Lecture 2

Dept. Of Electrical Engineering
Linköping University, Sweden

Outline

- Options
- Payoff diagram
- Trading strategies with options
- Put-call parity
- Rational pricing of options – some insights

Options

Forward/Futures

- No cost (except margin requirements)
- Holder obliged to trade underlying at maturity

Can one takes away the obligation?

Options : a piece of paper (financial instrument)

- gives the holder the right (but not obligation) to buy or sell a risky asset at an agreed price within a specified period

- Strike price (K) (for each unit of underlying)
- Maturity/ expiry date (T)
- holding requires an upfront payment (premium)
- Options available for different strike prices and expiry periods

Options types

Exchange traded : basic (or vanilla) options

- A call is an option to buy
- A put is an option to sell
- An European option can be exercised only at maturity
- An American option can be exercised at any time until maturity

Option positions

- Long : holder of the option
- Short : seller/writer of the option

Moneyness :

- At-the-money option
- In-the-money option
- Out-of-the-money option

Payoff diagram

The Payoff Function

- Example: European long call option

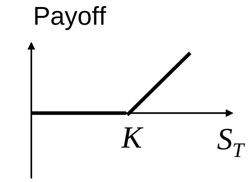
right but no obligation to buy at price K at maturity

K = Strike price,
S_T = Price of underlying at expiry

When $S_T \leq K$
Payoff = 0 (option worthless)

When $S_T > K$
Payoff = S_T - K (option exercised)

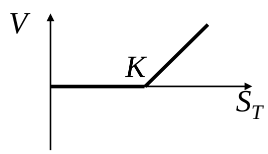
Note : Transaction costs ignored



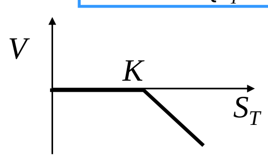
Payoff = max $\{S_T - K, 0\}$

- Payoffs from European Options

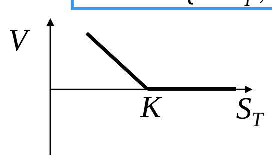
V = max $\{S_T - K, 0\}$



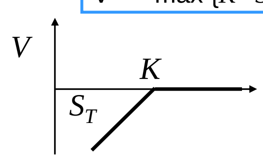
V = - max $\{S_T - K, 0\}$



V = max $\{K - S_T, 0\}$



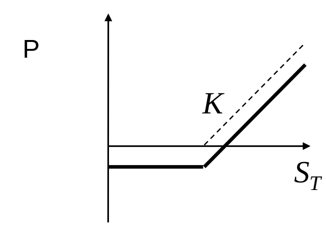
V = - max $\{K - S_T, 0\}$



Profit diagram :

- Initial upfront payment
- Time value of money

European long call



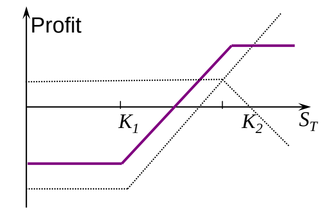
Trading strategies

Trading strategies with Options

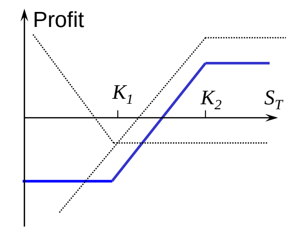
- **Spreads** (Two or more options of the same type)
  - Bull Spreads
  - Bear Spreads
  - Butterfly Spreads
  - ………..
- **Combinations** (Two or more options of different types)
  - Straddle
  - Strangles
  - ……….

Spreads

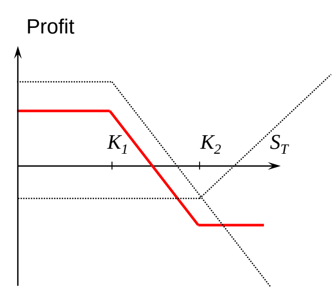
Bull Spreads using Calls



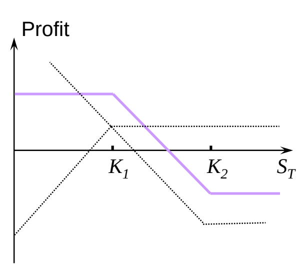
Bull Spreads using Puts



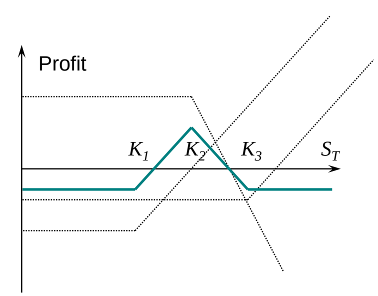
Bear Spreads using Calls



Bear Spreads using Puts

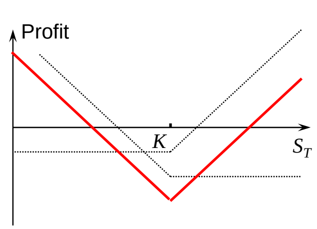


Butterfly Spreads using Calls

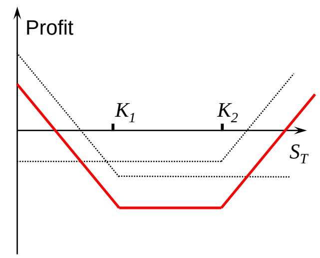


Combinations

Straddle



Strangle



Asymmetric risk (leverage)

Seller: risking a large loss to make a probable small profit !

- writing option very risky!

- Risk of default
- Margin account with clearing house

Option	Upside	Downside
Buying	Unlimited potential gain	Initial premium
Selling	Limited gain	Huge potential obligation

Market Makers

- Exchanges use market makers to promote options markets liquidity
- A market maker quotes bid and ask prices if requested
- The market maker does not know whether the individual wants to buy or sell

Option price before expiry ?

- No idea of future movement of the underlying
- Option worth is known at expiry
- Option has certain market price

c	European call option price
p	European put option price
C	American call option price
P	American put option price
r	Risk free interest rate (r >0)

Option price

A priori Bounds

- An American option is worth at least as the corresponding European option

  $C \geq c$ and $P \geq p$

- ***Upper Bounds***

  Call option: $C \leq S_0$ and $c \leq S_0$

  Put option: $P \leq K$ and $p \leq Ke^{-rT}$

Put-Call parity

Put – Call Parity

At $t = 0$

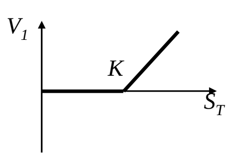
Portfolio 1:
long one European call and short one European put with same K and T

Worth = $(c - p)$

Portfolio 2:
long one stock and short cash worth Ke^{-rT} *in a risk free bank account with interest rate r (or a zero coupon bond worth* Ke^{-rT}*)*

Worth = $(S_0 - Ke^{-rT})$

At $t = T$, worth of portfolio1 = $(S_T - K)$ (*see the diagrams below*)
worth of portfolio2 = $(S_T - K)$



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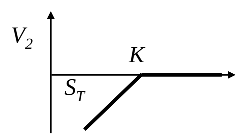
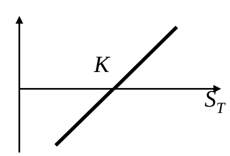
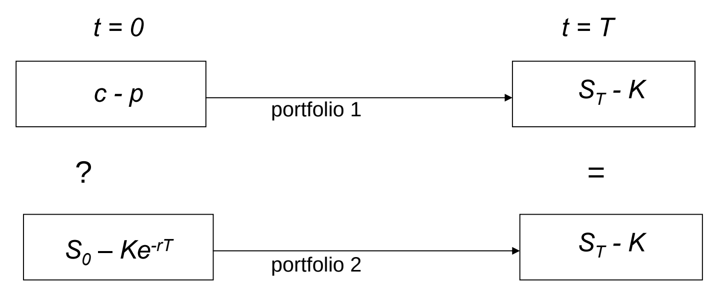


= $V = V_1 + V_2$



Payoff = $\{S_T - K\}$



No arbitrage => $c - p = S_0 - Ke^{-rT}$

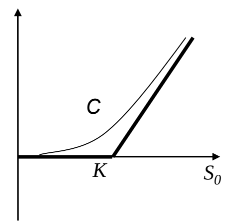
- Early exercise for American Option

American Call option : never optimal to exercise early !

Reason: $c \geq S_0 - Ke^{-rT}$ (P-C parity) and $C \geq c$
leads to $C \geq S_0 - Ke^{-rT}$

Now with $r > 0$ and $T > 0$, $C > S_0 - K$

- Better to sell the option than to exercise
- Possibility that stock price falls below K



Option price?

- Asymmetric risk between writer & holder of the options (!!!)
- Some *a priori* bounds (independent of the model for underlying)
- Market decides on the value of an option

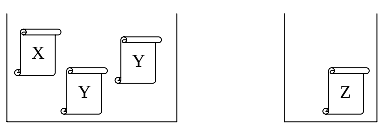
'**Fair value**' of an option before expiry ??

- requires mathematical model for the underlying (subject for next lecture)

Simple scenario & insights …….

Fair price of option

- No arbitrage ⇒ Pricing is linear operation !



if payoffs are equal, P(z) = P(X) + 2P(Y)

- Option & underlying driven by same source of randomness !

Can we replicate the option payoff ?

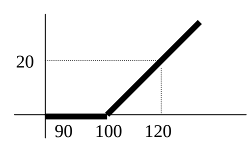
Single Period Binomial Option Model

- Finite state model with only 2 states

S 100 → 120 / 90 **B** 100 → 104 / 104

- **S**tock can become 90 or 120 with probabilities p, 1-p
- Cash in **B**ank A/c with risk free interest rate 4%

- Price of Call Option with strike K = 100 with 1 year to maturity ?



Payoff at maturity

$$\begin{cases} S - K & S \geq K \\ 0 & \text{otherwise} \end{cases}$$

Replication of option payoff

S 100 → 120 / 90   B 100 → 104 / 104   C ? → 20 / 0

- Replicate call using stock and cash in bank a/c
- Solution: $$\phi\begin{pmatrix}120\\90\end{pmatrix}+\psi\begin{pmatrix}104\\104\end{pmatrix}=\begin{pmatrix}20\\0\end{pmatrix}$$
- No arbitrage then implies:
  value option (C) = value replicating portfolio = $100\phi+100\psi$

The probability of stock going up/down (p and 1-p) never entered in calculations !

Single period replication: general model

More generally:

S → uS / dS 1 → $e^{r\Delta t}$ / $e^{r\Delta t}$ p_c → $\max\{0,uS-K\}=C_u$ / $\max\{0,dS-K\}=C_d$

$$\phi(uS)+\psi(e^{r\Delta t})=C_u$$
$$\phi(dS)+\psi(e^{r\Delta t})=C_d$$
$$\phi S(u-d)=C_u-C_d \quad\Rightarrow\quad \phi=\frac{C_u-C_d}{S(u-d)}$$

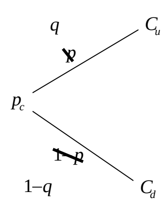
$$\psi=e^{-r\Delta t}(C_u-\phi uS)=e^{-r\Delta t}\left(\frac{uC_d-dC_u}{u-d}\right)$$

Price today : $p_c=\phi S+\psi 1$

So

$$\begin{aligned}p_c&=\phi S+\psi 1\\&=\frac{C_u-C_d}{u-d}+\frac{(uC_d-dC_u)e^{-r\Delta t}}{u-d}\\&=e^{-r\Delta t}\left[\frac{e^{r\Delta t}-d}{u-d}C_u+\frac{u-e^{r\Delta t}}{u-d}C_d\right]\\&=e^{-r\Delta t}\left[qC_u+(1-q)C_d\right]\end{aligned}$$

where $q=\dfrac{e^{r\Delta t}-d}{u-d}\in(0,1)$



- price = discounted version of expected payoff <u>under new probability</u> q instead of p

$$p_c=e^{-r\Delta t}\left[qC_u+(1-q)C_d\right],\qquad q=\frac{e^{r\Delta t}-d}{u-d}\in(0,1)$$

- Option price depends on *u, d,* and *r*
- but <u>not</u> on p (risk preference)
- hence <u>not</u> on mean growth rate of stock

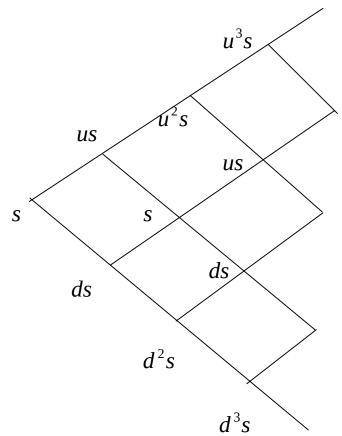
- Stock can attain more than 2 terminal values

  - divide the single time period into many small time periods

  Example: consider symmetric tree

  $$u = \frac{1}{d}$$

  - replication can be made dynamic



- If perfect replication possible:

  - selling option, while buying replicating portfolio is entirely risk free !

  - holding portfolio of only options is very risky

  - re-assess the risk asymmetry !!

Pricing possible due to dynamic replications

Perfect replication may not be always possible !!

More rigorous treatments of pricing – next few lectures

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