EXAM IN CONTROL THEORY (TSRT09)

SAL: Take-home exam

TID: Tuesday 24th August 2021, kl. 14.00–18.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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TILLÅTNA HJÄLPMEDEL: You are allowed to use any available help material (exercises, old exams, notes, etc.), but not to communicate with other persons.

LANGUAGE: You can write your exam in both English (preferred) or Swedish

HAND-IN OF SOLUTION: Upload your solution on Lisam (under the "Submission" menu). Recommended format is a single document bundling together matlab code, text, graphs and/or scans/photos of some hand written parts. Call the file with your personal number YYYYMMDDXXXX.pdf.

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: Difficult to organize. Online meetings can be set up upon request.

PRELIMINÄRA BETYGSGRÄNSER:	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) What is the RGA of the system

$$\begin{bmatrix} \frac{2}{s+5} & \frac{1}{s+1} \\ 0 & \frac{3}{s+4} \end{bmatrix}$$

at frequency $\omega = 0$? What can you deduce on its regulation?[2p]

(b) Consider the system

$$\dot{x} = -3x + v$$

where v is a 0-mean unit-variance white noise. What is the variance of x at stationarity? [2p]

(c) For which values of a, b, c and d is the system

$$\dot{x} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} x + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} u$$

controllable? [2p]

(d) A MIMO system has state space model

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$$

i) Verify that the feedback $u = -Lx + \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix}$ with L which is either

$$L_1 = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

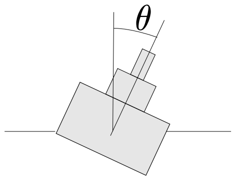
or

$$L_2 = \begin{bmatrix} 3 & 3 \\ 1 & 3 \end{bmatrix}$$

gives a closed loop system with the same poles. [2p]

ii) Show that one of the regulators above has the property of being decoupling. [2p]

2. (a) A ship has rolling motion described by the roll angle θ



and equation

$$J\ddot{\theta} = -b\dot{\theta} - f(\theta)$$

where J is a moment of inertia, f is a torque (straightening up the ship again) and b is a damping. Assume

$$J = 1, \quad b = 0.1, \quad f(\theta) = 3\theta - 4\theta^3$$

Introduce the state variables $x_1 = \theta$ and $x_2 = \dot{\theta}$. What are the eigenvalues of the linearization around the origin? What type of equilibrium is it (saddle/node/..., stable/unstable/..)? [4p]

(b) A multivariable system has transfer function

$$G(s) = \begin{bmatrix} \frac{2s+1}{(s+1)(s+3)} & \frac{1}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+4} \end{bmatrix}$$

What are the poles and the zeros of the system? (*Note: avoid using matlab functions* `pole()`, `zero()`, *etc.*) Do these poles/zeros lead to any limitation in the control design? [3p]

(c) Consider the system

$$\dot{x}_1 = -2x_1 + u + w$$

where w is a noise of spectrum

$$\phi(\omega) = \frac{1}{\omega^2 + 9}$$

Compute a spectral factorization of ϕ and then write the system in the form

$$\dot{x} = Ax + Bu + Nv$$

where x is a state vector with x_1 as first component, v is a white noise and A, B and N suitably chosen matrices. [3p]

3. Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} u$$

(a) Compute a state feedback $u = -Lx$ so that the evolution of the system from the initial condition

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has the following properties:

- The absolute value of the state x_1 is less than 0.1 after at most 1 time unit.
- The absolute value of the inputs is $|u_i| < 3$ at all times, $i = 1, 2$.
- the input u_2 does most of the job at $t = 0$: $|u_1(0)| \leq 0.1|u_2(0)|$.

Design the regulator L so that the closed loop satisfies these constraints. [8p]

(b) Do you see any advantage/disadvantage in adding a Kalman filter to the regulator design (even though it is not necessary in this case)? [2p]

4. Consider the system

$$y = G(s)u + w, \qquad G(s) = \frac{1}{(s+1)(s+3)}$$

and the following weight functions

$$W_S = \frac{1}{s+2}, \qquad W_T = \frac{s+4}{s+3}, \qquad W_u = 1$$

(a) Construct the extended system of eq. (10.4) of the book

$$\begin{bmatrix} z \\ y \end{bmatrix} = G_e \begin{bmatrix} w \\ u \end{bmatrix}$$

and show that it is in innovation form. [2p]

(b) Design an $\mathcal{H}_\infty$ controller for the system. Write down the resulting controller and the corresponding value of γ. [6p]

(c) Plot the resulting S, T and G_{wu}. [2p]

[Hint: use the `hinfsyn` *function of matlab. Hint #2:* `hinfsyn` *relies on state space. Whenever you have a transfer function transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s')`*,* `WS=minreal(ss(1/(s+2)))`*.]*

5. Consider the system

$$G(s) = \frac{1}{(s+1)^3}$$

with input u and output y.

(a) Assume the system is feedback coupled with a P controller $u = -Ky$. For which values of K ($K > 0$) is the closed loop system stable? [3p]

(b) Assume instead that the feedback is given by the nonlinear function $u = -K\phi(y)$ where K is a gain as before and $\phi(y)$ is a static sector nonlinearity s.t.

$$k_1 y \leq \phi(y) \leq k_2 y, \qquad \text{with} \quad k_1 = 0.5 \quad \text{and} \quad k_2 = 2$$

i) Can you show stability of the closed loop system using the small gain criterion? If yes, for which values of K? [3p]

ii) Can you show stability of the closed loop system using the circle criterion? If yes, for which values of K? [3p]

iii) Which one is more conservative and why? [1p]