EXAM IN CONTROL THEORY (TSRT09)

SAL: Take-home exam

TID: Friday 26th March 2021, kl. 8.00–12.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 5 + 10 + 10 + 15 +10 = 50)

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TILLÅTNA HJÄLPMEDEL: You are allowed to use any available help material (exercises, old exams, notes, etc.), but not to communicate with other persons.

LANGUAGE: You can write your exam in both English (preferred) or Swedish

HAND-IN OF SOLUTION: Upload your solution on Lisam (under the "Submission" menu). Recommended format is a single document bundling together matlab code, text, graphs and/or scans/photos of some hand written parts. Call the file with your personal number YYYYMMDDXXXX.pdf.

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: Difficult to organize. Online meetings can be set up upon request.

PRELIMINÄRA BETYGSGRÄNSER:	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. Below is a list of 5 tasks that you are asked to solve as a Control engineer. Below the 5 tasks is a list of 5 methods you learnt in this course. Associate each task with a method, motivating (briefly) your choice.

   Tasks:

   1 You must “measure” a signal for which it is impossible to place a sensor.

   2 You are asked to design a closed-loop system in which $|S(i\omega)| < 10^{-2}$ for all ω.

   3 You are given a quadratic penalty function in the state and input and asked to design a regulator which is optimal in some sense for that penalty.

   4 You must show global asymptotic stability for a nonlinear system.

   5 You are asked to design a closed-loop system in which $|S(i\omega)| < 2/(i\omega)$ and $|T(i\omega)| < \frac{(i\omega)^2+0.1(i\omega)+0.04}{(i\omega)^2+(i\omega)+0.04}\ \forall\, \omega$.

   Methods:

   A Lyapunov function

   B $\mathcal{H}_\infty$ design

   C Kalman filter

   D Bode integral theorem

   E LQ design

[5p]

2. (a) Consider the system

$$\dot{x}_1 = \sin(x_1 - 3x_2) + 2x_1x_2$$
$$\dot{x}_2 = \sin(2x_1) + x_1x_2^4$$

What type of equilibrium point (node, focus, saddle point, ...) is the origin? What can you say of the stability of the origin in the linear approximation and in the nonlinear system (locally around the origin)? [3p]

(b) Consider the system

$$\dot{x}_1 = \sin x_1 + x_2 + 2u$$
$$\dot{x}_2 = 2x_1 - u$$

Consider an output signal $y = \alpha x_1 + \beta x_2$, and choose α and β so that the relative degree of the system becomes 2. Compute then the feedback linearization that transforms the system into a linear second order system. [4p]

(c) In a closed-loop system we want the disturbance at the output to be damped by two orders of magnitude (i.e., two powers of 10) in the frequency range $0 \leq \omega \leq 1$, and the measurement disturbance to be damped also by two orders of magnitude but in the frequency range $a \leq \omega < \infty$, where $a > 1$.

i. Translate the specifications into specifications for S and T.
ii. Translates the specifications in S and T into specifications for the loop gain GF_y.
iii. Explain why it is difficult or impossible to satisfy the specifications when a is close to 1.

[3p]

3. Consider the system

$$G(s) = \begin{bmatrix} \frac{s+2}{(s-1)(s+1)} & \frac{s+4}{(s+2)(s+3)} \end{bmatrix}$$

(a) Compute the poles of the system. [2p]

(b) Compute the zeros of the system. [2p]

(c) How many singular values does this G have? Compute them. [2p]

(d) Compute $\|G\|_\infty$. [2p]

(e) If you could feedback regulate only one of the two inputs, which pairing would you choose $u_1 \leftrightarrow y$ or $u_2 \leftrightarrow y$? Why? [2p]

4. Consider again the system

$$G(s) = \begin{bmatrix} \frac{s+2}{(s-1)(s+1)} & \frac{s+4}{(s+2)(s+3)} \end{bmatrix}$$

This exercise consists of designing 3 different regulators. Note that you do not need to do calculations by hand, matlab is enough. Submit the code you use (and, as usual, justify your answers). Note further that each task is independent from the others (i.e., if you cannot solve one you can still try the others).

(a) Use PI controllers to stabilize the system. Report the values you use for the parameters of the PIs, the transfer function F_y and the resulting closed loop transfer function G_c. Produce plots of the resulting step response, of the sensitivity S and complementary sensitivity function T. Is the system also internally stable? *[Hint: see Definition 6.1 of the book.]* [5p]

(b) Solve the same problem using LQG. You can take R_1, R_2, Q_1 and Q_2 all equal to the identity (and $N = B$, $M = I$). In this case report L, K, a plot of the step response and plots of S and T. [5p]

(c) Solve the same problem with an $\mathcal{H}_2$ regulator, taking as weight functions the following:

$$W_S = \frac{1}{s+5}, \qquad W_T = \frac{s}{s+5}, \qquad W_u = [1 \;\; 1]$$

Report the extended system, the controller and observer gains and the plots of S and T. *[Hint: use the* `h2syn` *function of matlab. Hint #2:* `h2syn` *relies on state space. Whenever you have a transfer function transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s'), WS=minreal(ss(1/(s+5)))`*.]* [5p]

5. Consider the system in the left panel of Fig. 1, where

$$G(s) = \frac{1}{s(s+2)}$$

and K is a P gain.

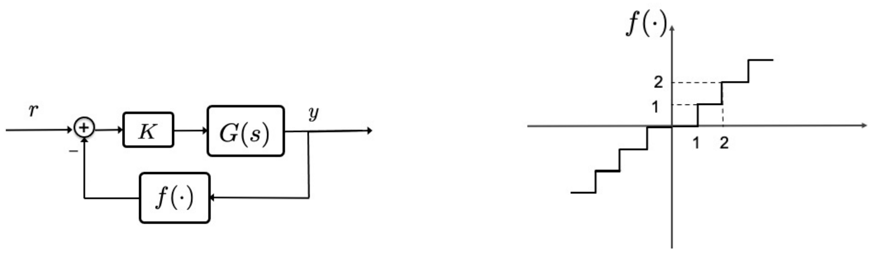

Figure 1: Exercise 5.

(a) For what values of K is the closed loop system stable if we disregard $f(\cdot)$ (i.e., assume $f(y) = y$)? [2p]

From now on assume $f(\cdot)$ is the quantizer given in the right panel of Fig. 1.

(b) Choose $K = 1$, and use the circle criterion to show stability. [4p]

(c) Can you use the small gain theorem to show stability in this case? [2p]

(d) What is the max value of K for which you have stability according to the circle criterion? And according to the small gain theorem? [2p]