EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Friday 25th March 2022, kl. 14.00–18.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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BESÖKER SALEN: cirka kl. 15 och kl. 17

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TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: ”Reglerteori. Flervariabla och olinjära metoder”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2022-04-08 kl 12.30-13:00 in Ljungeln, B-huset, entrance 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER:	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Print out relevant plots and your own matlab code if useful.

Lycka till!

1. (a) Describe (briefly) what a Lyapunov function is and what it is used for. [2p]

   (b) Consider the transfer function

$$G(s) = \begin{bmatrix} \frac{s+5}{s+1} & \frac{1}{s+3} \\ \\ \frac{1}{s+3} & \frac{3}{s+5} \end{bmatrix}$$

   i) Compute the pole polynomial of $G(s)$. [2p]

   ii) Compute the singular values of $G(s)$. [2p]

   iii) Use RGA($G(0)$) to pair inputs and outputs. [2p]

   iv) Compute a state space realization of $G(s)$ (in matlab). Is it stabilizable? And detectable? [2p]

2. Consider the system

$$y = G(s)u + w, \qquad G(s) = \frac{s+1}{(s+0.1)(s+20)}$$

and the following weight functions

$$W_S = \frac{1}{s+1}, \qquad W_T = \frac{10(s+2)}{s+20}, \qquad W_u = 0.1$$

   (a) Construct the extended system of eq. (10.4) of the book

$$\begin{bmatrix} z \\ y \end{bmatrix} = G_e \begin{bmatrix} w \\ u \end{bmatrix}$$

   and design an $\mathcal{H}_\infty$ controller for it. Write down the resulting controller F_y, the corresponding value of γ, and plot the resulting S, T and G_{wu}. [7p]

   (b) Assume that you are not satisfied with the sensitivity S at low frequencies, and would like to improve it (i.e., render the closed loop less sensitive at low frequencies). What would you modify and how? [3p]

[Hint: the function `hinfsyn` *of matlab relies on state space. Whenever you have a transfer function, transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s'), WS=minreal(ss(1/(s+1)))`*.]*

3. Consider an aircraft flying at a constant altitude with horizontal speed x_1, engine thrust x_2 and throttle input u. If the air drag depends quadratically on the speed and the variables are suitably scaled, the following model is obtained:

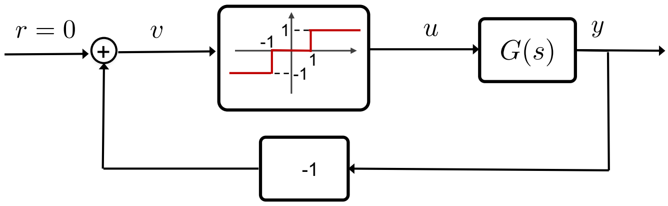
$$\dot{x}_1 = -x_1^2 + x_2$$
$$\dot{x}_2 = -x_2 + u$$

We only consider the first quadrant: $x_1 \geq 0$, $x_2 \geq 0$.

(a) Let the control signal be a constant $u = u_0 > 0$. Compute the equilibria, the linearization around the equilibria, and the eigenvalues of the linearization, all as functions of u_0. Also determine the type of the equilibria. What can you say on the equilibria of the nonlinear system? [7p]

(b) For $u_0 = 1$, use the result from (a) together with the information you can extract from dx_2/dx_1 to sketch the phase portrait for $x_1 \geq 0$, $x_2 \geq 0$. (You can also use simulations if you want). [3p]

4. A system is driven by a relay with deadzone as in the following figure



(a) Only one of the 3 transfer functions below may lead to sustained oscillations according to the describing function criterion. Which one?

$$G_1(s) = \frac{1}{s(s+1)}, \quad G_2(s) = \frac{100}{s(s+2)^2}, \quad G_3(s) = \frac{100}{(s+3)(s+6)}$$

[8p]

(b) For the transfer function admitting oscillations, what can you say on the stability in amplitude of these oscillations? [2p]

5. Consider the nonlinear system of state x, input u, and output y

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \sin x_3 + u \\
\dot{x}_3 &= x_1^3 + x_2 + x_3 \\
y &= x_1
\end{aligned}
$$

(a) What is the relative degree of the system? [1p]

(b) Compute a state feedback $u = p(x) + r$ so that the relationship between reference signal r and output y is linear and equal to the transfer function

$$\frac{1}{(s^2 + s + 1)}$$

[4p]

(c) What is the problem with this regulator? [1p]

(d) Assume that the output equation is instead the following

$$\bar{y} = \alpha x_1 + \beta x_2 + \gamma x_3$$

Compute the values of α, β and γ for which the relative degree of the system is 3. For these values, compute a regulator $u = q(x)+r$ that gives the transfer function

$$\frac{1}{(s^3 + s^2 + s + 1)}$$

between r and $\bar{y}$. [4p]