Exercise set # 2 for the course
"Networked Dynamical Systems"

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1. **Leontief input-output model**. Consider a group of n industries each of which is producing only one commodity, and to do so requires to use commodities from the other industries (for example to produce books you need paper and ink produced by other industries, etc.). Assume that a part of the produced commodities is for satisfying external demand. Denote $d \in \mathbb{R}^n_+$, $d \geqslant 0$ the demand vector. Call x_i the amount of product of the industry i and $t_{ij} \geqslant 0$ the units of product j needed per unit of product i. The Leontief model is given by the static relationship

$$x_i = \sum_j t_{ij} x_j + d_i \qquad i = 1, \ldots, n$$

or in matrix form

$$x = Tx + d$$

- Under what conditions a production vector x exist?

- Is is unique?

- What kind of properties do T and d have to obey in order for x to be economically meaningful?

- What can happen when $d = 0$?

- Show that if a Leontief model with matrix T is feasible, then the sum of the entries of T is less than one in at least one column.

- If you were to replace the matrix T with a nonlinear dependence from x (i.e., $T(x)$) what kind assumption on $T(x)$ (if any) would you impose to guarantee existence and uniqueness of x?