EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Wednesday 8th June 2022, kl. 8.00–12.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: ”Reglerteori. Flervariabla och olinjära metoder”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2022-06-22 kl 12.30-13:00 in Ljungeln, B-huset, entrance 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER
(PRELIMINARY GRADE THRESOHLDS):

betyg 3	23 poäng
betyg 4	33 poäng
betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) What is the difference between controllability and stabilizability in a linear system? Do you know any test that can be used to check both conditions? [2p]

   (b) A system with a real zero in $z > 0$ and a real pole in $p > 0$ is to be controlled. (The remaining poles and zeros are in the left half plane.) Which case is more difficult, $z > p$ or $p > z$? Give a motivation. [2p]

   (c) A position sensor is mounted on top of a platform which vibrates with a frequency of approximately 4 rad/s, and produces a disturbance $n(t)$ on the position measurement. You would like to model $n(t)$ as the output of a low-pass filter driven by white noise. Which of the following transfer functions should you use?

$$(i): \quad \frac{1}{s + 0.001}$$
$$(ii): \quad \frac{32}{s^2 + s + 400}$$
$$(iii): \quad \frac{32}{s^2 + 0.4s + 16}$$

   Motivate your answer. [3p]

   (d) Consider the system

$$y = \frac{1}{s+2}u + \frac{2}{s+3}v$$

   where y is the output, u is the control and v is a disturbance. The control signal is limited by the condition $|u| \leq u_0$, and the disturbance has the form $v = 2\sin 3t$. Compute the lowest value of u_0 for which it is possible to eliminate the influence of v on y completely. [3p]

2. Consider the following system

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x$$

(a) Denoting G the transfer function from u to y, compute the pole polynomial, the poles and the zeros of G. [3p]

(b) What is $\|G\|_\infty$? [1p]

(c) What is RGA at frequency 0? [1p]

Assume the system is controlled by a P-controller $u = F(r - y)$, where

$$F = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

and $r - y$ is the control error. We assume that $r = 0$.

(d) Compute the sensitivity function of the system. [2p]

Assume that the system is affected by an additive disturbance w at the output of G.

(e) What is the frequency range in which the gain from w to the control error is less than -20 dB? [2p]

(f) Assume that $w = [w_1 \; w_2]^T$ is constant. Compute the value of the quotient between w_1 and w_2 for which the gain from w to the control error has its maximum. [1p]

3. Consider the following system

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + v_2$$

where v_1 and v_2 are uncorrelated white noises of covariance $R_1 = 48$ and $R_2 = 1$, respectively.

(a) In order to estimate the state vector, the following observer is used:

$$\dot{\hat{x}} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \hat{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \left(y - \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{x} \right)$$

What are the observer poles? [1p]

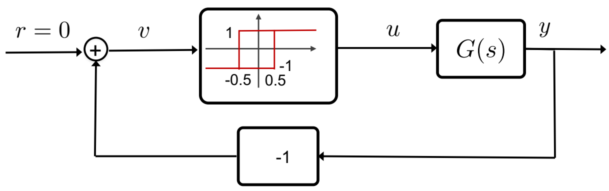
(b) Determine the covariance matrix $P_1 = E[\tilde{x}\tilde{x}^T]$ of the estimation error $\tilde{x} = x - \hat{x}$ for the observer in (a). [3p]

(c) The optimal estimator is the Kalman filter. Compute the Kalman filter of the system. What are the poles of the associated observer? What is the covariance matrix of the associated estimation error?

[4p]

(d) Show that indeed the estimation error of the Kalman filter has a lower variance than that of the observer in (a). [2p]

4. A servomotor is driven by a relay with hysteresis as in the following figure



where

$$G(s) = \frac{10}{s(s+1)}$$

(a) Compute the approximate amplitude and frequency of the possible sustained oscillations. [8p]

(b) Are the oscillations stable? [2p]

5. The system

$$G(s) = \frac{1}{s}$$

with input u and output y is controlled using the PI-controller (with saturation)

$$\begin{aligned} \dot{x}_1 &= e \\ u &= \text{sat}(x_1 + e) \end{aligned}$$

where $e = r - y$ is the control error and "sat" is the saturation function:

$$\text{sat}(v) = \begin{cases} -1 & v < -1 \\ v & |v| \leq 1 \\ 1 & v > 1 \end{cases}$$

which affects the control signal at the output of the PI-controller. Consider the case $r = 0$.

(a) Determine a state space model for the closed loop system created by G and by the saturated PI-controller. Determine the equilibrium points, their stability properties and their type (focus, saddle point,.....). Draw a sketch of the phase plane. What is the effect of the saturation? [6p]

(b) Modify the PI-controller so that x_1 is frozen to a constant value when the control is saturated:

$$\begin{aligned} \dot{x}_1 &= \begin{cases} e & |x_1 + e| \leq 1 \\ 0 & |x_1 + e| > 1 \end{cases} \\ u &= \text{sat}(x_1 + e) \end{aligned}$$

Show how the phase portraint changes, and in particular, show that this feedback solution gives "chattering", i.e. along certain curves (which you should determine) in the phase plane the velocity alternates infinitely fast between two values. Discuss what will happen in practice to the system.

[4p]