = \underbrace{\left[\begin{pmatrix} N & \sum_{k=1}^{N} c_k(x) \\ \sum_{k=1}^{N} c_k(x) & \left(\sum_{k=1}^{N} c_k(x)\right)^2 \end{pmatrix}\right]^{-1}}_{R(x)} \times \underbrace{\begin{pmatrix} \sum_{k=1}^{N} y_{k,i} \\ \sum_{k=1}^{N} c_k(x) y_{k,i} \end{pmatrix}}_{f_i(x)} \tag{7}$$

Note that the parameter estimate depends on the target location x. The matrix $R(x)$ and vector $f_i(x)$ are introduced in (7) to get more compact notation in the following. Note also that the matrix $R(x)$ is just a function of sensor geometry and target position. The matrix inversion can be eliminated to get

$$R(x) = \frac{1}{N\sum_{k=1}^{N} c_k^2(x) - \left(\sum_{k=1}^{N} c_k(x)\right)^2} \times \begin{pmatrix} \sum_{k=1}^{N} c_k^2(x) & -\sum_{k=1}^{N} c_k(x) \\ -\sum_{k=1}^{N} c_k(x) & N \end{pmatrix} \tag{8}$$

With this matrix defined, the covariance matrix is given by

$$\operatorname{Cov}(\hat{\theta}_i(x)) = \sigma_{P,i}^2 R(x). \tag{9}$$

The variance of the LRLM after plugging in the parameter estimate is thus given by

$$\operatorname{Var}\big(h(c_k(x),\hat{\theta}(x))\big) = \sigma_{P,i}^2 (1,\, c_k(x)) R(x) (1,\, c_k(x))^T. \tag{10}$$

B. Sensor Noise Variance Estimation

Further, the minimum of the sum of least squares for sensor type i can be taken as an estimate of the measurement variance as

$$\widehat{\sigma^2}_{P,i}(x) = \frac{1}{N-2}\sum_{k=1}^{N}\left(y_{k,i} - h(c_k(x),\hat{\theta}_i(x))\right)^2 \tag{11}$$

$$= \frac{1}{N-2}\left(\sum_{k=1}^{N} y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)\right), \tag{12}$$

where the normalization with $N-2$ accounts for the number of freedom lost by the minimization, and is needed to get an unbiased variance estimate. The last equality is a consequence of the LS theory, and will be used in the NLS formulation below.

C. LRLM NLS Formulation

The NLS formulation in (6) is now algebraically equivalent to the following reduced NLS problem in target location x

$$\hat{x} = \arg\min_{x,\theta} V(x,\theta,\sigma_P) = \arg\min_{x} V(x,\hat{\theta}(x),\sigma_P), \tag{13a}$$

$$V(x,\hat{\theta}(x),\sigma_P) = \sum_{i=1}^{M}\sum_{k=1}^{N}\frac{\left(y_{k,i} - h(c_k(x),\hat{\theta}_i(x))\right)^2}{\sigma_{P,i}^2},$$

$$= \sum_{i=1}^{M}\frac{\sum_{k=1}^{N} y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)}{\sigma_{P,i}^2}, \tag{13b}$$

The new weighting in the sum of least squares accounts for both measurement noise and the estimation uncertainty in the nuisance parameters. Typically, far away sensor nodes k or uncertain sensor types i get larger uncertainty in the parameters and thus automatically a smaller weight in the criterion.

Note that the noise variance has to be known in the NLS approach above. The simple idea of plugging in the estimate does not work, since

$$V(x,\hat{\theta}(x),\widehat{\sigma^2}_P) = M(N-2). \tag{14}$$

The maximum likelihood (ML) approach can be used to circumvent this problem, as described in Section IV-B.

D. Example

Figure 5 visualizes the cost function (13) for the example in Section II-D. The true position is located at the clearly distinguished minimum.

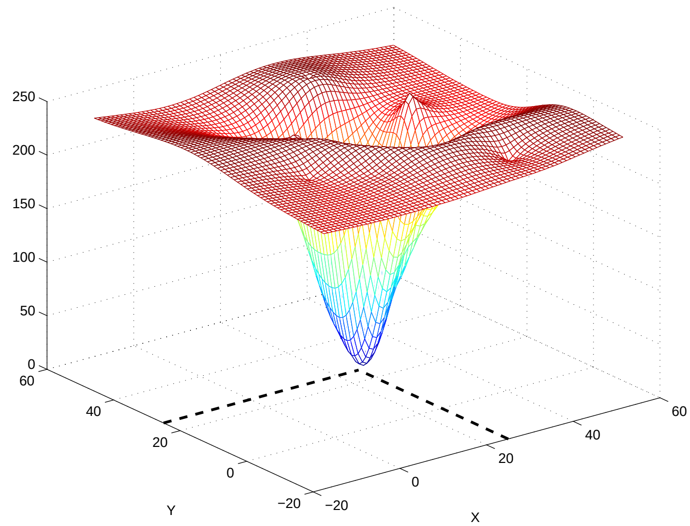

Fig. 5. NLS cost function $V(x,\hat{\theta}(x),\sigma_P)$ for the example. The true position is located at the marked minimum.

IV. Target Localization

In summary, in the previous section we have derived the LR model

$$\mathbf{y} = \mathbf{h}(x,\hat{\theta}(x)) + \mathbf{e}, \tag{15a}$$

$$\mathbf{h}_{k,i} = h(c_k(x),\hat{\theta}_i(x)), \tag{15b}$$

$$\mathrm{Cov}(\mathbf{e}) = \mathbf{R} = \mathrm{diag}\left(\sigma_{P,1}^2(x)I_N,\ldots,\sigma_{P,M}^2(x)I_N\right). \tag{15c}$$

Here, $\hat{\theta}_i(x)$ is given in (7), $c_k(x)$ in (6d), $h(c_k(x),\hat{\theta}_i)$ in (6c), and $R(x)$ in (8). The purpose here is to outline possible implementation strategies.

A. Estimation Criteria

The derivation in Section III was motivated by NLS. However, the same elimination of nuisance parameters can be applied to more general maximum likelihood (ML) approaches, with a Gaussian assumption or with other assumptions on sensor error distributions, as summarized in Table I. The Gaussian ML (GML) approach is useful when the variances of the individual measurements include important localization information in themselves.

NLS	$V^{NLS}(x) = (\mathbf{y}-\mathbf{h}(x))^T\mathbf{R}^{-1}(x)(\mathbf{y}-\mathbf{h}(x))$
GML	$V^{GML}(x) = (\mathbf{y}-\mathbf{h}(x))^T\mathbf{R}^{-1}(x)(\mathbf{y}-\mathbf{h}(x)) + \log\det\mathbf{R}(x)$
ML	$V^{ML}(x) = \log\ p_{\mathbf{e}}(\mathbf{y}-\mathbf{h}(x))$

TABLE I

Optimization criteria $V(x)$ for estimating position x from uncertain measurements $\mathbf{y} = \mathbf{h}(x) + \mathbf{e}$.

B. Eliminating the noise variance

The GML approach is also needed in the case where the noise variance is unknown. Minimizing the GML cost with respect to $\sigma_{P,i}$ gives a result similar to (13b),

$$\min_{\sigma_P} V^{GML}(x,\sigma_P) = \sum_{i=1}^{M} N\log\left(\sum_{k=1}^{N} y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)\right). \tag{16}$$

The logarithm intuitively decreases the difference in weighting between the different sensor types compared to the case of known noise variances in (13b).

C. Estimation Algorithms

As in any estimation algorithm, the classical choice is between a gradient and Gauss-Newton algorithm, see [10]. The basic forms are given in Table II. These local search algorithms generally require good initialization, otherwise the risk is to reach a local minimum in the loss function $V(x)$. Today, simulation based optimization techniques may provide an alternative.

Grid-based	$\hat{x} = \arg\min_{x\in\{x^{(1)},x^{(2)},\dots,x^{(L)}\}} \sum_{i=1}^{M} \widehat{\sigma^2}_{P,i}(x)$
Steepest descent	$\hat{x}_k = \hat{x}_{k-1} + \mu_k$ $\cdot \mathbf{H}^T(\hat{x}_{k-1})R^{-1}(y - \mathbf{H}(\hat{x}_{k-1})\hat{x}_{k-1})$
Newton-Raphson	$\hat{x}_k = \hat{x}_{k-1} + \mu_k \left(\mathbf{H}^T(\hat{x}_{k-1})R^{-1}\mathbf{H}(\hat{x}_{k-1})\right)^{-1}$ $\cdot \mathbf{H}^T(\hat{x}_{k-1})R^{-1}(y - \mathbf{H}(\hat{x}_{k-1})\hat{x}_{k-1})$

TABLE II

ESTIMATION ALGORITHMS APPLICABLE TO OPTIMIZATION CRITERIA IN TABLE I. HERE, $\mathbf{H}(x) = \nabla_x \mathbf{h}(x)$ FOR NLS AND GML AND $\mathbf{H}(x) = \nabla_x \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))$ FOR ML.

D. Gradient Derivation

In these numerical algorithms, the gradient $\mathbf{H}(x) = \nabla_x \mathbf{h}(x)$ of the model with respect to the position is instrumental, and it is the purpose here to derive the necessary equations.

First, it is easier to apply the chain rule to the expression

$$c_k(x) = \log\left(\|x - p_k\|\right) = \frac{1}{2}\log\left(\|x - p_k\|^2\right), \tag{17}$$

though the result is the same in the end. The gradient is then immediate as

$$\frac{\mathrm{d}c_k(x)}{\mathrm{d}x} = \frac{x - p_k}{\|x - p_k\|^2}. \tag{18}$$

The gradient of the NLS loss function $\bar{V}(x, \hat{\theta}(x))$ becomes a function of the gradients of $\hat{\theta}_i(x)$ and $R(x)$. These are all tedious but straightforward applications of the chain rule, not reproduced here. However, the point is that everything that is needed in the optimization algorithms surveyed in the next section are symbolic functions in target location x and sensor locations p_k only.

E. Model Validation and Target Detection

Assume that the noise in the LRLM is Gaussian distributed. The NLS loss function at the true target location x^o is then $\chi^2(M(N-2))$ distributed. This can be used for model validation, and also for testing the hypotheses that there is a target.

F. Fundamental Performance Bounds

The *Fisher Information Matrix (FIM)* provides a fundamental estimation limit for unbiased estimators referred to as the *Cramér-Rao Lower Bound (CRLB)* [11]. This bound has been analyzed thoroughly in the literature, primarily for AOA, TOA and TDOA, [1]–[3], but also for RSS [6], [7] and with specific attention to the impact from non-line-of-sight [8], [9].

For the log range model 4, the 2×2 Fisher Information Matrix $J(x)$ is defined as

$$J(x) = \mathrm{E}\left(\nabla_x^T \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x)) \nabla_x \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))\right) \tag{19a}$$

$$\nabla_x \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x)) = \begin{pmatrix} \frac{\partial \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))}{\partial x_1} & \frac{\partial \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))}{\partial x_2} \end{pmatrix} \tag{19b}$$

where p is the two-dimensional position vector and $p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))$ the likelihood given the error distribution.

In case of Gaussian measurement errors $p_{\mathbf{e}}(e) = \mathrm{N}(0, \mathbf{R}(x))$, the FIM equals

$$J(x) = \mathbf{H}^T(x)\mathbf{R}(x)^{-1}\mathbf{H}(x), \tag{20a}$$

$$\mathbf{H}(x) = \nabla_x \mathbf{h}(x). \tag{20b}$$

This form is directly applicable to the log range linear model (4).

In comparison, if the nuisance parameters were known, the FIM can be obtained in the Gaussian measurement error case as the following sum over sensors

$$\begin{aligned} J(x) &= \sum_{i=1}^{M}\sum_{k=1}^{N} \nabla_x h(c_k(x), \theta_i) \\ &= \sum_{i=1}^{M}\sum_{k=1}^{N} \frac{\theta_{i,1}^2}{\sigma_{P,i}^2 \|x - p_k\|^2}(x - p_k)(x - p_k)^T \end{aligned} \tag{21}$$

Plausible approximative scalar information measures are the trace of the FIM and the smallest eigenvalue of FIM

$$J_{\mathrm{tr}}(x) \triangleq \mathrm{tr}\, J(x), \quad J_{\min}(x) \triangleq \min \mathrm{eig}\, J(x). \tag{22}$$

The former information measure is additive as FIM itself, while the latter is an under-estimation of the information useful when reasoning about whether the available information is sufficient or not. Note that in the Gaussian case with a diagonal measurement error covariance matrix, the trace of FIM is the squared gradient magnitude.

The Cramér-Rao Lower Bound is given by

$$\mathrm{Cov}(\hat{x}) = \mathrm{E}(x^o - \hat{x})(x^o - \hat{x})^T \geq J^{-1}(x^o), \tag{23}$$

where x^o denotes the true position. The CRLB holds for any unbiased estimate of $\hat{x}$, in particular the ones based on minimizing the criteria in the previous sub-section. The lower bound may not be an attainable bound. It is known that asymptotically in the number of sensor nodes, the ML estimate is $\hat{x} \sim \mathrm{N}(x^o, J^{-1}(x^o))$ [12] and thus reaches this bound, but this may not hold for finite amount of data.

The right hand side of (23) gives an idea of how suitable a given sensor configuration is for positioning. It can also be used for *sensor network design*. However, it should always be kept in mind though that this lower bound is quite conservative and relies on many assumptions.

In practice, the root mean square error (RMSE) is perhaps of more importance. This can be interpreted as the achieved position error in meters. The CRLB implies the following bound:

$$\begin{aligned} \mathrm{RMSE} &= \sqrt{(x_1^o - \hat{x}_1)^2 + (x_2^o - \hat{x}_2)^2} \\ &= \sqrt{\mathrm{tr}\,\mathrm{Cov}(\hat{x})} \geq \sqrt{\mathrm{tr}\, J^{-1}(x^o)} \end{aligned} \tag{24}$$

If RMSE requirements are specified, it is possible to include more and more measurements in the design until (24) indicates that the amount of information is enough.

G. Example

The CRLB will vary spatially, revealing the relation between accuracy and sensor locations. For the example in Section II-D, and with known nuisance parameters, the CRLB is illustrated in Figure 6. This is important when determining the sensor unit locations, if the practical application allows.

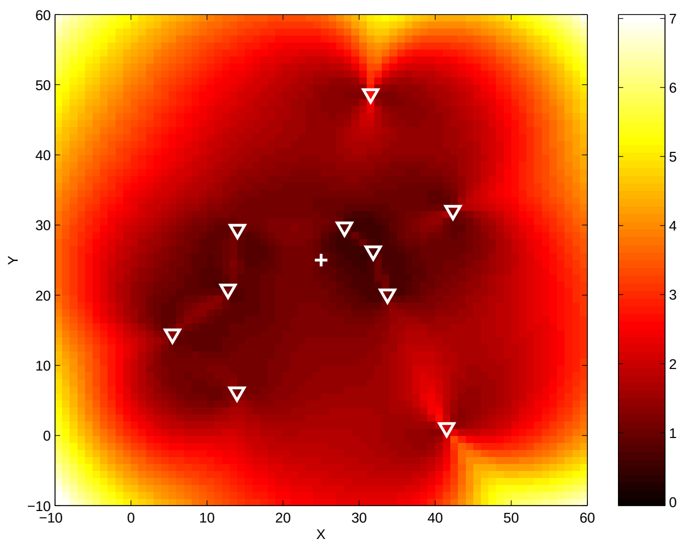

Fig. 6. Spatial variations of CRLB indicating how positioning accuracy will vary with the position.

V. Target Tracking

We here extend the LR model (15) with a time dimension

$$\mathbf{y}_t = \mathbf{h}(x_t, \hat{\theta}(x_t)) + \mathbf{e}_t, \tag{25}$$

in order to follow motion in the target location $x(t)$. One can here either assume that the parameters also change over time, so they are estimated as outlined above for each batch of data $\mathbf{y}(t)$, or they can be constant over time, in which case the least squares estimate $\hat{\theta}$ is averaged over time in a straightforward extension of the results above using the recursive least squares principle. Here, the more general Kalman filter is used for tracking.

A. Motion Models

The key idea with filtering is to include the target velocity in a dynamic model, so a prediction of the next position can be computed. The simplest possible motion model for target localization is the random walk model, see Table III.

For navigation where external measurement of the motion of the platform might be available, other motion models are possible. Here, the actual platform velocity v_t might be completely unknown, partly known or measurable (the natural example is a car-mounted system) or estimated in the model. Any model suggested in the target tracking literature is plausible also for this application, see for instance the survey [13], but we focus on the simplest cases. Higher order models can decrease estimation error slightly, but will not drastically change our conclusions.

The sample interval T_s determines the measurement update rate in the filter. Here, a bit care is required, since the LR measurements in (25) are correlated over time, while a filter requires uncorrelation measurements. The most immediate solution is to match the sample interval T_s to the coherence time of the measurements, which includes down-sampling (using an anti-alias filter) the measurements.

The filtering CRLB depends on $J(x_t^o)$, but also on the motion model selection and the user mobility assumptions. Due to averaging effects, the bound will decrease considerably compared to the static case. Another advantage with temporal data and filtering compared to estimation in the static case is that it is possible to handle an under-determined equation system in the measurement equations.

As a possible improvement to the models in Table III, the range to one base station may be introduced as an auxiliary state variable as discussed in Section IV-C. Then, the TDOA measurements can be expressed linearly in the state vector, and the problem with convergence to local minima is avoided.

B. Position Filtering

The simplest approach would be to use adaptive filters similar to the numerical search schemes in Table II. An adaptive algorithm based on the steepest descent principle is the Least Mean Square (LMS) algorithm. Similarly, the Recursive Least Squares (RLS) algorithm is an adaptive version of a Gauss-Newton search. The tuning parameter μ or λ controls the amount of averaging and reflects our belief in user mobility.

The last two models in Table III require state estimation, and the natural first try is the extended Kalman filter (EKF), where measurement errors are assumed Gaussian, and the non-linear measurement equation $y_t = h(x_t) + e_t$ is linearized around the current position estimate. In case of a highly non-linear measurement equation $h(x_t)$, or non-Gaussian error distribution, the position estimate using the EKF is far from the CRLB. The computer intensive particle filter (PF) [14], [15] has been proposed for high-performance positioning [16]. The advantage is that all currently available information is easily incorporated, including power attenuation maps and street maps. Some examples are provided in [17].

C. Fundamental Performance Bounds

Recently, location performance in the dynamic case in terms of the Cramér-Rao Lower Bound (CRLB) has been studied in [18]–[20] for the TDOA case. Remember that the CRLB is asymptotic in the number of samples, and thus it is a conservative bound also in filtering. The CRLB for non-linear filtering was derived in [21]. In short, they presented a recursion for non-linear models similar to the information filter version of the Riccati equation that computes a lower estimation bound x_t. We here present a general result for the random walk model and velocity sensor model in Table III. With process noise covariance matrix $\mathrm{Cov}(w_t) = Q_t$, both result in the same recursion

$$\mathrm{Cov}(x_t) \geq P_t, \tag{26a}$$

$$P_{t+1} = \left((P_t + T_s Q_t)^{-1} + J(x_t^o)\right)^{-1}. \tag{26b}$$

Random walk model:	$x_{t+1} = x_t + T_s w_t$
Velocity sensor model:	$x_{t+1} = x_t + T_s v_t + T_s w_t$
Random force model:	$\begin{pmatrix} x_{t+1} \\ v_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} I & T_s \cdot I \\ 0 & I \end{pmatrix}}_{F} \begin{pmatrix} x_t \\ v_t \end{pmatrix} + \underbrace{\begin{pmatrix} T_s^2/2 \cdot I \\ T_s \cdot I \end{pmatrix}}_{G} w_t$
Inertial sensor model:	$\begin{pmatrix} x_{t+1} \\ v_{t+1} \end{pmatrix} = \begin{pmatrix} I & T_s \cdot I \\ 0 & I \end{pmatrix} \begin{pmatrix} x_t \\ v_t \end{pmatrix} + \begin{pmatrix} T_s^2/2 \cdot I \\ T_s \cdot I \end{pmatrix} a_t + \begin{pmatrix} T_s^2/2 \cdot I \\ T_s \cdot I \end{pmatrix} w_t$

TABLE III

EXAMPLES OF SIMPLE MOTION MODELS, WHERE T_s IS THE SAMPLE INTERVAL, $k = t/T_s$ SAMPLE NUMBER, AND w_t THE PROCESS NOISE DESCRIBING THE MOBILITY VARIATIONS. VELOCITY AND ACCELERATION ARE DENOTED v_t AND a_t, RESPECTIVELY. IN THE LATTER MODEL, SUPPORT SENSORS AS GYROSCOPES, ACCELEROMETERS AND SPEEDOMETERS CAN BE INCLUDED IN THE SENSOR FUSION FILTER.

LMS	$\hat{x}_t = \hat{x}_{t-1} + \mu H^T(\hat{x}_{t-1})(y_t - H(\hat{x}_{t-1})\hat{x}_{t-1})$
RLS	$\hat{x}_t = \hat{x}_{t-1} + P_t H^T(\hat{x}_{t-1})(y_t - H(\hat{x}_{t-1})\hat{x}_{t-1})$ $P_t = \frac{1}{\lambda}\left(P_{t-1} - P_{t-1}H^T(\hat{x}_{t-1})\left(\lambda R + H(\hat{x}_{t-1})P_{t-1}H^T(\hat{x}_{t-1})\right)^{-1} H(\hat{x}_{t-1})P_{t-1}\right)$
EKF	$\hat{x}_t = \hat{x}_{t-1} + P_{t\|t-1}H^T(\hat{x}_{t-1})(y_t - H(\hat{x}_{t-1})\hat{x}_{t-1})$ $P_{t\|t} = P_{t\|t-1} - P_{t\|t-1}H^T(\hat{x}_{t-1})\left(R + H(\hat{x}_{t-1})P_{t\|t-1}H^T(\hat{x}_{t-1})\right)^{-1} H(\hat{x}_{t-1})P_{t\|t-1}$ $P_{t+1\|t} = FP_{t\|t}F^T + GQ_tG^T$
PF	Initialize N particles x_0^i, $i = 1, 2, \ldots, N$. For each time t: 1. Likelihood computation $\pi^i = p_E(y_t - h(x_t^i))$. 2. Resample N particles with replacement according to likelihood weight π^i. 3. Diffusion step: randomize w^i from p_W and let $x_{t+1}^i = Fx_t^i + Gw^i$.

TABLE IV

ADAPTIVE FILTERS APPLICABLE TO THE DYNAMIC MOTION MODELS IN TABLE III. THE LEAST MEAN SQUARE (LMS) AND RECURSIVE LEAST SQUARES (RLS) ALGORITHMS ARE APPLICABLE TO THE FIRST TWO MODELS IN TABLE III, WHILE THE EXTENDED KALMAN FILTER (EKF) AND PARTICLE FILTER (PF) APPLY TO MORE GENERAL MODELS.

In the cases with a velocity state in Table III, define $H = (I, 0)$. This gives

$$\mathrm{Cov}(x_t) \geq HP_tH^T, \tag{27a}$$

$$P_{t+1} = \left((FP_tF^T + GQ_tG^T)^{-1} + H^TJ(x_t^o)H\right)^{-1}. \tag{27b}$$

These expressions clearly show the compromise between mobility (Q) and information (J), and further down a few specific cases of special interest will be pointed out.

The position dependent information matrix $J(x^o)$ is under mild conditions on the measurement relation $h(x)$ a smooth function, and can be considered constant for movements in a small neighborhood of x^o. The recursion will then converge to a stationary point. We will in the sequel analyze the first two cases in Table III in more detail. For (26b), the stationary value is given by

$$\bar{P} = \left((\bar{P} + T_sQ)^{-1} + J(x^o)\right)^{-1}, \tag{28}$$

which has the solution (see Appendix A in [3])

$$\bar{P} = -\frac{1}{2}T_sQ + J^{-1/2}(x^o)\left(J^{1/2}(x^o)(T_sQ + \frac{T_s^2}{4}QJQ)J^{1/2}(x^o)\right)^{1/2} J^{-1/2}(x^o). \tag{29}$$

The following procedure thus yields the bound for any energy-based tracking algorithm:

1) Compute the Fisher Information Matrix using (20a) and (20b),
2) Select a motion model (random walk or velocity sensor model) from Table III,
3) Compute (29).

We can point out a couple of important special cases:

- For the case with large mobility uncertainty, $Q \to \infty$, and (28) gives $P_t = J^{-1}(x^o)$, which is the static case.
- For the case of symmetric movements in more than one dimension, we can assume that $Q = qI$. The measurement equation can also be assumed to be symmetric in the different dimensions (it can always be transformed to this form anyway), which means $J = J_{min}(x^o)I$. This assumption can also be seen as a conservative circular bound on the confidence ellipsoid. Then, the asymptotic CRLB according to (29) is

$$\bar{P} = \frac{qT_s}{2}\left(\sqrt{\frac{4}{J_{min}(x^o)qT_s} + 1} - 1\right)I \tag{30a}$$

$$\approx \begin{cases} \frac{1}{J_{min}(x^o)}I & \text{if } \frac{J_{min}(x^o)qT_s}{4} >> 1 \\ \sqrt{\frac{qT_s}{J_{min}(x^o)}}I & \text{if } \frac{J_{min}(x^o)qT_s}{4} << 1. \end{cases} \tag{30b}$$

  The second case is the normal case, as will be pointed out when plugging in realistic values. The first case of very rapid movements and/or very informative measurements, basically corresponds to the static case.
- For the special case of no movement at all $Q = 0$, the solution is found from (26b) directly (assuming $P_0^{-1} = 0$) as

$$P_t = \frac{1}{t}J^{-1}(x^o) = \frac{T_s}{t}J^{-1}(x^o). \tag{31}$$

VI. Conclusions

Energy-based measurements in sensor networks are linear functions of the logarithm of range when measured in decibels or any other logarithmic unit. The log range model is linear in the two nuisance parameters path loss exponent and transmitted power. The theoretical contribution is this paper is a non-linear least squares localization framework, where these parameters are eliminated algebraically. The resulting NLS criterion in position coordinates can be minimized by any sensor node that receives energy measurements for several other nodes with known location. The practical contribution is to adopt standard algorithms for localization, tracking and Cramér-Rao analysis to these new results.

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