# A Data-driven Method for Monitoring Systems that Operate Repetitively

*Applications to Wear Monitoring in an IRB Joint*

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# Outline



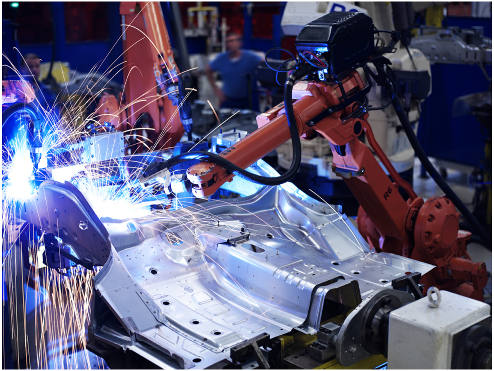
Diagnosis of Repetitive Systems

Basic Framework

Extensions

Conclusions

Systems that perform a task in a fixed pattern repetitively



Typical in automation

Systems that perform a task in a fixed pattern repetitively


Can be forced with a test-cycle

Systems that perform a task in a fixed pattern repetitively

Basic idea for diagnosis

> Compare how the task is executed now
> to how it is executed when healthy

Diagnosis of Repetitive Systems

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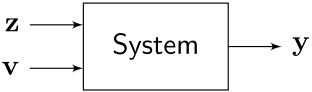
# Setup Description 

Consider a general system




$\mathbf{v}$ : random unknown.

$$\mathbf{z} : \begin{cases} \mathbf{r}, \text{ known} \\ \mathbf{d}, \text{ unknown} \\ \mathbf{f}, \text{ unknown of interest} \end{cases}$$

$\mathbf{y}$ : data (e.g. meas / ctrl inputs)
collected in regular batches

$$\mathbf{y}^k = [y_1^k, \cdots, y_i^k, \cdots, y_N^k]^T$$
$$\mathbf{Y}^M = [\mathbf{y}^0, \cdots, \mathbf{y}^k, \cdots, \mathbf{y}^{M-1}]$$

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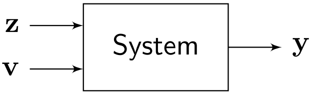


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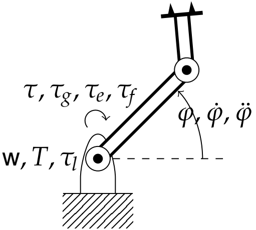



$$\tau = M(\varphi)\ddot{\varphi} + C(\varphi, \dot{\varphi}) + \tau_g(\varphi) + \tau_e + \tau_f(\dot{\varphi}, \tau_l, T, \text{w})$$

$$\mathbf{f} = \text{w}; \; \mathbf{d} = \tau_g, \tau_e, \tau_l, T, \ddot{\varphi}$$

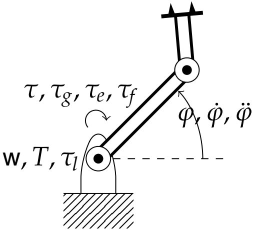
$$\mathbf{r} = \mho; \; \mathbf{y} = \tau, \varphi, \dot{\varphi}$$

More...

# Setup Description

Assumptions:

1. Faults affect $\mathbf{y}$ (necessary)

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Assumptions:

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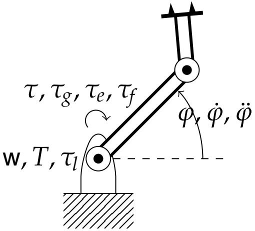


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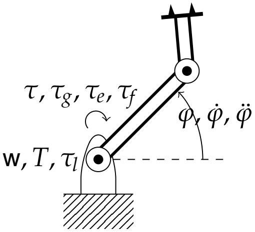


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Conditions for 2.

2.1. Regularity of $\mathbf{r}^k$ over $k$
2.2. Regularity of $\mathbf{d}^k$ over $k$

Mostly affected by $\mho$. Set

$\mho^{k-1} = \mho^k$ (same trajectory)

$\mathbf{d}^{k-1} = \mathbf{d}^k$ (hard)



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Assumptions:

1. Faults affect $\mathbf{y}$ (necessary)
2. Regularity of $\mathbf{y}^k$ over $k$ if $\mathbf{f}=0$ (comparable, repetitive)
3. Nominal data $\mathbf{y}^0$ available

Basic idea:

**Compare $\mathbf{y}^0$ with $\mathbf{y}^k$**

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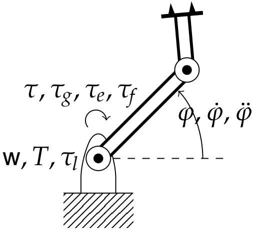


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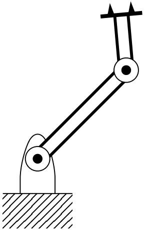

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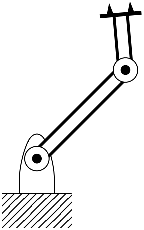

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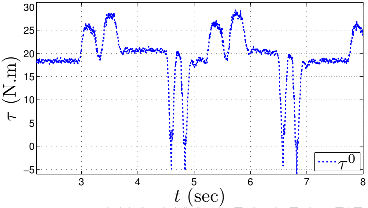


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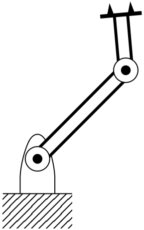

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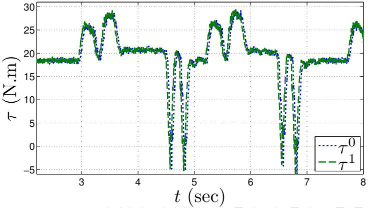

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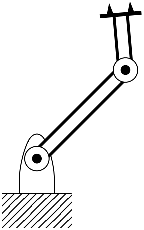

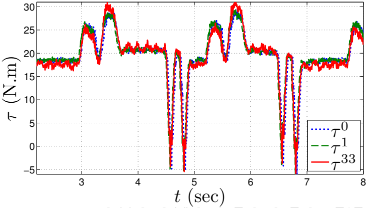

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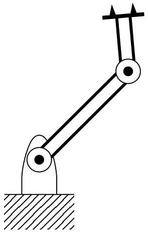

1. How to characterize $\mathbf{y}^k$?
2. How to compare $\mathbf{y}^0$ with $\mathbf{y}^k$?
3. How to relax the Assumps.?

**Characterizing the data $\mathbf{y}^k$**

smooth density estimate

$$\hat{p}^k(y) \triangleq \frac{1}{N}\sum_{i=1}^{N} k_h(y - y_i^k)$$

**Comparing $\mathbf{y}^0$ with $\mathbf{y}^k$**

Kullback-Leibler distance

$$\mathsf{KL}\left(\hat{p}^0 || \hat{p}^k\right) \triangleq D_{\mathsf{KL}}\left(\hat{p}^0 || \hat{p}^k\right) + D_{\mathsf{KL}}\left(\hat{p}^k || \hat{p}^0\right)$$

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# Basic Framework

## Characterizing the data $\mathbf{y}^k$

smooth density estimate

$$\hat{p}^k(y) \triangleq \frac{1}{N}\sum_{i=1}^{N} k_h(y - y_i^k)$$

+ changes in amplitude
+ no ordering
+ simple (no model)
+ little tuning

## Comparing $\mathbf{y}^0$ with $\mathbf{y}^k$

Kullback-Leibler distance

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– requires $\mathbf{y}^0$
– data should be from same $\mho$
– “nice” disturbances
– no physical meaning

Characterizing the data $\mathbf{y}^k$

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Comparing $\mathbf{y}^0$ with $\mathbf{y}^k$

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# Live demo

Backup...

Diagnosis of Repetitive Systems

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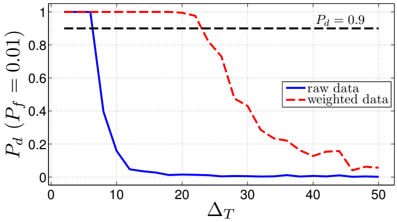
Reducing sens. to disturbances $\mathbf{d}$

apply weights to the data

$$\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$$

$\mathbf{w}$ is chosen based on data

More w... More ℧...



# Extending the Framework

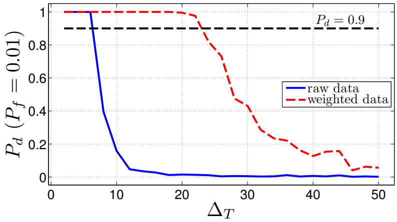
## Reducing sens. to disturbances $\mathbf{d}$

apply weights to the data

$$\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$$

$\mathbf{w}$ is chosen based on data

▸ More w... ▸ More $\mho$...



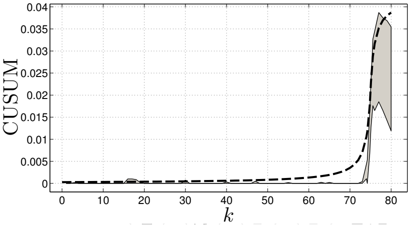

## Handling several $\mho$'s / need for $\mathbf{y}^0$

Monitor pairwise increments

Only $\mho^{k-1} = \mho^k$ needed

$$\sum_{j=1}^{k} \mathrm{KL}\left(\hat{p}^{j-1} || \hat{p}^{j}\right)$$

and a CUSUM filter to avoid drifts



## Some remarks

- Overlooked problem, commonly found in automation
- No interruption of the system (batch)
- Can handle the use of several tasks and disturbances
- Simple (no model) and easy to implement, with little tuning

## Future Work

- Multivariate case
- Study the choice of kernel and distances
- Fault isolation and alarm generation (ongoing)
- Conditions on the data and disturbances

Thank you!

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BITTENCOURT (2012). On Modeling and Diagnosis of Friction and Wear in Industrial Robots. *Licentiate thesis*, (get one here).

(Binary) Hypothesis testing

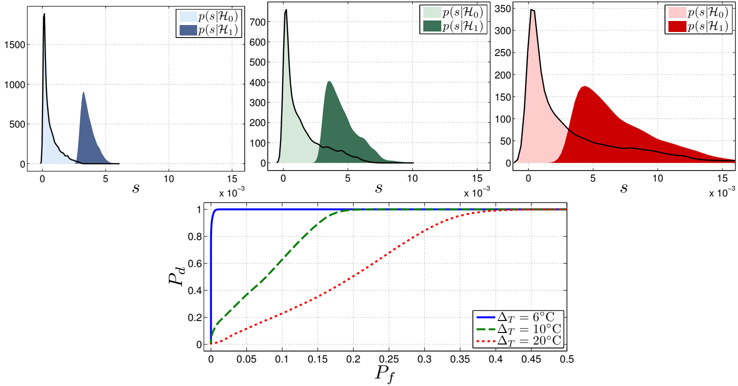
Given $s(k)$ decide behavioral mode presence, $\mathcal{H}_0$ or $\mathcal{H}_1$

$$s(k) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \hbar$$

$$P_f = \int_{\hbar}^{\infty} p(s|\mathcal{H}_0)\,\mathrm{d}s, \text{ assigns } \mathcal{H}_1 \text{ when } \mathcal{H}_0 \text{ is present.}$$
$$P_d = \int_{\hbar}^{\infty} p(s|\mathcal{H}_1)\,\mathrm{d}s, \text{ assigns } \mathcal{H}_1 \text{ when } \mathcal{H}_1 \text{ is present.}$$

# Evaluation of test quantities





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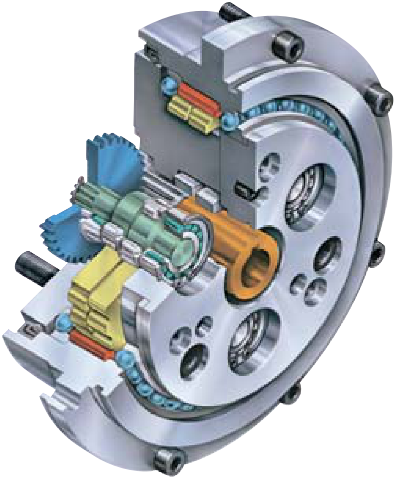
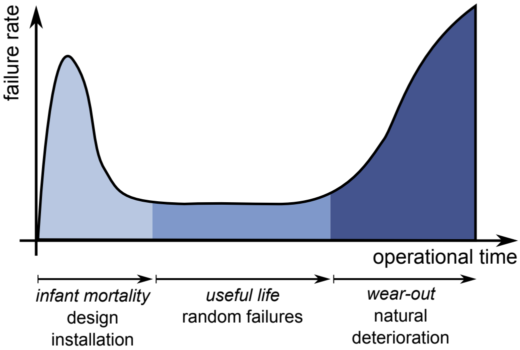


Which joint?
Not which bearing!

# Which faults to begin with?

Failure of equipments. The bathtub curve




For increased availability

- do service before failure
- $\rightarrow$ diagnose before failure

Failures due to wear are

- certain
- gradual

Wear diagnosis is a good candidate to allow for CBM!



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Where does it happens?

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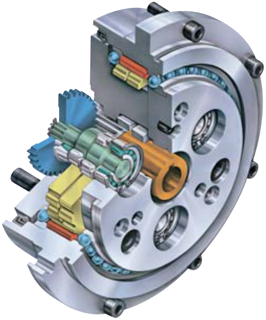

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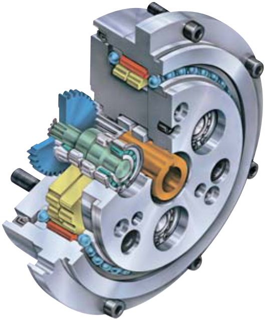

How we can diagnose wear?

Inspection allows for CBM (not suitable)

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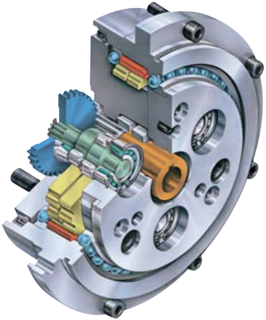

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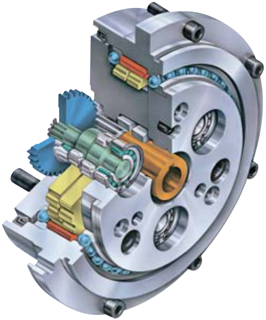
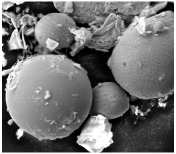
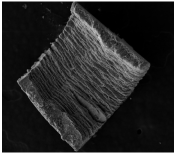
# Wear Diagnosis - Where and How

Where does it happens?



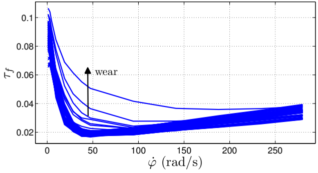
Wear debris particles accumulate in the lubricant



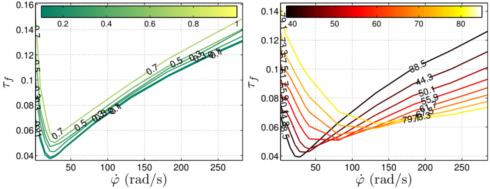


Ferrography allows for CBM (not suitable)

## Wear affects friction!




## ... however!




- BITTENCOURT ET AL. (2012). Static Friction in a Robot Joint - Modeling and Identification of Load and Temperature Effects. *ASME Journal of Dynamic Systems, Measurement, and Control*, (to appear).
- BITTENCOURT ET AL. (2011). Static Friction in a Robot Joint - Modeling and Identification of Load and Temperature Effects. *Proc. of the 18th IFAC World Congress*, 2011.

Back...

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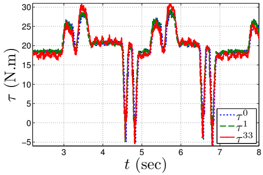
An experimental wear fault (acc. wear tests)

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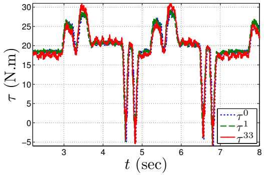


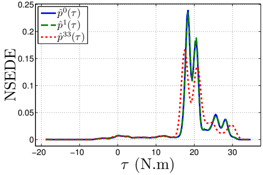


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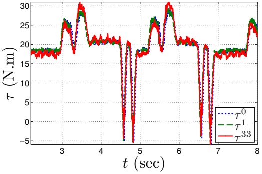
Comparing $\mathbf{y}^0$ with $\mathbf{y}^k$

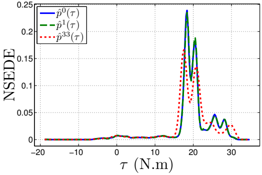
Kullback-Leibler distance

$$\text{KL}\left(\hat{p}^0||\hat{p}^k\right) \triangleq D_{\text{KL}}\left(\hat{p}^0||\hat{p}^k\right) + D_{\text{KL}}\left(\hat{p}^k||\hat{p}^0\right)$$

$$D_{\text{KL}}\left(\hat{p}^0||\hat{p}^k\right) \triangleq -\int_{-\infty}^{\infty} \hat{p}^0(y) \log \frac{\hat{p}^k(y)}{\hat{p}^0(y)}\, \mathrm{d}y$$

An experimental wear fault (acc. wear tests)




Back...

Characterizing the data $\mathbf{y}^k$
smooth density estimate

$$\hat{p}^k(y) \triangleq \frac{1}{N}\sum_{i=1}^{N} k_h(y - y_i^k)$$

Comparing $\mathbf{y}^0$ with $\mathbf{y}^k$
Kullback-Leibler distance

$$\text{KL}\left(\hat{p}^0 || \hat{p}^k\right) \triangleq D_{\text{KL}}\left(\hat{p}^0 || \hat{p}^k\right) + D_{\text{KL}}\left(\hat{p}^k || \hat{p}^0\right)$$

$$D_{\text{KL}}\left(\hat{p}^0 || \hat{p}^k\right) \triangleq -\int_{-\infty}^{\infty} \hat{p}^0(y) \log \frac{\hat{p}^k(y)}{\hat{p}^0(y)}\, \mathrm{d}y$$

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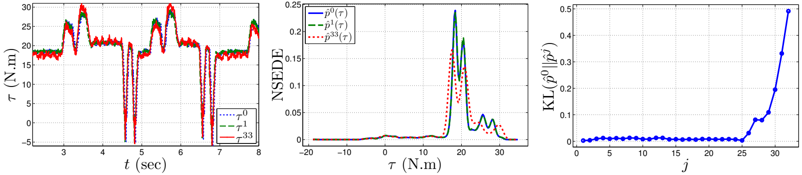


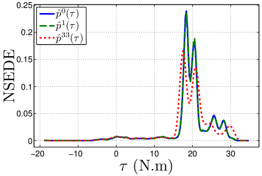

Back...

No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \mathsf{KL}\left(\hat{p}^{j-1} || \hat{p}^{j}\right)$$

An experimental wear fault (acc. wear tests)




Back...

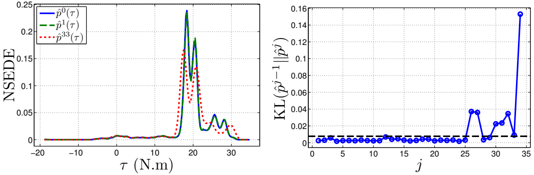
# Extending the Framework

No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \mathrm{KL}\left(\hat{p}^{j-1} || \hat{p}^{j}\right)$$

An experimental wear fault (acc. wear tests)




Back...

# Extending the Framework

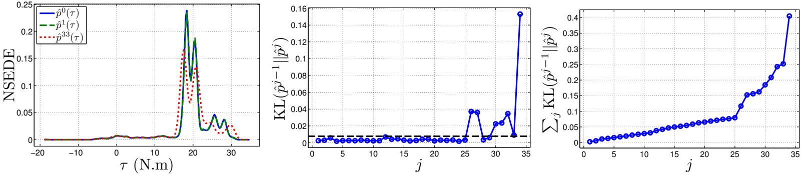


No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \mathsf{KL}\left(\hat{p}^{j-1}||\hat{p}^{j}\right)$$

An experimental wear fault (acc. wear tests)




Back...

No assignment of $\mathbf{y}^0$

accumulated changes

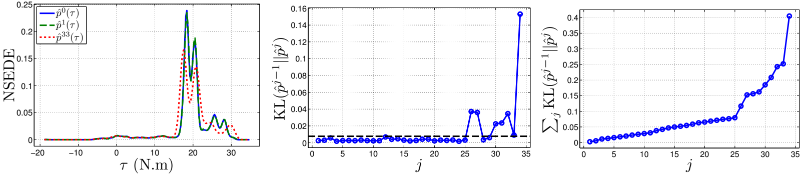
$$\sum_{j=1}^{k} \text{KL}\left(\hat{p}^{j-1}||\hat{p}^{j}\right)$$

CUSUM filter

$$\begin{cases} g^k &= g^{k-1} + \text{KL}\left(\hat{p}^{k-1}||\hat{p}^{k}\right) - \nu \\ g^k &= 0 \text{ if } g^k < 0 \end{cases}$$

with $\nu = \kappa\sigma + \mu$

An experimental wear fault (acc. wear tests)




Back...

No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \mathsf{KL}\left(\hat{p}^{j-1} || \hat{p}^{j}\right)$$

CUSUM filter

$$\begin{cases} g^k &= g^{k-1} + \mathsf{KL}\left(\hat{p}^{k-1} || \hat{p}^{k}\right) - \nu \\ g^k &= 0 \text{ if } g^k < 0 \end{cases}$$

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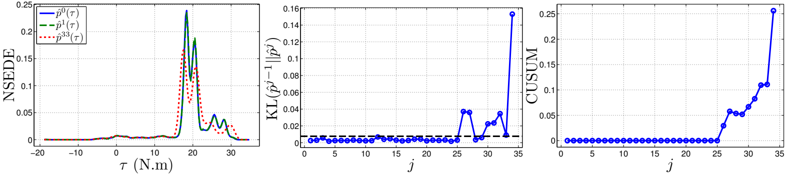



Back...

No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \text{KL}\left(\hat{p}^{j-1} || \hat{p}^{j}\right)$$

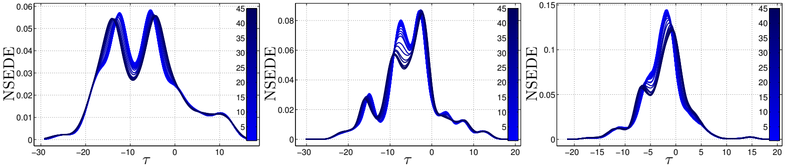
How to handle several ℧?

CUSUM filter

$$\begin{cases} g^k &= g^{k-1} + \text{KL}\left(\hat{p}^{k-1} || \hat{p}^{k}\right) - \nu \\ g^k &= 0 \text{ if } g^k < 0 \end{cases}$$

with $\nu = \kappa\sigma + \mu$

Behavior of the data differs with ℧ (simulation)




Back...

No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \mathsf{KL}\left(\hat{p}^{j-1} || \hat{p}^{j}\right)$$

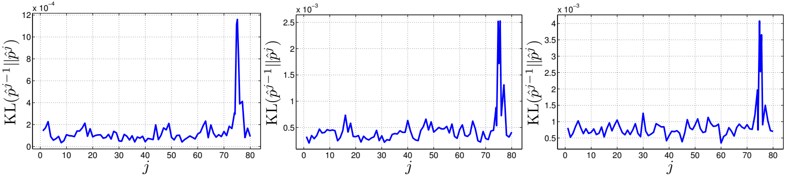
How to handle several $\mho$?

CUSUM filter

$$\begin{cases} g^k & = g^{k-1} + \mathsf{KL}\left(\hat{p}^{k-1} || \hat{p}^{k}\right) - \nu \\ g^k & = 0 \text{ if } g^k < 0 \end{cases}$$

with $\nu^k = \kappa\sigma(\mho^k) + \mu(\mho^k)$

Idea: mix the increments in the CUSUM




Back...

No assignment of $\mathbf{y}^0$

accumulated changes

$$\sum_{j=1}^{k} \text{KL}\left(\hat{p}^{j-1}||\hat{p}^{j}\right)$$

How to handle several $\mho$?

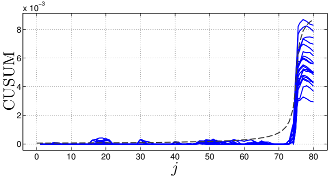
when $\mho^{k-1} = \mho^k$, same!

CUSUM filter

$$\begin{cases} g^k & = g^{k-1} + \text{KL}\left(\hat{p}^{k-1}||\hat{p}^{k}\right) - \nu \\ g^k & = 0 \text{ if } g^k < 0 \end{cases}$$

with $\nu^k = \kappa\sigma(\mho^k) + \mu(\mho^k)$

Simulations with several $\mho$'s




Back...

Handling disturbances $\mathbf{d}$

apply weights to the data

$$\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$$

Idea: choose $\mathbf{w}$ to $\max$ sens. to $\mathbf{f}$ and $\min$ sens. to $\mathbf{d}$

Back...

Handling disturbances $\mathbf{d}$

apply weights to the data $\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$

Idea: choose $\mathbf{w}$ to $\max$ sens. to $\mathbf{f}$ and $\min$ sens. to $\mathbf{d}$

Let the data decide!

$$\mathbf{Y}^M \triangleq \Big[ \underbrace{\mathbf{y}^0, \ldots, \mathbf{y}^{M_0}}_{\mathcal{C}_0}, \underbrace{\mathbf{y}^{M_0+1}, \ldots, \mathbf{y}^{M_1+M_0}}_{\mathcal{C}_1} \Big]$$

Back...

## Handling disturbances $\mathbf{d}$

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Back...

### Handling disturbances $\mathbf{d}$

apply weights to the data $\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$

Idea: choose $\mathbf{w}$ to $\max$ sens. to $\mathbf{f}$ and $\min$ sens. to $\mathbf{d}$

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$$\bar{\mathbf{Y}}^M \triangleq \Big[ \underbrace{\bar{\mathbf{y}}^0, \ldots, \bar{\mathbf{y}}^{M_0}}_{\mathcal{C}_0}, \underbrace{\bar{\mathbf{y}}^{M_0+1}, \ldots, \bar{\mathbf{y}}^{M_1+M_0}}_{\mathcal{C}_1} \Big]$$

Optimal criteria would be

$$\mathbf{w}^* = \arg\max_{\mathbf{w}} \mathsf{KL}\left( \hat{p}(\bar{\mathbf{y}}^{\mathcal{C}_0}) || \hat{p}(\bar{\mathbf{y}}^{\mathcal{C}_1}) \right)$$

which is not so easy to solve

Back...

## Handling disturbances $\mathbf{d}$

apply weights to the data $\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$

Idea: choose $\mathbf{w}$ to $\max$ sens. to $\mathbf{f}$ and $\min$ sens. to $\mathbf{d}$

Let the data decide!

$$\bar{\mathbf{Y}}^M \triangleq \Big[ \underbrace{\bar{\mathbf{y}}^0, \ldots, \bar{\mathbf{y}}^{M_0}}_{\mathcal{C}_0}, \underbrace{\bar{\mathbf{y}}^{M_0+1}, \ldots, \bar{\mathbf{y}}^{M_1+M_0}}_{\mathcal{C}_1} \Big]$$

Alt., $\max$ average distance while $\min$ variability within class

$$\frac{(\bar{m}^1 - \bar{m}^0)^2}{\bar{s}_1 + \bar{s}_0} \propto \frac{\mathbf{w}^T(\mathbf{m}^1 - \mathbf{m}^0)(\mathbf{m}^1 - \mathbf{m}^0)^T\mathbf{w}}{\mathbf{w}^T(\mathbf{S}^1 + \mathbf{S}^0)\mathbf{w}},$$ which has optimum

$$\mathbf{w}^* \propto (\mathbf{S}^1 + \mathbf{S}^0)^{-1}(\mathbf{m}^1 - \mathbf{m}^0)$$

Back...

Optimal weights

$$\mathbf{w}^* \propto (\mathbf{S}^1 + \mathbf{S}^0)^{-1}(\mathbf{m}^1 - \mathbf{m}^0)$$

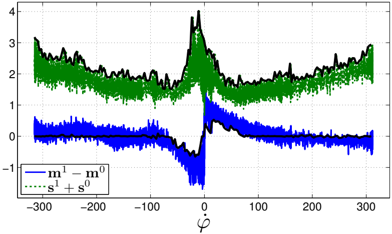
for an industrial robot under temperature disturbances (simulation)

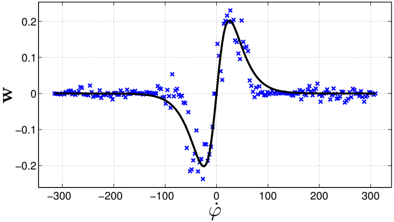
Back...

Optimal weights

$$\mathbf{w}^* \propto (\mathbf{S}^1 + \mathbf{S}^0)^{-1}(\mathbf{m}^1 - \mathbf{m}^0)$$

for an industrial robot under temperature disturbances (simulation)



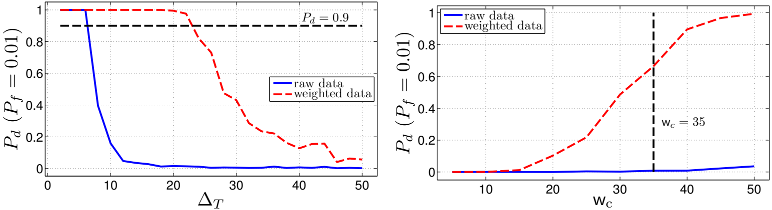

Optimal weights correlate with speed!

Back...

Optimal weights

$$\mathbf{w}^* \propto (\mathbf{S}^1 + \mathbf{S}^0)^{-1}(\mathbf{m}^1 - \mathbf{m}^0)$$

for an industrial robot under temperature disturbances (simulation)




The use of weights considerable improves the detection performance

Back...