EXAM IN CONTROL THEORY (TSRT09)

SAL: Take-home exam

TID: Tuesday 9th June 2020, kl. 8.00–12.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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TILLÅTNA HJÄLPMEDEL: You are allowed to use any available help material (exercises, old exams, notes, etc.), but not to communicate with other persons.

LANGUAGE: You can write your exam in both English (preferred) or Swedish

HAND-IN OF SOLUTION: Upload your solution on Lisam (under the Submissionmenu). Recommended format is a single document bundling together matlab code, text, graphs and/or scans/photos of some hand written parts. Call the file with your personal number YYYYMMDDXXXX.pdf.

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: Difficult to organize. Online meetings can be set up upon request.

PRELIMINÄRA BETYGSGRÄNSER:	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) What is the practical meaning of the Bode integral theorem? [2p]

   (b) Consider

$$\dot{x} = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

   Can you stabilize the system via state feedback? [2p]

   (c) Consider

$$G(s) = \begin{pmatrix} \frac{1}{(s+1)(s+2)} & \frac{5(s+2)}{s+1} \\ \frac{s+1}{s+5} e^{-0.2s} & \frac{5}{s+3} \end{pmatrix}$$

   Compute RGA($G(0)$). [2p]

   (d) Compute an exact linearizing feedback for the system

$$\begin{aligned} \dot{x}_1 &= -2x_2 \\ \dot{x}_2 &= -x_2 + x_1 + \tan x_1 - u \\ y &= x_1 \end{aligned}$$

[2p]

   (e) A model of a system is given by

$$\dot{x} = x + 2u + n$$

$$y = 3x + u$$

   where n is a low-frequency noise with spectrum

$$\phi_n(\omega) = \left(\frac{1}{j\omega + 1}\right) \left(\frac{1}{j\omega + 1}\right)^*$$

   Augment the system model with a model of the noise, so that the resulting system has the form

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + Bu + N\tilde{n} \\ y &= C\tilde{x} + Du \end{aligned}$$

   where $\tilde{n}$ is a white noise of intensity 1.

[2p]

2. Given the system

$$G(s) = \frac{s+2}{(s+1)^3}$$

   (a) Design an Internal Model Controller (IMC) such that the closed loop system has the poles in the same location as the open loop system G. *[Note: it is enough to consider the "nominal case" discussed in the book, i.e., $G = G_0$, and $\tilde{F}_r = 1$.]* [4p]

   (b) Will the closed-loop system be able to follow a constant reference signal without stationary error? Why? [2p]

   (c) Assume now that the system is affected by various disturbances (see Fig. 6.1 of the book): system disturbance (at the output) w; input disturbance w_u; measurement disturbance n. Which of the following scenarios is likely to perform better with your current design:

      i. High frequency w, low frequency n and no w_u.

      ii. Low frequency w, high frequency w_u and n.

      iii. Low frequency w and w_u, and n which is a white noise (i.e., has all frequencies).

      iv. High frequency w and n, low frequency w_u.

   Motivate your answer. [4p]

3. Consider the following linear MIMO system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

where

$$A = \begin{bmatrix} -1 & 0 & -2 \\ 0 & -0.4 & -1 \\ -1 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & -2 \end{bmatrix}$$

   (a) Compute zeros and poles of $G(s) = C(sI - A)^{-1}B$. [2p]

   (b) Compute the static gain of G. What are stationary values of u that gives the max and min of the static gain? [2p]

   (c) Compute a static decoupling matrix F of the form $u = F\tilde{u}$. [2p]

   (d) Design a linear-quadratic regulator from the output y. [3p]

   (e) What can you say on the robustness of the LQ regulator computed at the previous point? [1p]

4. Consider the system

$$y = G(s)u + w, \qquad G(s) = \frac{s+2}{(s+0.2)(s-1)}$$

and the following weight functions

$$W_S = \frac{1}{s+1}, \qquad W_T = 1, \qquad W_u = 1$$

(a) Construct the extended system of eq. (10.4) of the book

$$\begin{bmatrix} z \\ y \end{bmatrix} = G_e \begin{bmatrix} w \\ u \end{bmatrix}$$

and check if it is in innovation form. [2p]

(b) Design a controller $u = -F_y y$, so that the $\mathcal{H}_\infty$ norm of the closed-loop system $z = G_{ec} w$ is minimized. Write down the resulting F_y. *[Hint: use the* `hinfsyn` *function of matlab. Hint #2:* `hinfsyn` *relies on state space. Whenever you have a transfer function transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s')`, `WS=minreal(ss(1/(s+1)))`*.]* [4p]

(c) Do there exist an F_y such that $\|G_{ec}\|_\infty \leq \gamma$ with $\gamma = 1$? What is the least value of γ for which an $\mathcal{H}_\infty$ controller exists? [2p]

(d) For the optimal value of γ, which of the 3 components of z is the most critical one (i.e., the corresponding component in G_{ec} has $\mathcal{H}_\infty$ norm closer to γ)? [2p]

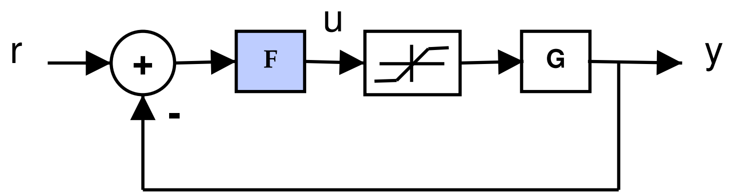
5. Given the linear system

$$G(s) = \frac{20}{(s+1)^2(s+3)}$$

with the input saturation

$$f(u) = \begin{cases} 1 & u > 1 \\ u & -1 \leq u \leq 1 \\ -1 & u < -1 \end{cases}$$

consider the closed-loop system in the following figure



where $F(s)$ is a P regulator, i.e., $F(s) = K$.

(a) Compute the maximal value of the gain K in the P controller for which the small gain theorem is satisfied for the closed loop system. [2p]

(b) Compute the maximal value of the gain K in the P controller, but this time using the circle criterion (an approximate upper bound for K is enough). [3p]

(c) Compute the maximal value of the gain K in the P controller, but this time using the describing function method. [3p]

(d) Discuss the various bound for K that you have obtained in (a), (b), and (c). Which are conservative and which are not? [2p]