Excercises

1. Find the price of a european long call option on a stock (current value S_t) with infinite maturity. The strike price for the option is K and the risk free interest rate is r. Mention explicitly any other assumption.

2. (a) Earlier, in the lectures, we have assumed that the stock pays no dividend. A dividend is a cash payment made to the owner of the stock. If a dividend is declared on the stock, the date on which the dividend is paid, the stock price is reduced by the amount of the dividend. Establish a put-call parity relation when the underlying stock pays a dividend. (Hint: use principle of no arbitrage.)

(b) Consider a stock 'X' which pays a dividend of \$5.5 in 5 months. The spot price of the stock is \$52. A 1 year european call has price \$6 and a 1 year european put has price \$4 respectively. Both options have strike price of \$50. Find the amount of cash that must be lent at the risk free rate of return in order to replicate the stock.

3. Find the solution for the second-order Langevin equation

$$\ddot{X}(t) = K(t, X(t), \dot{X}(t)) + \sigma N(t)$$

interpreting it as a vector stochastic equation.

4. From equation

$$mV'(t) + fV(t) = W'(t),\ t \geq 0,\ V(0) = v_0\ \epsilon\mathbf{R}$$

describe the SDE of Ito and find the solution. (Hint: Solution is an Ornstein-Uhlenbeck process).

5. Show for $U(t) = X_1(t)X_2(t)$ with

$$\begin{aligned} dX_1(t) &= f_1(t, X_1)dt + g_1(t, X_1)dW(t) \\ dX_2(t) &= f_2(t, X_2)dt + g_2(t, X_2)dW(t) \end{aligned}$$

that the following formula is valid

$$dU(t) = dX_1(t)X_2(t) + X_1(t)dX_2(t) + g_1(t, X_1)g_2(t, X_2)dt$$

6. a)Use the Ito rule to prove that $d[f(t)W(t)] = f'(t)W(t)dt + f(t)dW(t)$ with $f \epsilon C'$deterministic.

b)Verify that for any $n \geq 2 : d[W^n(t)] = nW^{n-1}(t)dW(t) + \frac{1}{2}n(n-1)W^{n-2}(t)dt$

7. Consider a continuous-time market model with a bond and one stock, where the stock price is only influenced by a one dimension Brownian motion, i.e., $d = m = 1$.Given the price of the stock at a time t

$$P(t) = p \cdot exp((b - \frac{1}{2}\sigma^2)t + \sigma W_t)$$

model the Stock Price Equation.

The deadline for this exercises is the 22nd of November at 23.00, you can send it by email.