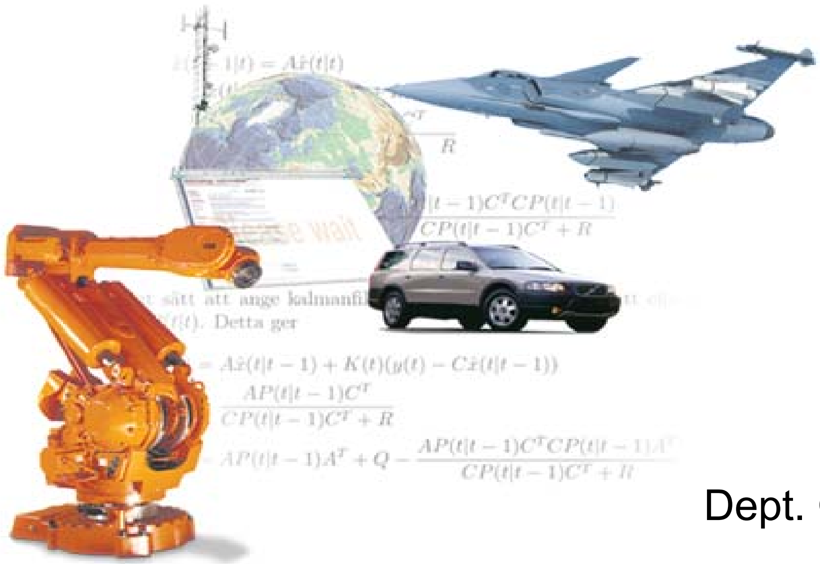
Lecture 9



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Cointegration: Basic ideas

Weak (or Covariance) stationary : process $\{x_t\}$

if (a) $E(|x_t|^2) \prec \infty$

(b) $E(x_t)$ independent of *t*

(c) $E(x_t x_{t+k})$ independent of *t*

$\Rightarrow$ Mean reverting process

(a) long term equilibrium, (b) short term deviation

Cointegration: Basic ideas

- If price process weak stationary
  - construct buy and sell signals !

Unfortunately price process not stationary, but
log price (may be) integrated !!

$\{x_t\}$ integrated of order 1 (i.e., $I(1)$)

(a) nonstationary

(b) $(x_t - x_{t-1})$ is stationary

Cointegration: Basic ideas

Two time series $\{x_t\},\{y_t\}$ are cointegrated if

$$\exists \beta \in R^d \text{ such that}$$

$$y_t + \beta x_t \text{ is stationary (loose def. !!)}$$

Objective : given two *I*(1) price series, find a linear combination

s. t. $z_t = x_t - \beta y_t = \mu + \varepsilon_t ; (\varepsilon_t \sim I(0)),$ stationary

Intuition: two closely related assets tend to "move together"

- the drunk and her dog (# M.P.Murray, 1994)

- Stocks from same industries more likely to be subjected to same systematic risk!

Cointegration: Basic ideas

- e.g., common trend

$$y_t = \beta_1 w_t + \varepsilon_{y,t} \qquad w_t \sim I(1)$$

$$z_t = \beta_2 w_t + \varepsilon_{z,t} \qquad \varepsilon_{y,t}, \varepsilon_{z,t} \sim I(0)$$

$\lambda = \frac{\beta_1}{\beta_2}$ implies $(y_t - \lambda z_t)$ is stationary

- Generalization: a linear collection of assets yielding an (approx.) stationary portfolio

- Test for stationarity ???? Augmented Dickey fuller test !

Cointegration: Basic ideas

- Vector error correction model (VECM)

  regression approach: one series chosen as dependent variable

  - choice is arbitrary !

Alternative,

VECM: treats the series symmetrically

Idea : when stationary linear combination deviates from mean, it is reverted towards its mean

Cointegration: Basic ideas

- Simple VAR(1) example

$$y_t = a_{11} y_{t-1} + a_{12} z_{t-1} + \varepsilon_{y,t}$$

$$z_t = a_{21} y_{t-1} + a_{22} z_{t-1} + \varepsilon_{z,t}$$

for VAR(1) to be cointegrated, certain restrictions on parameters (roots of charac. eqn. lie on or outside unit disc)

$$a_{11} = \frac{(1 - a_{22}) - a_{12} a_{21}}{1 - a_{22}};$$

$$a_{22} \succ -1; \; a_{12} a_{21} + a_{22} \prec 1$$

Cointegration: Basic ideas

VECM

$$\begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix} = \begin{bmatrix} a_{11} - 1 & a_{12} \\ a_{21} & a_{22} - 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{bmatrix}$$

VECM (alternative form)

$$\Delta y_t = \alpha_y (y_{t-1} - \lambda z_{t-1}) + \varepsilon_{y,t};$$

$$\Delta z_t = \alpha_z (y_{t-1} - \lambda z_{t-1}) + \varepsilon_{z,t};$$

$$\alpha_y = \frac{-a_{12}a_{21}}{1-a_{22}}; \alpha_z = a_{21};$$

$$\lambda = \frac{1-a_{22}}{a_{21}};$$

Generalized vector form

$$\Delta y_t = \alpha \beta^T y_{t-1} + \varepsilon_t$$

Cointegration: Basic ideas

Application of Kalman filter(KF):

- Compute pair wise cointegration relationship
- For each pair, validate stationarity
- For strongly mean reverting pair, design trading strategies
- For high frequency trading, we need fast computation of β : use KF

$$y_t = \beta_t x_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_y^2)$$
$$\beta_t = \beta_{t-1} + \upsilon_t. \qquad \upsilon_t \sim N(0, \sigma_\beta^2)$$

σ_y, σ_β : online MLE