EXAM IN CONTROL THEORY (TSRT09)

SAL: Take-home exam

TID: Tuesday 8th June 2021, kl. 8.00–12.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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TILLÅTNA HJÄLPMEDEL: You are allowed to use any available help material (exercises, old exams, notes, etc.), but not to communicate with other persons.

LANGUAGE: You can write your exam in both English (preferred) or Swedish

HAND-IN OF SOLUTION: Upload your solution on Lisam (under the "Submission" menu). Recommended format is a single document bundling together matlab code, text, graphs and/or scans/photos of some hand written parts. Call the file with your personal number YYYYMMDDXXXX.pdf.

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: Difficult to organize. Online meetings can be set up upon request.

PRELIMINÄRA BETYGSGRÄNSER:	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) What is the complication in feedback controlling a non-minimum phase system? [2p]

(b) What are the poles (and with what multiplicity) for the system

$$G(s) = \begin{bmatrix} \frac{s+7}{s+3} & \frac{s+2}{(s+1)(s+3)(s+7)} \end{bmatrix}$$

[2p]

(c) Compute the linear system whose output has spectrum

$$\frac{1}{\omega^2 + 4}$$

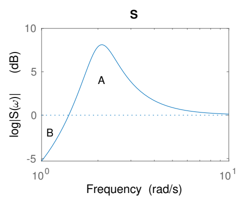
when the input is a white noise of intensity 1. [2p]

(d) Consider the system

$$y = \frac{1}{s+1}u + \frac{2}{s+3}d$$

where y is the output signal, u is the control signal and d is a disturbance. Assume that d is a constant taking value in the interval $[-1,\ 3]$ and that u is bounded: $|u(t)| \leq 1$. Is it possible to construct a regulator that completely eliminate the influence of d on y? Motivate your answer. [2p]

(e) A physical system under feedback regulation has the following sensitivity function:



What can you say on the loop gain if the area A (w.r.t. the 0 baseline) is bigger than the area B? [2p]

2. Consider the multivariable system

$$Y(s) = G(s)U(s)$$

where

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{3}{s+2} \\ \frac{\alpha}{(s+1)^2} & \frac{3}{s+1} \end{bmatrix}$$

and $\alpha > 0$ is a parameter.

(a) Compute the zeros of the multivariable system. How are the zeros depending on α? [4p]

(b) Assume that one wants to do an exact dynamical decoupling for the system, for instance design $F(s)$ such that

$$\tilde{G}(s) = G(s)F(s) = \begin{bmatrix} \frac{1}{(s+1)^2} & 0 \\ 0 & \frac{1}{(s+1)^2} \end{bmatrix}$$

Can you anticipate what kind of complication one can expect to encounter and when it is not a good idea? Motivate! [2p]

(c) Assume instead that one is satisfied with a steady-state decoupling, that is, that the system $\tilde{G}(0)$ is decoupled. Compute the matrix that achieves this. [4p]

3. Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

(a) Compute a state feedback that minimizes the criterion

$$\int_0^\infty (3x_1^2 + 2x_3^2 + u^2)dt$$

What are the poles of the closed loop system? [4p]

(b) Assume that only the states x_1 and x_3 are measured. Compute a regulator based on a Kalman filter and on the gain you have computed in (a). Covariance matrices of the process and measurements disturbances are given by

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{bmatrix} \quad \text{and} \quad R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Choose the value of the parameter ρ so that the poles of the Kalman filter are all faster than those of the regulator in (a).

[5p]

(c) Without doing any calculation, what can you say on the robustness of the regulators in (a) and (b)? [1p]

4. Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_2 + x_1 x_2 + u_1 \\ \dot{x}_2 &= 2x_1 - x_2 + 3x_1 x_2 + u_2 \\ y_1 &= x_1 \\ y_2 &= x_2 \end{aligned}$$

(a) Compute the equilibrium points of the system when $u = 0$. What is their character (stable/unstable node, focus, saddle point, etc.)?

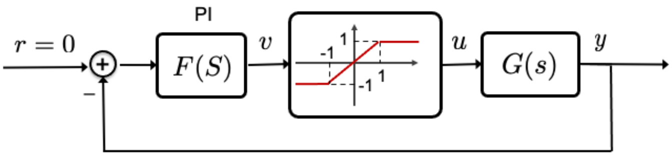
[4p]

(b) Consider the unstable equilibrium point. Compute a feedback from y_1, y_2 that renders the linearized system stable. [3p]

(c) Consider the linearized system for the stable equilibrium point. You want to set up two PI regulators, each using one output to control one input. Which pairing $u_i \leftrightarrow y_j$, $i, j = 1, 2$, would you choose and why? It is enough to discuss the steady state case.

[3p]

5. The system



with reference $r = 0$ and

$$G(s) = \frac{1}{(s+2)^2}$$

is controlled by means of the PI

$$F(s) = K\left(1 + \frac{1}{\tau s}\right)$$

where $\tau = 0.1$. The signal v at the output of the PI passes through a saturation:

$$u(t) = \begin{cases} 1 & \text{if } v > 1 \\ v & \text{if } |v| < 1 \\ -1 & \text{if } v < -1 \end{cases}$$

(a) Use the describing function method to show that self-sustained oscillations are possible for the system, and compute for which values of K they can occur. [4p]

(b) What is the frequency of the oscillations? Is it changing with K? [2p]

(c) Are the oscillations stable in amplitude? [2p]

(d) Can you confirm that the system indeed oscillates by e.g. simulations? [2p]