Target Tracking: Lecture 6
Multiple Sensor Issues and Extended Target Tracking

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Lecture Outline

- Multiple Sensor Tracking Architectures
- Multiple Sensor Tracking Problems
- Extended Target Tracking

Multi Sensor Architectures cont’d

Figures taken from: M.E. Liggins and Kuo-Chu Chang
"Distributed Fusion Architectures, Algorithms, and Performance within a Network-Centric Architecture,"
Multi Sensor Architectures: Pros & Cons

- The traditional centralized architecture gives optimal performance but
  - Requires high bandwidth communications.
  - Requires powerful processing resources at the fusion center.
  - There is a single point of failure and hence reliability is low.
- For distributed architectures
  - Communications can be reduced significantly by communicating tracks less often.
  - Computational resources can be distributed to different nodes
  - Higher survivability.
  - It is a necessity for legacy systems e.g. purchased radars might not supply raw data.

Problems in Multi Sensor TT

- **Registration**: Coordinates (both time and space) of different sensors or fusion agents must be aligned.
- **Bias**: Even if the coordinate axis are aligned, due to the transformations, biases can result. These have to be compensated.
- **Correlation**: Even if the sensors are independently collecting data, processed information to be fused can be correlated.
- **Rumor propagation**: The same information can travel in loops in the fusion network to produce fake information making the overall system overconfident. This is actually a special case of correlation.
- **Out of sequence measurements**: Due to delayed communications between local agents, sometimes measurements belonging to a target whose more recent measurement has already been processed, might arrive to a fusion center.

Correlation

**Centralized Case**

Suppose the fusion center have the prediction $\hat{x}_{k|k-1}$ in both cases.

Centralized Case

$$\begin{bmatrix} y_1^{k} \\ y_2^{k} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x_k + \begin{bmatrix} e_1^{k} \\ e_2^{k} \end{bmatrix}$$

where $e_1^{k}$ and $e_2^{k}$ are independent.

**Decentralized Case**

$$\begin{bmatrix} \hat{x}^{1}_{k|k} \\ \hat{x}^{2}_{k|k} \end{bmatrix} = \begin{bmatrix} I \\ I \end{bmatrix} x_k - \begin{bmatrix} \hat{x}^{1}_{k|k-1} \\ \hat{x}^{2}_{k|k-1} \end{bmatrix}$$

Suppose the target follows the dynamics

$$x_k = A x_{k-1} + w_k$$

and the $i$th sensor measurement is given as

$$y_i^{k} = C_i x_k + e_i^{k}$$

Then with the KF equations

$$\begin{align*}
\hat{x}^{i}_{k|k} &= A \hat{x}^{i}_{k-1|k-1} + K_i^{i} (y_i^{k} - C_i \hat{x}^{i}_{k-1|k-1}) \\
&= A \hat{x}^{i}_{k-1|k-1} + K_i^{i} C_i (x_k - A \hat{x}^{i}_{k-1|k-1}) + K_i^{i} e_i^{k} \\
&= A \hat{x}^{i}_{k-1|k-1} + K_i^{i} C_i A (x_{k-1} - \hat{x}^{i}_{k-1|k-1}) + K_i^{i} C_i w_k + K_i^{i} e_i^{k}
\end{align*}$$
Correlation cont’d

Define $\tilde{x}_k^i \triangleq x_k - \hat{x}_k^i|k$, then

$$\tilde{x}_k^i = x_k - A\tilde{x}_{k-1}^{i|k-1} - K_k^i C_i A(x_{k-1} - \hat{x}_{k-1}^{i|k-1}) - K_k^i C_i w_k - K_k^i \epsilon_k^i$$

$$= Ax_{k-1} + w_k - A\tilde{x}_{k-1}^{i|k-1} - K_k^i C_i A(x_{k-1} - \hat{x}_{k-1}^{i|k-1}) - K_k^i C_i w_k - K_k^i \epsilon_k^i$$

$$= (I - K_k^i C_i)A\tilde{x}_{k-1}^i + (I - K_k^i C_i)w_k - K_k^i \epsilon_k^i$$

Hence

$$\bar{x}_k^i = (I - K_k^i C_i)A\tilde{x}_{k-1}^i + (I - K_k^i C_i)w_k - K_k^i \epsilon_k^i$$

We can calculate the correlation matrix $\Sigma_k^{ij} \triangleq E(\bar{x}_k^i \bar{x}_k^j^T)$ as

$$\Sigma_k^{ij} = (I - K_k^i C_i)A\Sigma_{k-1}^{ij}A^T (I - K_k^j C_j)^T + (I - K_k^i C_i)Q(I - K_k^j C_j)^T$$

Correlation Illustration

Maneuvers make this problem more dominant and visible.

Track Association: Testing

Test for Track Association [Bar-Shalom (1995)]:

- Two estimates $\hat{x}_{k|k}^i, \hat{x}_{k|k}^j$ and the covariances $\Sigma_{k|k}^i, \Sigma_{k|k}^j$ are given from ith and jth local systems.
- We calculate the difference vector $\Delta_{k}^{ij}$

$$\Delta_{k}^{ij} \triangleq \hat{x}_{k|k}^i - \hat{x}_{k|k}^j$$

- Then we calculate covariance $\Gamma_k^{ij} \triangleq E(\Delta_{k}^{ij} \Delta_{k}^{ij^T})$ as

$$\Gamma_k^{ij} = \Sigma_{k|k}^i + \Sigma_{k|k}^j - \Sigma_{k|k}^i - \Sigma_{k|k}^j$$

- Then test statistics $D_k^{ij}$ calculated as

$$D_k^{ij} = \Delta_{k}^{ij^T} (\Gamma_k^{ij})^{-1} \Delta_{k}^{ij} \leq \chi^2$$

can be used for checking track association.
What about the cross covariance $\Sigma_{ij}^{k}$?

- Simple method is to set it $\Sigma_{ij}^{k} = 0$.
- It can be calculated using Kalman gains if they are transmitted to the fusion center.

Approximation for cross covariance from [Bar-Shalom (1995)]:

- The following cross-covariance approximation was proposed:

$$\Sigma_{ij}^{k} \approx \rho \left( \Sigma_{i}^{k|k} \ast \Sigma_{j}^{k|k} \right)^{\frac{1}{2}}$$

where multiplication and power operations are to be done element-wise. For negative numbers, square root must be taken on the absolute value and sign must be kept.

- The value of $\rho$ must be adjusted experimentally. $\rho = 0.4$ was suggested for 2D tracking.

Track Association: Assignment Problem

Method proposed by [Blackman (1999)] for two local agents

Form the assignment matrix:

$$A_{ij} = \begin{bmatrix} T_{1}^{1} & T_{1}^{2} & T_{2}^{2} & T_{3}^{3} & \ldots \ & \ & \ & \ & T_{N}^{N} \end{bmatrix} \begin{bmatrix} T_{1}^{1} & T_{1}^{2} & T_{2}^{2} & T_{3}^{3} & \ldots \ & \ & \ & \ & T_{N}^{N} \end{bmatrix} \begin{bmatrix} NA_{1} & NA_{2} & NA_{3} \end{bmatrix}$$

where $\ell_{mn} = \log \frac{\beta_{ij} P_{i|j} N(\hat{x}_{m}^{k} - \hat{x}_{n}^{k}, 0, \Gamma_{mn}^{k})}{\beta_{NA}}$.

Then, use auction($A_{ij}$) to get track association decisions.

Track Association cont’d

Track association for more than two local agents.

- One way is to solve multi dimensional assignment problem.
- The simpler way is to do the so-called sequential pairwise track association.

Suppose we have $N_L$ local agents whose tracks need to be fused. Then, we order the local agents according to some criteria e.g. accuracy, priority, etc.

Track Fusion: Independence Assumption

Once we associate two tracks, we have to fuse them to obtain a fused track. This is called as track fusion.

Consider the track fusion at point A assuming $t_{CR} = t_{CT} = t$.

- Independence assumption gives

$$\left( \Sigma_{t}^{A} \right)^{-1} = \left( \Sigma_{t}^{B} \right)^{-1} + \left( \Sigma_{t}^{C} \right)^{-1}$$

$$\left( \Sigma_{t}^{A} \right)^{-1} \hat{x}_{t}^{A} = \left( \Sigma_{t}^{B} \right)^{-1} \hat{x}_{t}^{B} + \left( \Sigma_{t}^{C} \right)^{-1} \hat{x}_{t}^{C}$$

- This is simplistic and expected to give very bad results here.
- This is also called as naive fusion.
Track Fusion cont’d

Illustration of Correlation Independent Schemes.

\[ z^T (\Sigma_B^{-1}) z = 1 \]
\[ z^T (\Sigma_C^{-1}) z = 1 \]
\[ z^T (\Sigma_A^{-1}) z = 1 \]

Largest Ellipsoid
Algorithm
Covariance Intersection

Extended Target Tracking

Single extended target modeling
- Target state: kinematics and shape
- Number of measurements per target
- Motion modeling not covered. Typically possible to use point target models.

Multiple extended target tracking
- Data association for extended targets
- Measurement set partitioning

Extended Target Tracking

One measurement per target is often not valid, e.g.,
- Laser sensors, camera images, or automotive radar.

This leads us to the following definition:
- Extended targets are targets that potentially give rise to more than one measurement per time step.

Multiple measurements per target ⇒ possible to estimate the target’s extension.
- The spatial extension, i.e. size and shape.
- Hence the name extended target.

The extension estimate provides additional information, e.g. for classification and identification of different target types:
- Human?
- Bike?
- Car?
- etc...
Group targets

Closely related to extended targets is group targets.

- Closely spaced point targets
- Cannot be resolved
- Treated as single group target

Presented extended target models applicable to group targets.

Target shape – single ellipse

- Approximate shape as a single ellipse
- Vector \( x_k \) and positive semi-definite shape matrix \( X_k \),

\[
(z_k - x_k)^T X_k^{-1} (z_k - x_k) \leq 1
\]

Single extended target

- Airplane tracked with radar
- Reflection points, or sources, \( \bigtriangleup \) spread across surface. Locations and number vary with time.
- Measurements \( z_k \) may fall "outside" target due to noise
- Extended target state \( \xi_k \) contains parameters for
  - Kinematics
  - Extension

Not interested in sources, want to estimate the state \( \xi_k \) given the sets of measurements

\[
p(\xi_k|Z^k), \quad Z_k = \{z_1^k, \ldots, z_m^k\}, \quad Z^k = \{Z_1, \ldots, Z_k\}
\]

Target shape – single ellipse

“Extended Object and Group Tracking with Elliptic Random Hypersurface Models”, Baum et al.

- Cholesky decomposition of shape matrix \( X_k = L_k L_k^T \)

\[
L_k = \begin{bmatrix} a_k & 0 \\ b_k & c_k \end{bmatrix}
\]

- Extended state vector \( \xi_k = [x_k^T \ a_k \ b_k \ c_k]^T \)
- Assumed Gaussian models

\[
p(\xi_k|Z^k) = \mathcal{N}(\xi_k; m_k|k, P_{kk})
\]

\[
p(z_k|\xi_k) = \mathcal{N}(z_k; h_k(\xi_k), R_k)
\]
Target shape – single ellipse

“Extended Object and Group Tracking with Elliptic Random Hypersurface Models”, Baum et al.

- Simulation results (plots copied from paper)

A simple random matrix model.

- State $\xi_k$ is combination of kinematical vector $x_k$ and extension matrix $X_k$, $\xi_k = (x_k, X_k)$. Gaussian inverse Wishart distributed,

$$p\left(\xi_k | Z^k \right) = \mathcal{N} \left( x_k ; m_{k|k}, P_{k|k} \otimes X_k \right) \mathcal{IW} \left( X_k ; v_{k|k}, V_{k|k} \right)$$

- Called random matrix model
- Measurement model

$$p \left( z_k | \xi_k \right) = \mathcal{N} \left( z_k ; H_k x_k, X_k \right),$$

i.e. the covariance is given by the extension matrix.

Target shape – single ellipse

“Bayesian Approach to Extended Object and Cluster Tracking using Random Matrices”, Koch.

- Pros: Linear measurement update equations for extension state.
- Cons: Difficult to model sensor noise. The measurement model

$$p \left( z_k | \xi_k \right) = \mathcal{N} \left( z_k ; H_k x_k, sX_k + R_k \right)$$

implies that sensor noise is much smaller than extension.

“Tracking of extended objects and group targets using random matrices”, Feldmann et al.

- Alternative measurement model

$$p \left( z_k | \xi_k \right) = \mathcal{N} \left( z_k ; H_k x_k, X_k \right),$$

i.e. the covariance is given by the extension matrix.
A simple random matrix model.

- **Prediction Update**
  \[
  x_{k+1|k} = \Lambda x_{k|k}, \\
  P_{k+1|k} = A P_{k|k} A^T + BQ, \\
  v_{k+1|k} = \Lambda v_{k|k}, \\
  V_{k+1|k} = \Lambda V_{k|k},
  \]

  where \(0.5 < \lambda < 1\) is the forgetting factor.

- **Measurement model**
  \[
  p(z_k | \xi_k) = \mathcal{N}(z_k; H_k x_k, X_k),
  \]
  i.e. the covariance is given by the extension matrix.

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A simple random matrix model.

- **Measurement Update**
  \[
  p(x_k, X_k | Z^k) \propto p(Z_k | x_k, X_k)p(x_k, X_k | Z^{k-1})
  \]

- **Inverse Wishart** defines a conjugate prior for the Gaussian likelihood.
- A family of prior distributions is conjugate to a particular likelihood function if the posterior distribution belongs to the same family as the prior.
- Hence the posterior is in the same form.

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A simple random matrix model.

- **Measurement Update**
  \[
  \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - C \hat{x}_{k|k-1}), \\
  P_{k|k} = P_{k|k-1} - K_k S_k K_k^T,
  \]

  where,
  \[
  S_k = CP_{k|k-1}C^T + \frac{\hat{X}_{k|k-1}}{m}, \\
  K_k = P_{k|k-1} C^T S_k^{-1}
  \]

- **Extent State**
  \[
  v_{k|k} = v_{k|k-1} + m, \\
  V_{k|k} = V_{k|k-1} + \hat{Z}_k,
  \]

  “Tracking of extended objects and group targets using random matrices”, Feldmann et al.

- **State of the art filter**
  \[
  p(z_k | \xi_k) = \mathcal{N}(z_k; H_k x_k, X_k + R_k)
  \]

  Better sensor noise model, but the measurement update is heuristic and no longer linear.
Target shape – single ellipse

- Random matrix model popular in the literature, see e.g.
  - “Probabilistic tracking of multiple extended targets using random matrices”, Wieneke and Koch.
  - “A PHD filter for tracking multiple extended targets using random matrices”, Granström and Orguner.
  and the reference therein.

- Single ellipse also used by Degerman et al in “Extended Target Tracking using Principal Components”.
  - The shape matrix decomposed into rotation, major axis, and minor axis.

Target shape – multiple ellipses

- A single ellipse can be a crude model, multiple ellipses gives better approximation, see e.g.
  - Arbitrary number of ellipses, not same center point.
  - “Tracking of Extended Object or Target Group Using Random Matrix – Part II: Irregular Object”, Lan and Rong Li.

Target shape – Star convex

- A set $S(x_k)$ is called star-convex if each line segment from the center to any point is fully contained in $S(x_k)$.

- Figure: Description of star-convex shapes using a radius function $r = f(\theta)$.

Target shape – general shapes

- Single, or multiple, ellipses can be a crude model of the shape. Need for more general shape model.

- “Shape Tracking of Extended Objects and Group Targets with Star-Convex RHMs”, Baum and Hanebeck.

- The shape is given by a radial function $r(\theta)$ – distance from center of target to edge of shape at angle $\theta$.
  - Circle: $r = f(\theta)$
  - Canonical ellipse: $r(\theta) = \sqrt{a^2 \cos(\theta)^2 + b^2 \sin(\theta)^2}$
  - General case: truncated Fourier series expansion
    $$r(\theta) = a_0^2 + \sum_{j=1}^{N_F} a_j \cos(j\theta) + b_j \sin(j\theta)$$

- Shape parameter: $p = [a_0, a_1, \ldots, a_{N_F}, b_1, \ldots, b_{N_F}]^T$
Target shape – general shapes

- "Shape Tracking of Extended Objects and Group Targets with Star-Convex RHMs", Baum and Hanebeck.
- Compare with ellipse (plots copied from paper):

Multiple extended target tracking

- Assume that we have chosen the random matrix model for the extended targets.
- How can we handle multiple extended targets, when there is clutter and detection uncertainty?
- As with point target tracking, the data association problem must be solved.
- Different because each target can generate multiple measurements.
  - Which measurements were caused by clutter sources?
  - Which measurements were caused by targets? Of those, which were caused by the same target?

Data association for extended targets

- One approach: Partitioning of the measurement set

- A partition $p$ is a division of the set $Z_k$ into non-empty subsets, called cells $W$.

- Interpretation: One cell = one source.

Partitioning the measurements — example

Partition the measurement set $Z_k = \{ z_k^{(1)} , z_k^{(2)} , z_k^{(3)} \}$

- Data association for extended targets
- Partitioning the measurements — example
Partitioning method

- For optimality we must consider all possible partitions.
- Number of possible partitions for \( n \) measurements given by \( n \text{th Bell number } B_n \).
- The Bell numbers \( B_n \) increase very fast when \( n \) increases, e.g. \( B_3 = 5 \), \( B_5 = 52 \) and \( B_{10} = 115975 \).
- Computationally infeasible to consider all partitions.
- Necessary to approximate the full set of partitions with a subset of partitions.

Partitioning method – laser example

- Intuition: Measurements are from same source if they are close, with respect to some measure or distance.

Partitioning method

- Method: Measurements are in same cell \( W \) if distance is “small”.
- Distance between two cells \( W_j \) and \( W_i \) is measured as
  \[
  \min \{ d(z_i, z_j) : z_i \in W_i, \; z_j \in W_j \}
  \]
- Partitions \( p_i \) where all pairwise cell distances is \( < d_i \).
- Limit to partitions for thresholds \( d_i \) that satisfy
  \[
  d_{\text{min}} \leq d_i < d_{\text{max}}
  \]
- If possible, use scenario knowledge to choose distance measure and to determine bounds.
- Important to choose \( d_{\text{min}} \) and \( d_{\text{max}} \) conservatively.
Partitioning example

\[ p_1 = \{ W_{11}^1, W_{12}^1, W_{13}^1 \} \]

\[ p_2 = \{ W_{21}^2, W_{22}^2, W_{23}^2, W_{24}^2 \} \]

- Reasonable to discard most partitions as highly unlikely.
- Additional methods given in:
  "Extended Target Tracking using a Gaussian-Mixture PHD filter", Granström et al.
  "A PHD filter for tracking multiple extended targets using random matrices", Granström and Orguner.

Extended Target Models

- Extended target models applicable in scenarios where there are multiple measurements per target.
- Also applicable when multiple targets move in tight groups.
- Shape models range from simple ellipse to general shapes.
- Multiple extended target tracking possible using, e.g., the extended target PHD filter.

Extended Multi-target Tracking - Recent Works

- "A PHD filter for tracking multiple extended targets using random matrices", Granström and Orguner.
- "A Multiple-detection joint probabilistic data association filter", Habtemariam et al.
- "A multiple hypothesis tracker for multitarget tracking with multiple simultaneous measurements", Sathyan et al.

Extended Target Examples

- Laser Pedestrian
- Video Pedestrian
- Laser - Mix


Granström et al., Extended Target Tracking using a Gaussian-Mixture PHD filter

Granström and Orguner, A PHD filter for tracking multiple extended targets using random matrices