EXAM IN CONTROL THEORY (TSRT09)

SAL: Take-home exam

TID: Tuesday 25th August 2020, kl. 14.00–18.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

ANSVARIG LÄRARE: Claudio Altafini, 073-9931092, e-mail: claudio.altafini@liu.se

KURSADMINISTRATÖR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL: You are allowed to use any available help material (exercises, old exams, notes, etc.), but not to communicate with other persons.

LANGUAGE: You can write your exam in both English (preferred) or Swedish

HAND-IN OF SOLUTION: Upload your solution on Lisam (under the "Submission" menu). Recommended format is a single document bundling together matlab code, text, graphs and/or scans/photos of some hand written parts. Call the file with your personal number YYYYMMDDXXXX.pdf.

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: Difficult to organize. Online meetings can be set up upon request.

PRELIMINÄRA BETYGSGRÄNSER:	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) Consider the linear system in state space form and a state feedback. What are the requirements in order to be able to place the poles of the closed loop system arbitrarily? [2p]
   (b) What is the difficulty in controlling a system with a pole in the right half of the complex plane? [2p]
   (c) Assume that you want to regulate the system

$$G(s) = \frac{-s+1}{(s+2)(s+3)}$$

   with an internally stable controller so that the closed-loop system has only one zero in $s = -3$. Can you solve the problem or not? Motivate your answer. [3p]
   (d) The number of poles in -1 for the system

$$\frac{1}{s+1}\begin{bmatrix} 1 & \beta \\ 1 & 1 \end{bmatrix}$$

   depends from β. How? [3p]

2. Consider the system in Fig. 1, where v_1 and v_2 are zero-mean white noises with intensities $R_1 = 6$ and $R_2 = 1$ and cross-intensity $R_{12} = 1$.

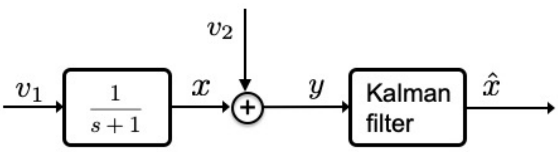

Figure 1: Exercise 2

   (a) Design a Kalman filter such that $\hat{x}$ is the optimal estimate of x. [2p]
   (b) Show that the transfer function from y to $\hat{x}$ of the resulting Kalman filter is $\frac{2}{s+3}$. [2p]
   (c) Compute the stationary variance of x, Ex^2, and the spectral density of x. [2p]

(d) Compute the stationary variance of $\hat{x}$, $E\hat{x}^2$, and the spectral density of $\hat{x}$. [2p]

(e) Show that indeed $Ex^2 = E\hat{x}^2 + E\tilde{x}^2$ where $\tilde{x} = x - \hat{x}$. [2p]

3. Consider the system

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{10}{s+2} \\ \frac{9}{s+3} & \frac{4}{s+4} \end{bmatrix}$$

One must choose between the following pairs of regulators: either

$$u_1 = K_1(r_1 - y_1), \quad u_2 = K_2(r_2 - y_2)$$

or

$$u_1 = K_1(r_2 - y_2), \quad u_2 = K_2(r_1 - y_1)$$

(a) Decide with the help of the RGA test which alternative is more suitable. [3p]

(b) Choose the regulator according to (a) and set $K_1 = K_2 = 2$. What are the poles of the closed-loop system? [4p]

(c) Plot the largest singular value of the sensitivity function with the regulator obtained in (b). How well are constant disturbances suppressed? In what frequency range can disturbances be suppressed? [3p]

4. Consider the control system shown in Fig. 2 where

$$f(e) = \begin{cases} 1 & e > 1 \\ 0 & |e| \le 1 \\ -1 & e < -1 \end{cases} \quad \text{and} \quad G(s) = \frac{K}{s(s+1)(s+2)}$$

(a) What are the requirements on K in order for the system not to have any sustained oscillation? [5p]

(b) Assume a self-sustained oscillation of amplitude 3 at the error e can be accepted. How big is K allowed to be? What is the corresponding angular frequency of the oscillations? [5p]

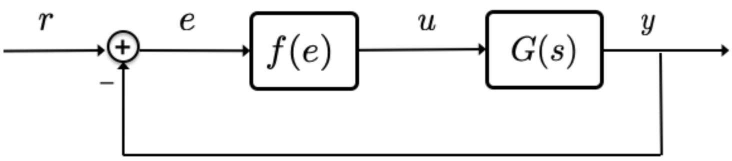

Figure 2: Exercise 4

5. Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

   (a) Compute a linear state feedback $u = -Lx$ so that the closed loop system has two distinct asymptotically stable eigenvalues in the origin. Sketch the resulting phase plane. [4p]

   (b) Assume that the feedback is based on measurement of the state through a sensor characterized by the nonlinear function

$$u = \begin{cases} -1 & \text{if } -Lx < -1 \\ -Lx & \text{if } \ |Lx| \leq 1 \\ 1 & \text{if } -Lx > 1 \end{cases}$$

   where $-Lx$ is the linear state feedback you obtained in (a). Sketch the new phase plane. What can you deduce of the behavior of the system, both near the origin and far away from it? [6p]