Particle Filters for Positioning in Wireless Networks

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ABSTRACT

Positioning of road vehicles in wireless radio networks is a highly non-linear multi-sensor problem. The radio measurements deliver snapshot information of position as circle, radial and hyperbolic lines with various reliability, and possibly also complex power attenuation maps. Further, fading and absence of line of sight give complicated disturbances on these measurements. Temporal and spatial prior knowledge may include maximal velocity and acceleration of the mobile unit, and for automotive applications also the constraint that most of the time is spent on roads, which are stored in a digital map. We outline a framework where all this information can be incorporated, and the true a posteriori distribution of position can be approximated with arbitrary accuracy to be traded off with real-time requirements. The algorithm is based on the particle filter, and we demonstrate it on a few simulation examples.

1 INTRODUCTION

The field of positioning in wireless networks is both driven by new market opportunities and laws for emergency call positioning. Various methods have been discussed in literature, and the aim here is to illuminate the potential of using particle filters for positioning.

The position is determined using measurements, which are either network-assisted or mobile-assisted. The former requires less user equipment modifications, but is relatively costly, while the latter is cheaper for the operators but requires dedicated routines in the user equipment. For a more thorough overview, see [6, 16]. Proposed approaches utilize different types of measurements, which include:

- **Distance** to base stations of known positions [7, 17, 18]. It is determined from time of arrival (TOA), time difference of arrival (TDOA) or enhanced observed time difference (E-OTD) measurements, see Figure 1.
- **Received Signal Strength** of signals with known powers, which are closely related to the travelled distance [17, 14].
- **Angle**, which relates the angle of arrival (AOA) at the receiver to a fixed coordinate system.
- **Map Information**, which is further described in [10].

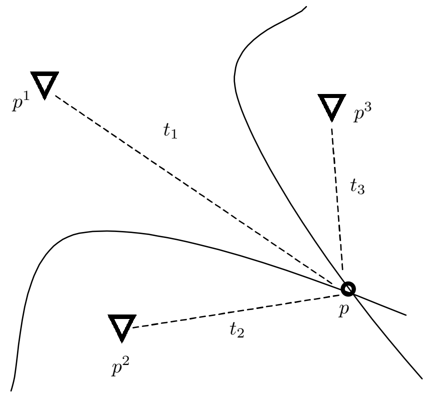

Figure 1: Distance measurements in wireless networks. In uplink TOA, the base stations p^i monitor the time of arrival t_i of a characteristic burst from the mobile p to extract the distance $|p - p^i|$. The mobile is more active in TDOA, where the time differences $\Delta t = t_i - t_1$ are measured, an possible locations describe hyperbolas with the other base stations p^i in foci. The effect of non-synchronized base stations is considered in E-OTD, where the network estimates the difference in transmission times between the base stations.

When regular samples of the measurements are assumed available, a popular solution to this sensor fusion problem is a model-based extended Kalman filter [2, 9, 14, 18]. An interesting alternative to this classical approach is recursive implementations of Monte Carlo based statistical signal processing [8], which are known as *particle filters*, see [5]. The more non-linear model, or the more non-Gaussian noise, the more potential particle filters have, especially in applications where computational power is rather cheap and the sampling rate slow. A framework for particle filtering in positioning, navigation and tracking applications is further discussed in [10].

Models of motion and measurements form a natural basis for the work and are presented in Section 2. The particle filter is introduced in Section 3, followed by simulations and a short discussion. Finally, Section 5 provides some conclusive remarks.

2 MODELS

Central for all position applications is the characterization of measurements. When considering model-based sensor fusion, also the motion model is vital. A rather generic model can be written as

$$x_{t+1} = f(x_t) + B_u u_t + B_w w_t \tag{1a}$$
$$y_t = h(x_t) + e_t. \tag{1b}$$

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Motion models (1a) are further discussed in Section 2.1, while Section 2.2 provides measurement equations (1b).

2.1 Motion Models

The signals of primary interest in positioning applications are related to position p_t, velocity v_t and acceleration a_t. Depending on whether the signals are measureable or not, they may be components of either the state vector x_t or the input signal u_t. Other parameterizations, however, might provide better understanding of design variables and algorithm tuning.

Depending on the context, the velocity and/or the acceleration might be measurable (the natural example is a car-mounted system) The state dynamics with/without velocity measurements can be modeled as

$$p_{t+1} = \underbrace{p_t}_{x_t} + \underbrace{T_s v_t}_{B_u u_t} + \underbrace{T_s w_t}_{B_w w_t} \tag{2a}$$

$$\begin{pmatrix} p_{t+1} \\ v_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} I & T_s \cdot I \\ 0 & I \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} p_t \\ v_t \end{pmatrix}}_{x_t} + \underbrace{\begin{pmatrix} T_s^2 \cdot I \\ T_s \cdot I \end{pmatrix}}_{B_w} w_t \tag{2b}$$

In (2b), the process noise w_t could be multimodal, where each mode represents a specific event having a certain probability. For example, when driving in an urban environment we know that the car most of the time follows a straight line, but occasionally makes sharp turns. Of course, even more complex models are possible, e.g. the coordinated turn model [9].

2.2 Measurement Equations

The main difference between the different positioning approaches is the measurements available. A generic model of a measurement equation is according to (1b), where the measurement noise contributions e_t are characterized by their distributions. If not explicitely mentioned, a Gaussian distribution is assumed. Every available measurement signal thus corresponds to a measurement equation. In this work, the focus is on measurements related to relative distance $r_t^i = |p_t - p^i|$ between the mobile position p_t and a number of base station positions p^i, $i = 1, \ldots, M$. Essentially, the measurements reflect the travelled distance of radio signals, either via time measurements or power measurements. In suburban and

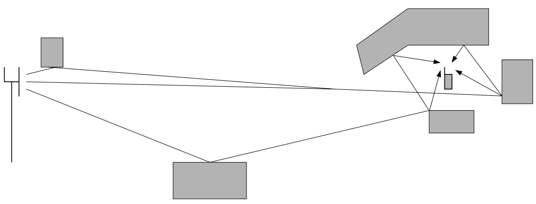

Figure 2: Far-field scattering and local scattering of transmitted signals in a wireless network.

urban areas, the *line-of-sight (LoS)* path is sometimes blocked by buildings. Due to this *far-field scattering*, where the signals might take a *non-line-of-sight* path (see Figure 2), the measurements can be biased. Instead, the measurements reflect the travelled distance as

$$r_t^i = \left|p^i - p_t\right| + m_t^i, \quad i = 1, \ldots, M.$$

The signals are also subject to near-field scattering, where the signals are reflected by a large number of objects close to the receiver. This causes fast variations of the signal amplitude known as *fast fading*, but it has little effect on the travelled distance or low-pass filtered power measurements.

The base stations in a terrestrial wireless communications system act as beacons by transmitting pilot signals of known power. The mobile station monitors the M (in GSM, $M = 5$, and in the upcoming WCDMA standard [1], $M = 6$) strongest signals, and reports regularly (or event-driven) the measurements to the network. Based on these measurements, the network centrally transfers connections from one base station to another (handover) when the mobile is moving during the service session. According to the empirical model by Okumura-Hata [11], this received power typically decays as $\sim K_1/r^\alpha$, $\alpha \in [2, 5]$, where K_1 and α are depending on the radio environment, antenna characteristics, terrain etc. One natural approximation is therefore to assume that K_1 and α are the same for similar base stations in a service area with roughly the same terrain. All in all, in a logarithmic scale, this provides M measurement equations, one for each available base station, according to

$$h_{a,i}(p_t) = K - \alpha \log_{10}(\left|p^i - p_t\right| + m_t^i), \tag{3a}$$

where $K = \log_{10} K_1$. Similar measurements, but without considering or modeling the bias, are used in [13]. Point-mass implementation of estimators based on RF measurements is also discussed in [4].

To provide more accurate positioning via RF measurements, future mobile stations will be able to estimate the traveled time (and hence the distance) of radio signals from a multitude of base stations with known positions. The aforementioned methods TOA, TDOA, OTD all are based on this principle. The resulting M (M is typically 1-3) measurement equations can thus be modeled by (3b).

$$h_{b,i}(p_t) = |p_i - p_t| + m_t^i, \tag{3b}$$

Any signaling aspects are neglected, but it is identified that network-assisted and mobile-assisted systems will result in different signaling requirements and situations. Note that any other available measurement signal relating to the position can be readily incorporated by adding a new measurement equation. Also highly nonlinear information such as map information can thus also be utilized [10].

2.3 Unknown Parameters

Positioning aims at estimating the position p_t. If the velocity is unknown, it can also be incorporated in the state vector of the motion model as in (2b). Furthermore, the biases m_t^i are unknown and can be assumed to change rather step-wise, when the signal path occasionally changes rapidly due to new reflections. A plausible state evolution model for the biases is therefore

$$m_{t+1}^i = m_t^i + n_t^i, \; p(n_t^i) = \sum_{k=0}^{1} \text{Pr}_k \mathcal{N}(0, \sigma_k), \tag{4}$$

where n_t^i takes on values from two Gaussian distributions, with a small and a large variance respectively ($\sigma_0 << \sigma_1$). The Pr_0 and Pr_1 reflect the probability for m_t^i making a jump or not. Additional information about the biases comes from the fact that they are always larger or equal to zero. These constraints

$$m_t^i \geq 0, \; i = 1, \ldots, M, \tag{5}$$

should of course be incorporated in the estimation problem.

In case of power measurements, also K and α are unknown. With the assumption of equal parameters for every base station, they can be modeled as two random-walk processes.

3 THE PARTICLE FILTER

3.1 Recursive Bayesian Estimation

Consider systems that are described by the generic state space model (1). The optimal Bayesian filter in this case is given below. For further details, consult [3].

We assume independent noise with probability densities p_{e_t} and p_{w_t}. Denote the observations at time t by $Y_t = \{y_0, \dots, y_t\}$. The Bayesian solution to compute the posterior distribution, $p(x_t|Y_t)$, of the state vector, given past observations, is given by [3]

$$p(x_{t+1}|Y_t) = \int p(x_{t+1}|x_t)p(x_t|Y_t)\,dx_t, \tag{6a}$$

$$p(x_t|Y_t) = \frac{p(y_t|x_t)p(x_t|Y_{t-1})}{p(y_t|Y_{t-1})}. \tag{6b}$$

For expressions on $p(x_{t+1}|x_t)$ and $p(y_t|x_t)$ in (6) we use the known probability densities p_{e_t} and p_{w_t}

$$\begin{aligned} &p(x_{t+1}|x_t) = \\ &|\det B_w^{-1}|p_{w_t}\big(B_w^{-1}(x_{t+1} - f(x_t) - B_u u_t)\big), && (7a)\\ &p(y_t|x_t) = p_{e_t}\big(y_t - h(x_t)\big), && (7b)\end{aligned}$$

where we have assumed an invertible B_w. For the case with a non-invertible B_w see [12].

3.2 Particle Filter Implementation

A numerical approximation to (6) is given by

$$p(x_t|Y_t) \approx \sum_{i=1}^{N} \bar{q}_t^{(i)} \delta_{x_t^{(i)}}(x_t), \tag{8}$$

where $\delta(\cdot)$ is the Dirac delta function. A straighforward way to recursively update the particles $x_t^{(i)}$ and the weights $\bar{q}_t^{(i)}$ is given by [10, 5]

$$x_{t+1}^{(i)} \sim p(x_{t+1}|x_t^{(i)}),\ i = 1, \dots, N, \tag{9a}$$

$$\bar{q}_{t+1}^{(i)} = \frac{p(y_{t+1}|x_{t+1}^{(i)})\bar{q}_t^{(i)}}{\sum_{j=1}^{N} p(y_{t+1}|x_{t+1}^{(j)})\bar{q}_t^{(j)}},\ i = 1, \dots, N, \tag{9b}$$

initiated at time $t = 0$ with

$$x_0^{(i)} \sim p(x_0),\ \bar{q}_0^{(i)} = \frac{1}{N},\ i = 1, \dots, N. \tag{10}$$

4 SIMULATIONS

Either power or distance measuremens are considered in these simulations. To focus on characteristic aspects of particle filtering, we simplify by assuming a known velocity (*i.e.* the car-mounted application with motion model according to (2a)) and known propagation parameters $K = 16.3$ and $\alpha = 3.5$ (typically estimated from operators drive tests). The mobile travels at constant speed

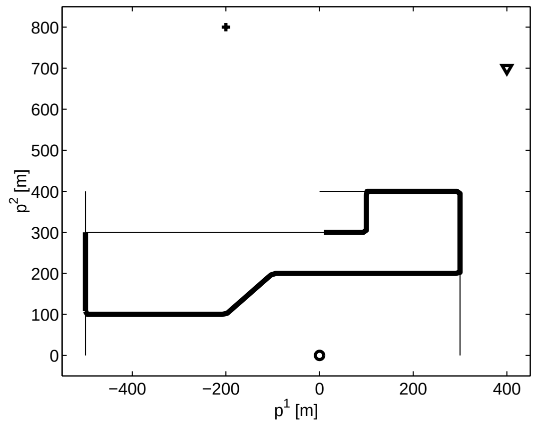

Figure 3: The true path (thick line), together with road map and the three base stations.

(= 12 m/s) along roads with sharp turns (45 to 90 degrees) in an area with three base stations, see Figure 3. The power measurements are more uncertain ($\sigma_e = 6$ dB, [4]) than the distance measurements ($\sigma_e = 3$ dB, [1]). The other simulation parameters are

$$\begin{aligned} p(n_t^i) &= 0.98 \cdot \mathcal{N}(0, 0.5) + 0.02 \cdot \mathcal{N}(0, 15),\\ p(m_0^i) &= \mathcal{U}(0, 50),\end{aligned} \tag{11}$$

for $i = 1, \dots, 3$ and

$$\begin{aligned} p(w_t) &= p(w_t^1)p(w_t^2) = \mathcal{N}(0, 3) \cdot \mathcal{N}(0, 3),\\ p(p_0) &= p(p_0^1)p(p_0^2) = \mathcal{U}(-500, 300) \cdot \mathcal{U}(0, 400).\end{aligned} \tag{12}$$

We have used not only a lower limit, see (5), but also an upper limit ($m_t^i \le 50,\ i = 1, \dots, 3$) on the biases to simplify the simulations. This means that we can use $p(m_0^i)$ to initialize each bias. In the case with no upper limit we can use $p(m_0^i|y_0^i, p_0)$ for initialization instead, see [15] for details.

All in all, we tested 6 different cases; 3 with distance measurements (D.1 – D.3) and 3 with power measurements (P.1 – P.3):

- D.1, P.1 Biases are applied to the measurements, and they are estimated.
- D.2, P.2 Biases are applied to the measurements, but they are not estimated.
- D.3, P.3 The ideal case, i.e. no biases added to the measurements and no estimation of biases.

For all simulations we used 2000 particles, i.e. $N = 2000$. More particles did not seem to improve the performance significantly.

The result based on 50 Monte-Carlo simulations on each of the 6 cases are summarized in Figure 4 and Figure 5. In the case with distance measurements (Figure 4), due to observability problems the estimates of the biases ($m_t^i,\ i = 1, \dots, 3$) do not converge until after 60 seconds. After that we get a significant improvement in the position estimate compared to not estimating the biases, except during the period 90 – 105 s when there are no biases contaminating the measurements. Using power measurements (Figure 5), we gain nothing estimating the biases. With power measurements we can

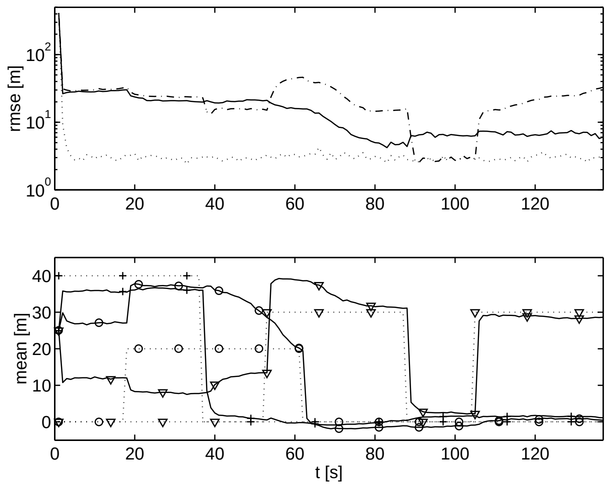

Figure 4: Upper plot: Monte-Carlo performance over time using distance measurements. D.1 (solid line) provide RMSE = 16 m, D.2 (dashed line) provide RMSE = 24 m and D.3 (dotted line) provide RMSE = 3 m. Lower plot: True biases (dotted line) and estimated mean(t) of the biases (solid line) for D.1.

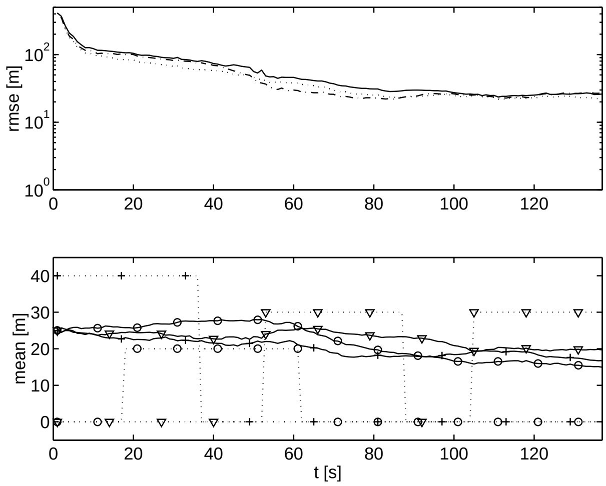

Figure 5: Upper plot: Monte-Carlo performance over time in the simulated scenario using power measurements. P.1 (solid line) provide RMSE = 76 m, P.2 (dashed line) provide RMSE = 69 m and P.3 (dotted line) provide RMSE = 64 m. Lower plot: True biases (dotted line) and estimated mean(t) of the biases (solid line) for P.1.

actually obtain worse position performance compared to not estimating the biases.

For the case with distance measurements we can try to estimate the biases separately, for each sequence of position samples, using for example the GPB or IMM filter [9]. Also, the observability problem encountered while estimating the biases using distance measurements can probably be eliminated by including for example map information. Due to lack of space, consult [15] for a more extensive simulation study, including a comparison to Kalman filter based estimation.

5 CONCLUSIONS

The problem of positioning using measurements relative a multitude of base stations can be seen as a sensor fusion problem. This work discusses the applicability of particle filters as an alternative to classical Kalman filter techniques. As shown, particle filters allow a great flexibility in the structure of the problem, and the chararcteristic nonlinearities and non-Gaussian noises of the studied application can be readily adopted. We investigate the plausibility of using more detailed multipath models for the signal propagation. The conclusion is that it is of little use with crude power measurements, but improves the performance with more accurate distance measurements.

REFERENCES

[1] 3GPP. Technical specification group radio access network. Standard Document Series 3G TS 25, Release 1999, 2001.

[2] B.D.O. Anderson and J.B. Moore. *Optimal filtering.* Prentice Hall, Englewood Cliffs, NJ., 1979.

[3] N. Bergman. *Recursive Bayesian Estimation: Navigation and Tracking Applications.* Dissertation nr. 579, Linköping University, Sweden, 1999.

[4] J. Blom, F. Gunnarsson, and F. Gustafsson. Estimation in cellular radio systems. In *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing.*, Phoenix, AZ, USA., March 1999.

[5] A. Doucet, N. de Freitas, and N. Gordon, editors. *Sequential Monte Carlo methods in practice.* Springer-Verlag, 2001.

[6] C. Drane, M. Macnaughtan, and C. Scott. Positioning GSM telephones. *IEEE Communications Magazine*, 36(4), 1998.

[7] S. Fischer, H. Koorapaty, E. Larsson, and A. Kangas. System performance evaluation of mobile positioning methods. In *Proc. IEEE Vehicular Technology Conference*, Houston, TX, USA, May 1999.

[8] W. Gilks, S. Richardson, and D. Spiegelhalter. *Markov Chain Monte Carlo in practice.* Chapman & Hall, 1996.

[9] F. Gustafsson. *Adaptive filtering and change detection.* John Wiley & Sons, Ltd, 2000.

[10] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, and P.-J. Nordlund. Particle filters for positioning, navigation and tracking. *IEEE Transactions on Signal Processing*, Feb 2002.

[11] M. Hata. Empirical formula for propagation loss in land mobile radio services. *IEEE Transactions on Vehicular Technology*, 29(3), 1980.

[12] A. H. Jazwinski. *Stochastic processes and filtering theory.* Academic Press, 1970.

[13] H. Jwa, S. Kim, X. Cho, and J. Chun. Position tracking of mobiles in a cellular radio network using the constrained bootstrap filter. In *Proc. National Aerospace Electronics Conference*, Dayton, OH, USA, October 2000.

[14] B. Mark and Z. Zaidi. Robust mobility tracking for cellular networks. In *Proc. IEEE International Communications Conference*, New York, NY, USA, 2002.

[15] P.-J. Nordlund, F. Gunnarsson, and F. Gustafsson. Particle filters for positioning in wireless networks. Submitted to *IEEE Transactions on Vehicular Technology*, 2002.

[16] M. Silventoinen and T. Rantalainen. Mobile station locating in GSM. In *Proc. IEEE Wireless Communication System Symposium*, Nov 1995.

[17] M.A. Spirito. Further results on GSM mobile station location. *IEE Electronics Letters*, 35(22), 1999.

[18] M.A. Spirito and A.G. Mattioli. On the hyperbolic positioning of GSM mobile stations. In *Proc. International Symposium on Signals, Systems and Electronics*, Sept 1998.