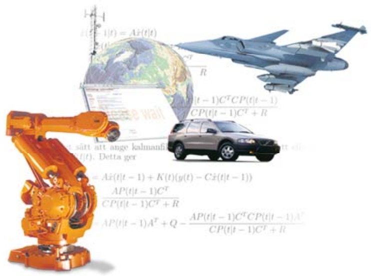
Digital Signal Processing, Lecture 14
Summary



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Outline Lecture 14

1. Summary of lecture 12
2. Summary of the course
   a) Signal approximations
   b) Signal models
   c) Signal model estimation
   d) Filtering
3. Sensor fusion (TSRT14)
4. Brief overview of our research

Summary of Lecture 12 (I/II)

The adaptive algorithms are in the form

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \varphi^T(t)\hat{\theta}(t-1)\right),$$

where

LMS	$K(t) = \mu\varphi(t)$
NLMS	$K(t) = \frac{\mu\varphi(t)}{\alpha + \varphi^T(t)\varphi(t)},$
RLS	$K(t) = P(t)\varphi(t),$ $P(t) = \frac{1}{\lambda}\left(P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{\lambda + \varphi^T(t)P(t-1)\varphi(t)}\right).$
KF	$K(t) = \frac{P(t-1)\varphi(t)}{R(t) + \varphi^T(t)P(t-1)\varphi(t)},$ $P(t) = P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{R(t) + \varphi^T(t)P(t-1)\varphi(t)} + Q(t).$

Summary of Lecture 12 (II/II)

Design considerations:

- The fundamental trade-off in adaptive filtering is between adaptation speed and accuracy (noise sensitivity)
- RLS & KF typically have significantly shorter transient compared to LMS.
- Computational complexity
  - LMS – Linear
  - RLS – Quadratic
- The estimates do not converge (bias error): Increase the model order
- Slow adaptation (adaption error): Increase the filter gain
- Large variance in the estimates (variance error): Decrease the filter gain
- Transient error: Increase P_0

Course Goal

“The student should after the course have the ability to ***describe*** *the* ***most important methods and algorithms*** *for signal processing, and be able to* ***apply*** *these on signals of various kinds.“*

Course Goal – More Specifically

- Compute the **discrete Fourier transform (DFT)** and understand scaling effects and practical limitations implied by finite data length and sampling.
- **Use the DFT** for filtering and know how circular convolution is avoided.
- Explain the **basic signal models** and their relationship.
- Perform transform-based and model-based **spectral analysis**, and understand the compromise between resolution and noise suppression.
- Describe the theory for **model estimation**, and be able to apply algorithms for this purpose and validate an estimated model.
- Describe the **basics in optimal filtering** and be able to compute a **Wiener filter** for simple examples.
- Describe the premises for **Kalman filtering**, and be able to **apply** a Kalman filter to data and tune it to compromise tracking speed, transient behavior and noise suppression.
- Describe the most important **adaptive filters**, some common applications, and be able to apply and tune an adaptive filter to compromise parameter tracking speed and noise suppression.

Course Contents

1. **Signal Approximations**
   a) Sampling and truncation
2. **Signal Models**
   a) Non-parametric (FT, DTFT, DFT, Spectrum, covariance function)
   b) Parametric (AR, MA, ARMA, state-space models)
3. **Signal Model Estimation**
   a) Non-parametric (spectral estimation)
   b) Parametric (LS for AR, NLS for ARMA and State-space)
4. **Filtering**
   a) Non-parametric (frequency-selective filters)
   b) Parametric (Wiener filter, Kalman filter)
   c) Adaptive filters

Transform Overview (I/II)

Tuncation leads to **leakage**, which in turn results in a **limited frequency separation**. Use windowing to limit this problem.

1. $X(i\omega)$ → 2. $X_T(e^{i\omega T})$ → 3. $X_T^{(N)}(e^{i\omega T})$ → 4. $X[n]$

FT — DTFT — Truncated DTFT — DFT

Time sampling, sample faster than the band width. Alternatively apply an **anti-alias filter** before the sampling in order to avoid aliasing (folding).

Frequency sampling, we will only see the DTFT in a discrete frequency grid. A problem here is **circular convolution**, which can be handled by **zero-padding**.

Transform Overview (II/II)

Operation	Transform	Limitation	Problem	Countermeasure
Time sampling	DTFT	$\omega_B < \frac{\pi}{T}$	Alias	Low-pass (LP) filtering
Truncation	DTFT	Frequency separation	Leakage	Windowing
Frequency sampling	DFT	$\omega = \frac{2\pi}{NT}n$	Circular convolution	Zero-padding

MSc Thesis Example 3

An Optimization Based Approach to Visual Odometry Using Infrared Images, by Emil Nilsson, performed at Autoliv Electronics, Linköping, Sweden.

Figure 4.26: Sequence I: Trajectory for EKF estimates (dashdotted) and SAM estimates (solid), using the basic vehicle process model.

Topics from this course:

- Kalman filters
- Solving estimation problems
- Models for motion and sensors

MSc Thesis Example 4

Observers for estimation of tool position for an industrial robot: Design simulation and experimental evaluation, by Robert Henriksson, performed at ABB Robotics, Västerås, Sweden.

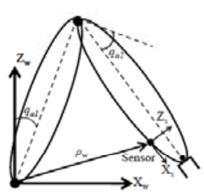

Fig. 1. Coordinate frames used in modeling the 2-DOF robot. The world (w) frame is attached to the robot base and the sensor (s) frame is attached to the accelerometer.

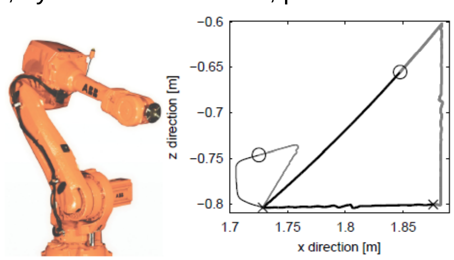

Fig. 2. Left: The ABB 6-DOF manipulator IRB4600 used in the experiments. Right: Illustration of the two different paths and their location in the robot workspace. Thick line: P1, thin line: P2. Gray line: selected part showed in Fig. 4–6, where 'o' indicates the start and 'x' the end of the selected part.

Robert Henriksson, Mikael Norrlöf, Stig Moberg, Erik Wernholt and Thomas B. Schön, **Experimental Comparison of Observers for Tool Position Estimation of Industrial Robots**. *Proceedings of the 48th IEEE Conference on Decision and Control (CDC)*, Shanghai, China, December 2009.

MSc Thesis Example 5

A Simulation Model for Detecting and Tracking Bio Aerosol Clouds using Elastic LIDAR, by Erika Jönsson, performed at the Swedish Defense Research Agency (FOI), Linköping, Sweden.

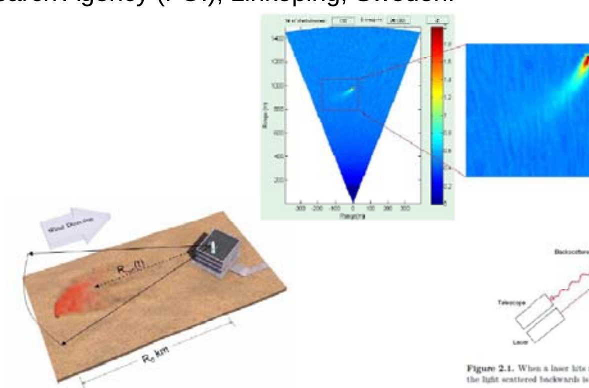

Figure 1.1. An overview of the final system. The laser will sweep back and forth to look for discharges [7].

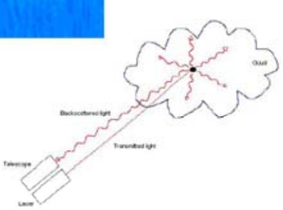

Figure 2.1. When a laser hits a cloud the light will be scattered in all directions. Only the light scattered backwards is the light detected by the telescope.

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Estimating AR Models using Least Squares

Let $\varphi(t) = \begin{pmatrix} -y(t-1) & -y(t-2) & \cdots & y(t-n) \end{pmatrix}^T$

$$\theta = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}^T$$

implying that the predictor can be written as $\widehat{y}(t|t-1,\theta) = \varphi^T(t)\theta$

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(y(t) - \varphi^T(t)\theta\right)^2$$

$$\widehat{\theta}_N = \arg\min_{\theta} V_N(\theta)$$

$$= \left(\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right)^{-1}\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t) = R_N^{-1}f_N$$

There are numerically more reliable and efficient techniques for solving the normal equations, based on matrix factorizations (e.g., QR).

Estimating ARMA and State-Space Models

Consider the following problem, where $\varepsilon(t,\theta)$ is a nonlinear function of θ

$$\widehat{\theta}_N = \arg\min_{\theta} V_N(\theta),$$

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon^2(t,\theta)$$

There does not exist an explicit solution to this problem. Hence, forced to numerical approximation.

A very common algorithm for handling this is the Gauss-Newton algorithm.

$$\theta^{(i+1)} = \theta^{(i)} - \mu R_N^{-1} f_N$$

- $\theta^{(i+1)}$: New iterate
- $\theta^{(i)}$: Previous iterate
- μ: Step length
- R_N^{-1}: Hessian approximation
- f_N: Gradient approximation

Kalman Signal Model

Signal model

$$x(t+1) = Ax(t) + w(t),$$

$$y(t) = Cx(t) + v(t),$$

where w(t) and v(t) are white noise, with

$$\mathsf{E}(w(t)) = \mathsf{E}(v(t)) = 0,$$

$$\mathsf{Cov}(w(t)) = \mathsf{E}(w(t)w^T(t)) = Q,$$

$$\mathsf{Cov}(v(t)) = \mathsf{E}(v(t)v^T(t)) = R,$$

$\mathsf{E}(w(t)v^T(t)) = S$. (We will assume that S = 0 for simplicity)

The initial state x(0) is also assumed to be a stochastic variable with

$$\mathsf{E}(x(0)) = x_0, \qquad \mathsf{Cov}(x(0)) = \Pi_0$$

Typically, all stochastic variables are assumed Gaussian.

Kalman Filter – Problem Formulation

Problem: Compute an estimate of the state vector x(t) for a state-space model using the information available in all the previous measurements

$$y(\tau), \qquad 0 \le \tau \le t$$

such that the covariance of the estimation error

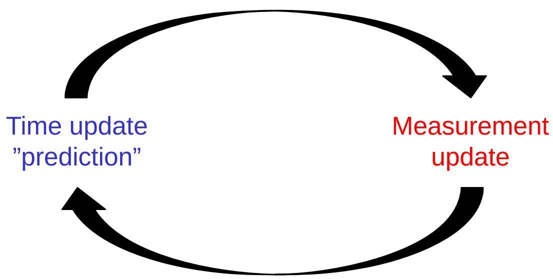
$$\tilde{x}(t \mid t) = x(t) - \hat{x}(t \mid t)$$

is minimized.

Kalman Filter Structure

Structure of the Kalman filter

$$x(t+1) = Ax(t) + w(t),$$
$$y(t) = Cx(t) + v(t).$$



Kalman Filter – the Solution

Time update

$$\hat{x}(t+1|t) = A\hat{x}(t|t),$$
$$P(t+1|t) = AP(t|t)A^T \underbrace{+Q}_{\text{Increase uncertainty}}.$$

Measurement update

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K(t)\overbrace{(y(t) - C\hat{x}(t|t-1))}^{\text{Innovation}},$$
$$L(t) = CP(t|t-1)C^T + R,$$
$$K(t) = P(t|t-1)C^T L(t)^{-1},$$
$$P(t|t) = P(t|t-1) \underbrace{-P(t|t-1)C^T L(t)^{-1} C P(t|t-1)}_{\text{Decrease uncertainty}},$$

Adaptive Algorithms

The algorithms are in the form

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \varphi^T(t)\hat{\theta}(t-1)\right),$$

where

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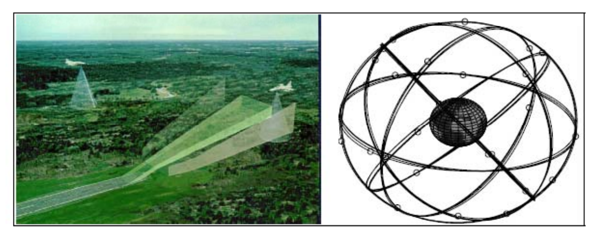
Sensor Fusion (TSRT14)

Deals with the problem of estimating the state of a dynamic system using measurements from several different sensors.

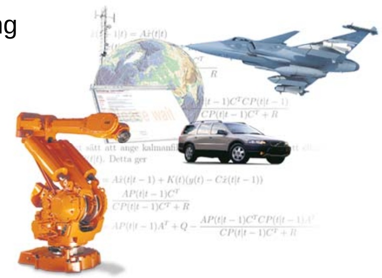
- Nonlinear estimation theory (particle filters, etc.)
- Simultaneous localization and mapping (SLAM)
- Detection theory and sensor network applications
- Motion modelling



Our Research Areas

Research areas at the Division of Automatic Control

- System identification and model building
- Signal processing and sensor fusion
- Control of nonlinear systems
- Optimization, e.g., MPC
- Specific applications (robotics, automotive, UAV, aircraft, …)



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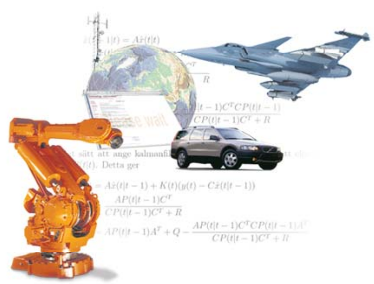

”Signal processing is the art of getting what you want from the signals”

Thank you for your attention and good luck in the future!!