1

Let (A, B, C, D) be a balanced realization of a system G. That is

$$A^*\Sigma + \Sigma A + C^*C = 0$$
$$A\Sigma + \Sigma A^* + BB^* = 0$$

(a) Is the balanced realization unique?

(b) If not, try to find a parametrization of all balanced realizations.

(c) Consider the balanced realization of $G(s) = \frac{120-60s+12s^2-s^3}{120+60s+12s^2+s^3}$. Try to find a balanced realization such that as many elements as possible are zeroed in (A, B, C, D).

2

Let $G = \tilde{M}^{-1}\tilde{N}$ be a normalized coprime factorization. Perform a model reduction of $\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix}$ such that

$$\left\| \begin{bmatrix} (\tilde{N} - \tilde{N}_r) & (\tilde{M} - \tilde{M}_r) \end{bmatrix} \right\| \leq \varepsilon.$$

Determine an upper bound for the relative error between $G_r = \tilde{M}_r^{-1}\tilde{N}_r$ and G.

3

A transfer function G is called positive real if G is stable and $\operatorname{Re} G(j\omega) > 0$ for all $\omega \in \mathbb{R}$. Determine if $G(s) = D + C(sI - A)B$ is positive real without making a frequency sweep.

4

We will here consider the problem of controlling and stabilizing the attitude of a rocket using thrust vector control. A hypothetical rocket is used and we consider the control of the second stage, which has its burn phase in the upper atmosphere. The vehicle is aerodynamically unstable due to the fact that the center of pressure is in front of the center of mass. The vehicle is stabilized by directing its movable nozzle. The velocity of the vehicle is assumed to be relatively high, and we use the approximation that attitude is identical to angle of attack, which allows us to use only two states in the model. Neglecting aerodynamic damping we use

$$\ddot{y} = ay + bu$$

as our model of the vehicle dynamics, where y is the attitude of the vehicle, u is the thrust vector deflection, and a and b are parameters.

The parameters a and b are uncertain due to uncertainties in dynamic pressure (caused by altitude, velocity and air density), gravimetrics, and aerodynamics. In addition, the a parameter depends on the angle of attack.

The parameter a can be modeled explicitly by an uncertainty while variations in b can be included in the phase and gain margin.

A delay of 0.06 seconds is included in the loop for modeling computational delay, sampling effects and actuator dynamics.

Design a controller that should satisfy the following requirements.

(i) $a = a_0 + a_1\Delta_1,\ \Delta_1 \in [0, 1]$;

(ii) The gain margin shall be 6 dB or better;

(iii) The phase margin shall be 35 degrees or better;

(iv) The compensator gain shall be less than –6 dB at frequencies above 50 rad/s.