TSRT14: Sensor Fusion
Lecture 10
Sensors and filter validation

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Spring 2015
Guest lecturer:

- Rickard Karlsson (NIRA Dynamics)

Slides:

- Examples of detecting sensors
- Physical sensors
- Filter validation
SIS PF Algorithm
Choose the number of particles $N$, a proposal density $q(x_k^{(i)}|x_{0:k-1}^{(i)}, y_1:k)$, and a threshold $N_{th}$ (for instance $N_{th} = \frac{2}{3} N$).

- **Initialization:** Generate $x_0^{(i)} \sim p_{x_0}$, $i = 1, \ldots, N$.

Iterate for $k = 1, 2, \ldots$:

1. **Measurement update:** For $i = 1, 2, \ldots, N$:
   $$w_{k|k}^{(i)} = w_{k|k}^{(i)} p(y_k|x_k^{(i)})$$
   and normalize $w_{k|k}^{(i)}$

2. **Estimation:** MMSE
   $$\hat{x}_{k|k} \approx \sum_{i=1}^{N} w_{k|k}^{(i)} x_k^{(i)}$$

3. **Resampling:** Resample with replacement when $N_{\text{eff}} = \frac{1}{\sum_i (w_{k|k}^{(i)})^2} < N_{th}$

4. **Prediction:** Generate samples $x_{k+1}^{(i)} \sim q(x_k^{(i)}|x_{k-1}^{(i)}, y_k)$,
   update the weights
   $$w_{k+1|k}^{(i)} = w_{k|k}^{(i)} \frac{p(x_k^{(i)}|x_{k-1}^{(i)}))}{q(x_k^{(i)}|x_{k-1}^{(i)}, y_k)}$$
   and normalize $w_{k+1|k}^{(i)}$
Radar Types

**CW radar** transmits a sinusoid where the Doppler shift gives range rate. All computations can be realized in analogue electronics.

**FM-CW radar** sweeps the frequency according to a saw-tooth (or sine), which makes it possible to also compute the range.

**PD radar** transmits a short pulse of duration $T_p$ with *pulse repetition frequency* ($PRF$) $f_{PRF}$ with an overlayed sinusoid.
Ambiguity in range and speed:

\[
\dot{R} = \frac{\Delta f c f_{\text{PRF}}}{4\pi f} \quad \text{modulo} \quad \frac{c f_{\text{PRF}}}{4f},
\]

\[
R = \frac{c T_d}{2} \quad \text{modulo} \quad \frac{c}{2f_{\text{PRF}}}. 
\]

<table>
<thead>
<tr>
<th>PRF</th>
<th>Velocity Ambiguity</th>
<th>Range Ambiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (2 kHz)</td>
<td>30 m/s</td>
<td>75 km</td>
</tr>
<tr>
<td>Medium (12 kHz)</td>
<td>180 m/s</td>
<td>12.5 km</td>
</tr>
<tr>
<td>High (200 kHz)</td>
<td>3000 m/s</td>
<td>750 m</td>
</tr>
</tbody>
</table>
Radar Equation

Energy decays quickly with range $R_t = R_r$

$$P_r = \frac{P_t G_t A_r \sigma F^4}{(4\pi)^2 R_t^2 R_r^2}$$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t$</td>
<td>transmitter power</td>
</tr>
<tr>
<td>$G_t$</td>
<td>gain of the transmitting antenna</td>
</tr>
<tr>
<td>$A_r$</td>
<td>effective aperture (area) of the receiving antenna</td>
</tr>
<tr>
<td>$s$</td>
<td>radar cross section, or scattering coefficient, of the target</td>
</tr>
<tr>
<td>$F$</td>
<td>pattern propagation factor</td>
</tr>
<tr>
<td>$R_t$</td>
<td>distance from the transmitter to the target</td>
</tr>
<tr>
<td>$R_r$</td>
<td>distance from the target to the receiver</td>
</tr>
</tbody>
</table>
Radar image in polar coordinates (thresholded pulse response for each bearing), sea chart, and radar overlay.
Pulse Thresholding (1/3)

The pulse response is thresholded to give a black and white image.

Problems:
False detections (clutter), missed detections, multiple detections.

Illustration 1: ideal case
**Illustration 2:** broad returned pulse

![Graph showing sample intensity](image.png)

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Illustration 3–5: multiple echoes

Need for distinguishing clutter (false echoes), moving objects (narrow pulses) and stationary objects (broad pulses).
Other Range Sensors

Other sensors with similar basic functionality as a radar:

- Active sonar
- Lidar (laser radar, ladar)
- Active IR
Physical Sensors

- Examples
  - Accelerometers
  - Gyroscopes
  - Geophones
  - Magnetometers

- Used to be mechanical devices
- Nowadays MEMS (micro-mechanical systems)
## MEMS accelerometer specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Automotive</th>
<th>Navigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>±2 g (ADAS) or ±50 g (airbag)</td>
<td>±1 g</td>
</tr>
<tr>
<td>Resolution</td>
<td>&lt; 10 mg (ADAS) or &lt; 100 mg (airbag)</td>
<td>&lt; 4 µg</td>
</tr>
<tr>
<td>Frequency range</td>
<td>0–400 Hz</td>
<td>0–100 Hz</td>
</tr>
</tbody>
</table>
### Gyroscope

#### MEMS gyroscope specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rate grade</th>
<th>Tactical grade</th>
<th>Inertial grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range (^\circ/\text{s})</td>
<td>50–1000</td>
<td>&gt; 500</td>
<td>&gt; 400</td>
</tr>
<tr>
<td>Angle random walk (^\circ/\text{h})</td>
<td>&gt;0.5</td>
<td>0.05–0.5</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Bias drift (^\circ/\text{h})</td>
<td>10–1000</td>
<td>0.1–10</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Scale factor accuracy (^\circ/\text{s})</td>
<td>0.1–1</td>
<td>0.01–0.1</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Frequency range [Hz]</td>
<td>0–70</td>
<td>0–100</td>
<td>0–100</td>
</tr>
</tbody>
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Geophones

Used in seismology for a long time.
Potential for surveillance: tracking pedestrians and vehicles.
Magnetometers

- Magnetometers more than a compass
- Electromagnetic field cannot be obstructed. Makes them suitable for tracking and detection of metallic/magnetic objects
- Limited range, signal decays cubically

\[ y_k = \frac{\mu_0}{4\pi \|r_k\|} \left( (r_k \cdot m)r_k - \|r_k\|^2 m \right) \]
Filter and Model Validation

A filter assumes correct
- motion model
- sensor model

How to validate these?

Key ideas:
- Sensitivity analysis
- Ground truth analysis
Sensitivity Analysis: example

Unknown parameter $\theta = a$ in

$$x_{k+1} = ax_k + v_k,$$
$$y_k = x_k + e_k,$$

$v_k \sim \mathcal{N}(0, 0.01)$
$$e_k \sim \mathcal{N}(0, 1)$$

- Deterministic sensitivity

$$\frac{\partial \hat{x}_k(\theta)}{\partial \theta_i} \quad \frac{\partial P_k(\theta)}{\partial \theta_i}$$

- Stochastic sensitivity (for instance using Monte Carlo samples $\theta^{(i)}$)

$$E(x_k) = E_\theta(\hat{x}_k(\theta)),$$
$$\text{cov}(x_k) = E_\theta(\text{cov}_{x_k}(x_k|\theta)) + \text{cov}_\theta(E_{x_k}(x_k|\theta))$$
$$= E_\theta(P_k(\theta)) + \text{cov}_\theta(\hat{x}_k(\theta))$$
Ground-Truth Analysis

Ground truth never available. However, a state sequence $x_{1:N}$ good enough can be obtained using

- better models (more states, less approximations and linearizations)
- better sensors (reference IMU)
- more sensors (GPS for non-GPS applications)
- better filters (smoothers where filters will be used, PF where EKF will be used, ...)

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Ground-Truth Analysis

How to use ground-truth to validate a filter?

- The “I’m feeling lucky” test statistic

\[ T = \sum_{k=1}^{N} (\hat{x}_k - \hat{x}_k^0)^T P_k^{-1} (\hat{x}_k - \hat{x}_k^0) \]

- Separate sensor model validation

\[ T = \sum_{k=1}^{N} (y_k - h(\hat{x}_k^0))^T R^{-1} (y_k - h(\hat{x}_k^0)) \]

- Separate motion model validation

\[ T = \sum_{k=1}^{N} (x_{k+1}^0 - f(\hat{x}_k^0))^T Q^{-1} (x_{k+1}^0 - f(\hat{x}_k^0)) \]

- Validate \( Q \) and \( R \)