Uplink Load in CDMA Cellular Systems

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Abstract

Cellular mobile systems designed to handle multimedia services are currently being developed. The primary goal for any mobile system is to provide a satisfactory quality of service, both to operators and customers. Providing good quality of service while maintaining system stability requires accurate knowledge of the system load. In particular, it is important to be able to predict how a resource management decision will affect the stability of the network. This thesis addresses the problem of characterizing and estimating the uplink load in a cellular system using CDMA (Code Division Multiple Access) in the radio interface.

Uplink load can be related directly to the uplink power control problem of finding transmitter powers to support the users’ quality of service requirements. This yields a rather theoretical view. Another way of looking at uplink load is to relate it to the total received power users induce in the base station antennas. Both of these views are handled and relations between them are established in this thesis.

The literature survey on uplink load provided in this work concludes that practical estimates of the uplink load are generally what can be referred to as decentralized estimates; they use information gathered only in the immediate vicinity of the estimates’ host node. As an alternative to these estimates, a number of centralized estimates based on information readily available to the system are proposed. The usage of information from several cells makes them more sensitive to the soft capacity inherent in all CDMA cellular systems.

Extensive simulations in an advanced WCDMA (Wideband Code Division Multiple Access) simulator show that the estimates statistically perform well and the performance is insensitive to non uniform traffic load.

In the time domain, the load approximation can be described as oscillations superimposed on a slowly varying bias. If occasional high loads can be accepted, a non-oscillative signal representing the uplink load can be used to further increase the utilization of the resources. An algorithm is proposed to estimate the bias and a prediction of it.
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## Notation

### Symbols

- $a_i$: AR parameter
- $B$: Number of cells in the entire system
- $B_{tgt}$: Diagonal matrix with the users’ target CTIR in the diagonal
- $\beta_i$: CTIR user $i$ experiences
- $\beta_{tgt}$: User $i$’s target CTIR ratio
- $\beta_0$: Target CTIR in a single service system
- $C_i$: Received carrier strength of user $i$
- $C_{i,j}$: Received carrier strength of signal from user $i$ received in cell $j$
- $E$: Identity Matrix
- $e$: Measurement noise
- $\epsilon$: Residual, innovation
- $f$: Interference expansion factor
- $\gamma_i$: CIR user $i$ experiences
- $\gamma_{tgt}$: User $i$’s target CIR
- $\gamma_{i,k}$: CIR user $i$ experiences in cell $k$
- $\Gamma_{tgt}$: Diagonal matrix with the users’ target CIR in the diagonal
- $G$: Path gain matrix
- $g_{i,j}$: Path gain between user $i$ and cell $j$
\( h \) \quad \text{Threshold in CUSUM detector}

\( I_{\text{tot}} \) \quad \text{Vector with the received power users experiences}

\( I_{\text{tot}}^i \) \quad \text{Total received power user } i \text{ experiences}

\( j_i \) \quad \text{The cell user } i \text{ is connected to in a system not utilizing soft handover}

\( p_i \) \quad \text{Transmission power of user } i

\( K_i \) \quad \text{Active set of user } i

\( K \) \quad \text{Matrix representing the link configuration between users and base stations}

\( \Lambda \) \quad \text{Uplink noise rise, } \Lambda_j = \frac{I_{\text{tot}}^j}{N}

\( L_j \) \quad \text{Uplink Relative Load in cell } j, \quad L = 1 - \frac{1}{\Lambda_j}

\( L_f \) \quad \text{Feasibility Uplink Relative Load}

\( L_s \) \quad \text{System Load, } L_s = \max_j L_j

\( L_s^{IMRC} \) \quad \text{System load estimate, } L_s^{IMRC} = \max_j L_j^{IMRC}

\( L_s^{ISEL} \) \quad \text{System load estimate, } L_s^{ISEL} = \max_j L_j^{ISEL}

\( M \) \quad \text{Number of users in the entire system}

\( N \) \quad \text{Vector with the background noise power in the different cells}

\( N_i \) \quad \text{Background noise power user } i \text{ experiences}

\( N_j \) \quad \text{Background noise power in cell } j

\( \nu \) \quad \text{Drift term in CUSUM detector}

\( Q \) \quad \text{Process noise covariance matrix}

\( R \) \quad \text{Measurement noise variances}

\( T \) \quad \text{Filter update rate}

\( T_{\text{frame}} \) \quad \text{Length of a frame in WCDMA (0.01 s)}

\( Z \) \quad \text{Normalized path gain matrix, } Z_{i;\ell} = \frac{g_{i;\ell}}{\bar{g}_{i;\ell}}

\( \bar{Z} \) \quad \text{Normalized path gain matrix, } \bar{Z}_{i;\ell} = \frac{\bar{g}_{i;\ell}}{\bar{g}_{i;\ell}}

### Approximative Expressions for Uplink Noise Rise

\[
\begin{array}{c|c|c|c|c|c}
\beta_i^{\text{gt}} & I_k = I_j & N_k = N_j & \Lambda_{\text{IMRC}} & \Lambda_{\text{NIMRC}} & \Lambda_{\text{ISEL}} & \Lambda_{\text{NISEL}} & \Lambda_{\text{IBOTH}} & \Lambda_{\text{NIBOTH}} \\
\hline
\beta_i^{\text{gt}} & = & \sum_{k \in K_i} \beta_{i;k} & & & & & & \\
\beta_i^{\text{gt}} & = & \max_{k \in K_i} \beta_{i;k} & & & & & & \\
\beta_i^{\text{gt}} & = & \max(\sum_{k \in K_{i;1}} \beta_{i;k}, \sum_{k \in K_{i;2}} \beta_{i;k}) & & & & & & \\
\end{array}
\]
Abbreviations and Acronyms

3GPP    Third Generation Partnership Project
AR      Auto Regressive
CDMA    Code Division Multiple Access
CIR     Carrier-to-Interference Ratio
CTIR    Carrier-to-Total-Interference Ratio
CUSUM  CUmulative SUM
FDMA    Frequency Division Multiple Access
GSM     Global System Mobile
GoS     Grade of Service
QoS     Quality of Service
RNC     Radio Network Controller
RRM     Radio Resource Management
SIR     Signal to Interference Ratio
TDMA    Time Division Multiple Access
UMTS    Universal Mobile Telecommunications Systems
WCDMA   Wideband Code Division Multiple Access
The purpose of this chapter is to provide the reader with an overview of the material as well as a short motivation for this study.

1.1 Background

Mobile telecommunications have grown to be quite far from simple speech services for just a few people. Today, not just voice services but also data services are provided at a cost suitable for the general public. The second generation mobile systems are today at their limit of their capacity. Furthermore, the second generation mobile systems have a limited data communication ability, something which is expected to be strongly requested in the future. Therefore new systems are currently being designed. In these systems more sophisticated resource management and higher data rates than in the second generation systems are considered through the entire design process.

Some examples of third generation mobile systems are *Universal Mobile Telecommunication System* (UMTS), *cdma2000* or *Time Division-CDMA* (TD-CDMA). Different systems will be used in different places of the world. In Europe and Asia, UMTS will be used. One of the radio interfaces of UMTS is *Wideband Code Division Multiple Access* (WCDMA).
1.2 Problem Statement

Radio resource management algorithms control the amount of resources allocated to each subscriber at every time instant. This is not an easy task since there are many factors to consider, both subjective and objective ones, see Figure 1.1. Crucial for good performance of these algorithms is an ability to feedback an estimate of the current load, $L$. The load of a system is naturally defined as the ratio between used resources and total resources. Properties of the used radio interface make the total capacity of the system both unknown and time varying. This, in turn, makes the question of how much of the system’s total resources that are currently used hard to answer.

This work is focused on estimating the current uplink load of a Code Division Multiple Access (CDMA) radio interface, where the true load in one area depends on the situation in surrounding areas. The goal with this work is to develop and evaluate load estimates using readily available information gathered in several nodes of the network. The algorithms and estimates are applicable in a general CDMA network or even in a general network, but simulations have been done in a WCDMA environment.

**Figure 1.1** A radio system from an automatic control point of view. The system is expected to provide high user satisfaction despite variations in external signals such as the background noise, $N$, and user activity.
1.3 Contributions

Main contributions of this work are

- The survey on uplink load in WCDMA systems in chapter 3
- The derivation and evaluation of a set of centralized uplink load estimates in Chapter 4 and 7, respectively.
- The work presented in Chapter 5 which establishes relations between different views on uplink load.
- The adaptive filtering applied to a biased auto regressive model in Chapter 6.

The main part of the material presented has been published earlier. The introduction to uplink load in Chapter 3 also appears in


An evaluation of the estimates was done in


Estimates especially designed for dealing with a problem of information distribution was proposed and evaluated in


The adaptive filtering presented in Chapter 6 also appeared in

1.4 Thesis Outline

The next chapter provides an introduction to cellular radio systems and radio resource management. This work is focused on uplink load in systems with CDMA radio interfaces. Chapter 3 is a survey on different ways of looking at uplink load and different ways of estimating it in such a system. A set of centralized uplink load estimates are derived in Chapter 4. In the following chapter, relations between different definitions of uplink load and also links to the estimates in Chapter 4, are established. In Chapter 6, advanced signal processing has been applied to one of the estimates. The statistical properties of the estimates in Chapter 4 have been investigated through simulations in Chapter 7. Finally, some conclusions regarding the entire work are made in Chapter 8.
This chapter gives a brief introduction to some fundamentals of radio communications. A requirement for applying math to wireless communication systems is obviously a way of describing how a signal is changed as it travels through the air. Since the exact behaviour of the propagation channel is far too complex to be described exactly, a model is used. The first section of this chapter describes how the propagation channel is modeled. Besides the propagation channel, the two other important parts of a radio system are the transmitter and the receiver. Section 2.2 describes the generic parts of these two components.

For many reasons, one being that the bandwidth available for radio communication is limited and therefore expensive, users have to communicate using the same frequencies. Methods for sharing the available bandwidth are presented in Section 2.3. Because of this sharing, users will interfere with each other. However, if they can be spatially separated, a user will share the available bandwidth with less users. This is one of the ideas behind cellular radio networks. Theory regarding cellular radio networks is further explored in Section 2.4. Different people will have different expectations on the system. What these expectations are depends on what kind of relation you have with the system. An attempt to characterize the performance of the system is done in Section 2.5. In order to utilize the available resources in an efficient manner, radio resource management algorithms are used. In
Section 2.6 some fundamental radio resource management algorithms are mentioned and their purpose explained.

Exactly how these algorithms are implemented is a choice of the individual system manufacturers. However, successful operation over manufacture borders requires standardization, both in terms of radio network architecture and in protocols between for example transmitters and receivers. The last section of this chapter presents some details of the architecture of a WCDMA system and, for the present work, interesting parts of the current standard.

2.1 Radio Wave Propagation

A signal propagating through the air is subject to attenuation. Given perfect knowledge of the surrounding environment, this attenuation can be calculated using Maxwell’s equations. This, of course, is not practically feasible for many reasons. Therefore, we have to rely on a simplified version of the reality, a model. A criteria for the model is that it will provide a statistically correct description of the attenuation. Instead of modeling the attenuation, path gain, \( g \), is often modeled. The received signal power can comprehensively be expressed as transmitted power times the path gain. The model is often separated into three components. The product of the three is the path gain

\[
g = g_p g_s g_f < 1,
\]

where \( g_p \) represents path loss, \( g_s \) shadow fading and \( g_f \) fast fading\(^1\).

**path loss** is the long term attenuation caused by the distance between transmitter and receiver. Path loss is the dominating factor in for example satellite communications. It is usually modeled as

\[
g_p = C_p r^{-\alpha},
\]

where \( C_p \) is a constant which depends on the gain at the receiving antenna and the wavelength of the radio signals, \( r \) is the distance between transmitter and receiver and \( \alpha \) is a radio environment dependent propagation exponent, ranging from 2 (free space propagation close to the antenna) to 5.5 (far from the antenna in a very dense urban environment). This model, with

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\(^1\)The transmitting antenna’s gain and performance of some of the algorithms in the receiver can also be incorporated in the path gain.
2.1 Radio Wave Propagation

terrain dependent $\alpha$ and $C_p$, was verified by Okumura et al. (1968) and Hata (1980). In our application, cellular radio systems, $\alpha$ is usually taken equal to 4 (Gilhousen et al., 1991).

**Shadow fading** is due to large obstacles in the radio environment, objects which may absorb the radio wave. This part of the path gain is not, unlike the free space propagation loss, strictly increasing with the distance. Shadow fading is usually modeled using a log-normal distribution (Hata, 1980; Okumura et al., 1968)

$$g_s = 10^{\xi/10}, \xi \in N(0, \sigma_s).$$

This model assumes that the user is standing still and thus experiences the same shadow fading over time. A user moving around in the environment will experience time varying shadow fading. The correlation between two consecutive samples of the shadow fading depends on how fast the user is moving. Gudmundson (1991) proposes a model where the correlation is expressed using a relation between the user’s speed, $v$, and a *correlation distance*, $d_0$. This distance is chosen together with an additional constant $\epsilon_D$ such that the correlation between the shadow fading at two points separated a distance $d_0$ should be $\epsilon_D$. The dependence between consecutive samples is then implemented by filtering the $\xi$-values through a first order low pass filter with a pole at $\epsilon_D$

$$y(t) = \frac{(1 - \frac{vT}{\epsilon_D})q_x(t)}{q - \frac{vT}{\epsilon_D}}$$

$$g_s = 10^{y/10}.$$

$T$ is the sampling time used.

**Fast fading** is caused by signals being reflected on obstacles in the radio environment. The reflections cause a signal to be received in several copies. Since these copies may arrive at different times and with different strength, they interfere either constructively or destructively. Fast fading depends on the user’s position relative the surrounding environment. Thus one position does not have a time constant fast fading due to a time varying radio environment. This contribution can change very rapidly, hence the name fast fading. Further details on fast fading can be found in Sklar (1997). Figure 2.1 shows how a user’s path gain may vary as he moves through the
environment. Notice the occasional deep fades and how the gain is sometimes larger than 1 (0 \, dB). Fast fading also causes local deep fades in the frequency spectrum. In case of narrow band communication this can be devastating.

One way of decreasing the variations in experienced path gain which the fast fading causes is to use a RAKE receiver. A RAKE receiver estimates the relative delay of separate signal copies (rays). The information from different rays can then be combined providing a more stable total path gain after the RAKE receiver. Figure 2.1 shows the fast fading gain after the RAKE receiver.

2.2 Radio Communication Systems

Figure 2.2 shows the generic parts of a radio system. A message given to the source encoder can be in practically any format, such as a text file, a picture or speech. The source encoder converts this information into a string of bits.

These bits are then given to a channel encoder. A channel encoder adds redundancy bits, which in the receiver will be used to correct errors induced between sender and receiver.
Both information bits and redundancy bits are then used to modulate a carrier signal. This process produces a high frequency signal which is suited for transmission over the air interface.

At the other end, on the receiver side, things are basically done in the opposite direction. However, algorithms here are much more complicated. For example, demodulation usually requires accurate synchronization between receiver and sender. The channel decoder uses the redundancy bits introduced by the channel encoder to detect and possibly correct bit errors. Finally, the source decoder converts the bits into the form of the original information. In order to provide a certain service to the users, the system has to provide each user with a received signal to interference ratio (SIR). A user’s SIR is the ratio between the received power of the user’s signal and the interference power. The interference power consists of the background noise power, $N$ and the signal power from all other users currently transmitting using the same frequency band (see Section 2.3). SIR is closely related to the more generally known signal-to-noise ratio, the difference lies in the fact that SIR considers the actual noise power, i.e., not just background noise but also noise originating from other users. Another user quality related quantity is a user’s carrier-to-interference ratio (CIR), denoted $\gamma_i$. This is a measure of the power of the signal received from the user versus the interfering noise power, when measured at the receiving antenna. Thus CIR is measured in the radio frequency band and SIR is measured in the base band, see Figure 2.2.

![Figure 2.2 The generic parts of a radio system.](image-url)
2.3 Multiple Access

Because of limited availability of radio spectrum, the radio spectrum has to be shared between several users. This is done using some sort of multiple access technique. Regardless of which algorithm is used, the multiple access is implemented as a part of the modulation and demodulation of Figure 2.2. Below is a description of the three most common techniques, divided into two groups based on whether they use orthogonal signals or not.

2.3.1 Orthogonal Signals

A rather simple, but yet in many areas wide spread, technique is called Frequency Division Multiple Access (FDMA). The idea here is simply to split the available radio bandwidth into a number of (possibly differently sized) parts and assign each user one part. A publicly known example of FDMA is radio broadcasting. A drawback when using FDMA is that each user is limited to a (narrow) frequency band. Each user is stuck at using his assigned frequencies, even if there is locally heavy interference on this frequency band or the path gain is locally exceptionally low on these frequencies due to fast fading.

The second generic technique, Time Division Multiple Access (TDMA), splits the radio spectrum in time, instead of in frequency. This technique requires precise synchronization between all users (which in some areas and applications even means taking the propagation time into account). An example of where TDMA is used is when several users share the same walkie talkie system. When using TDMA, each user is momentarily allocated the system’s entire available frequency band. This means transmission over a larger bandwidth, and therefore less sensitivity to local narrow band interference.

The signals in the above mentioned techniques are what is sometimes referred to as orthogonal-signals. Ideally the users do not interfere with each other.

2.3.2 Nonorthogonal Signals

The third technique, Code Division Multiple Access (CDMA), uses non-orthogonal-signals. Using this technique, the users transmit independently of each other, using the same frequency. This is the technique used in the third generation cellular system, which is why it will be explained in a bit more detail.
The idea behind CDMA is that each user is assigned an individual spreading code. The spreading code consists of a number of chips, each chip is either 1 or -1. The number of chips per second, the chip rate, is higher than the symbol rate at which the user intends to send the information. The ratio between chip rate and symbol rate is called processing gain, $PG$. Because of the processing gain, each user needs a carrier-to-interference ratio which is a factor $PG$ lower than he would need in for example FDMA to maintain the same signal-to-interference ratio. It is the ability to use such a low carrier-to-interference ratio that has made CDMA popular in military applications, the transmitted signal can easily be hidden in the background noise and detecting it requires knowledge of the spreading code. In cellular applications however, the interference power is far higher using CDMA compared to the two other multiple access techniques shown here due to the concurrent transmission of several users on the same frequency band. Thus the required transmission power is still not decimated a factor $PG$. Due to the spreading, which is done in time domain (see Figure 2.3), the modulating signal, $s$, has a bandwidth which is a factor $PG$ larger. With an increased bandwidth the transmitted signal is less sensitive to fast fading. The inherent usage of a large bandwidth makes CDMA more spectrum efficient than the orthogonal signal techniques.

2.4 Cellular Radio Networks

Because of the attenuation between user and the access point providing the service, a too large distance implies an impairment in service and in the extreme case lack of connection due to limited transmission power. Hence, the distance has to be kept short.

By dividing the entire service area into a number of cells, each served by
an access point (in our case referred to as a *base station*), the attenuation can be kept at moderate levels yielding lower requirements on transmission powers. The attenuation’s dependence on distance is thus utilized here since users far from each other will create negligible mutual interference if they belong to different cells. A system providing radio service over an area divided into several cells is called a *cellular radio network*.

The size of the cells depends on the radio environment, expected user density and type of traffic. In an area where several users are expected to use high bit-rate-services simultaneously, the cells are chosen to be smaller. Typical cell radii are between a couple of hundred of meters (in a dense urban environment) to several tenths of kilometers (in rural areas). In practice, a cell’s size is defined by the location of the users connected to the cell. A user chooses cell based on the pilot signal from different base stations; the user applies for a connection to the base station with the strongest pilot signal. Therefore, since the transmission power of the pilot signal is variable, a cell can actually be resized dynamically during operation in order to adapt to changing user densities. This technique is referred to as cell breathing (Jalali, 1998).

In radio systems in general one denotes the direction from user to the access point (i.e., base station in the case of cellular networks) as *uplink* and conversely the opposite direction as *downlink*.

The uplink performance of a cellular radio network is highly dependent on the path gains between each user and the base stations. A comprehensive way of representing these path gains is through the (time varying) path gain matrix

\[
G(t) = \begin{pmatrix}
g_{1,1}(t) & \cdots & g_{1,B}(t) \\
\vdots & \ddots & \vdots \\
g_{M,1}(t) & \cdots & g_{M,B}(t)
\end{pmatrix},
\]

where \(g_{i,j}\) is the path gain between user \(i\) and the base station serving cell \(j\). \(M\) and \(B\) are the number of users and the number of cells in the entire network, respectively. The \(G\)-matrix representing downlink is in general different from the one representing the uplink.

Using the above notation for path gain between users and base stations, a user’s uplink carrier-to-interference ratio can be expressed as

\[
\gamma_i(t) \triangleq \frac{C_i(t)}{I^{\text{tot}}_i(t) - C_i(t)} = \frac{p_i(t)g_{i,j}(t)}{\sum_{l \neq i} p_l(t)g_{l,j}(t) + N_j(t)},
\]

where \(I_i^{\text{tot}}(t)\) and \(p_i(t)\) are user \(i\)'s individual *total interference power* and transmission power, respectively at time \(t\), \(C_i(t)\) is the received carrier power
of user $i$ and $N_j(t)$ is the background noise in cell $j$. The sum is over all users using the same channel. A similar ratio is the carrier-to-total-interference ratio (CTIR), $\beta$, defined as

$$\beta_i(t) \triangleq \frac{C_i(t)}{I_{\text{tot}}(t)} = \frac{\sum_{l=1}^{M} p_l(t)g_{l,j}(t)}{\sum_{l=1}^{M} p_l(t)g_{l,j}(t) + N_j(t)} < 1,$$

where $M$ is the number of users sharing the same frequencies, i.e., all users in the entire network in the case of CDMA.

According to Equation (2.2), the system performance is dependent on the amount of induced interference from other users. Hence, it is of utmost importance to maintain a suitable transmission power at all times, a transmission power such that the required carrier-to-interference ratio is maintained while not inducing excessive interference to other users. Because of this, a fundamental mechanism of a cellular radio network is power control. Power control regularly decides what transmission power commands to send to each transmitter in the network (mobile phones and base stations). The intercell interference, which is due to the fact that all cells use the same frequencies, makes the power control problem much harder. However, since all cells share the same frequencies in a CDMA cellular network, there are no longer any hard frequency allocation problems to solve. In for example the second generation system used in Europe, GSM, huge amounts of efforts have been put into solving the complex optimization problem of allocating frequencies to different cells. In CDMA cellular system, the interference from other cells is dealt with in the same manner as the interference from the own cell.

As a user moves from one cell to another, the path gain to the cell he is connected to will decrease while the path gain to the neighbouring cell will increase. It is therefore natural to, at some point, change the cell the user is connected to. This maneuver is called hand over. Since communication takes place on the same frequency in all cells of a CDMA cellular system, a user can actually be connected to several cells at a time. So, signals to/from a user located in between two or more cells can be send/received in several nearby located base stations. A user connected to several cells is said to be in soft handover. The cells that user $i$ is connected to is called the active set, denoted $K_i$.

Using soft handover, the system combines the information from all the cells in the active set. A user who is in soft handover is power controlled from all cells in the active set, obeying only the cell requiring the lowest transmission power. A base station can be designed to serve several cells.
In case a user is in soft handover between two or all of these cells, the extra information regarding the correlation of the received signals can be utilized. Hence, one separates between the two cases of soft handover where the user is connected to one or several physical base stations. The user is said to be in softer handover between the cells belonging to the same base stations and soft handover when the cells are served by separate base stations. Of course, a combination of the two is possible.

User’s link configuration in a system can conveniently be visualized through a matrix, $K$, which is very much similar to the $G$-matrix. If user $i$ has a link to cell $j$ the element $K_{i,j}$ equals 1 otherwise it is 0. So, as an example, in a system with 2 users and 2 cells where user 1 is connected to both cells and user 2 is solely connected to cell 2 the link matrix would be

$$K = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$ 

Concerning the actual combining of the received signals, selection combining.

![Diagram](image)

**Figure 2.4** The combination of soft and softer handover in a WCDMA system.
is used on signals received from different base stations while *maximum ratio combining* is used in the base stations. Hence, soft handover yields selection combining and softer handover maximum ratio combining. Figure 2.4 can perhaps make the combining more clear. When combining signals using maximum ratio combining, the combined signal’s carrier-to-interference ratio is the sum of the ratios obtained in each cell

\[ \gamma_{i,b} = \sum_{k \in C_b} \gamma_{i,k}, \quad (2.4) \]

where \( C_b \) is the set of cells controlled by base station \( b \) and \( \gamma_{i,k} \) is user \( i \)’s carrier-to-interference ratio in cell \( k \). Selection combining, on the other hand, means that the central node simply chooses the signal associated with the highest carrier-to-interference ratio. Therefore the carrier-to-interference ratio after the selection combining is

\[ \gamma_i = \max_{b \in R} \gamma_{i,b}, \quad (2.5) \]

where \( R \) is the set of base stations controlled by the central node.

**Example 2.1 (Intercell Interference)**

Consider a system with two users and two cells. The path gain matrix is

\[ G = \begin{pmatrix} g_{1,1} & g_{1,2} \\ g_{2,1} & g_{2,2} \end{pmatrix}. \]

User 1 is solely connected to cell 1 while user 2 is initially not connected to the system at all. User 1 has a service requiring a carrier-to-interference ratio of \( \gamma_1 \). User 1’s transmission power is then simply

\[ \gamma_1 = \frac{p_1 g_{1,1}}{N} \iff p_1 = \frac{\gamma_1 N}{g_{1,1}}, \]

where \( N \) is the background noise power common to both cells. If user 2 is provided another type of service requiring a carrier-to-interference ratio of \( \gamma_2 \) through a connection with cell 2, the required transmission powers satisfy the following system of equations

\[
\begin{cases} 
\gamma_1 = \frac{p_1 g_{1,1}}{N + p_2 g_{2,2}} \\
\gamma_2 = \frac{p_2 g_{2,2}}{N + p_1 g_{1,2}}
\end{cases}
\]
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Solving for \( P = (p_1 p_2)^T \) yields

\[
P = \begin{pmatrix} g_{1.1} & -g_{1.2} \gamma_{21} \\ -\gamma_{21} g_{1.2} & g_{2.2} \end{pmatrix}^{-1} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} N.
\]

Explicitly,

\[
p_1 = \gamma_1 \frac{g_{2.2} + \gamma_{21} g_{1.2}}{g_{21} g_{1.1} + \gamma_1 \gamma_{21} g_{1.2}} N
\]

\[
p_2 = \gamma_2 \frac{g_{1.1} + \gamma_{12} g_{2.2}}{g_{12} g_{2.2} - \gamma_2 \gamma_{12} g_{1.2}} N.
\]

Thus, user 1’s transmission power is affected by the decision to admit another user, even if it is in a neighbouring cell. Furthermore, since the relation between the users’ transmission powers and their CIR values is nonlinear, the factor by which user 1’s transmission power is increased highly depends on both \( \gamma_1 \) and \( \gamma_2 \).

User 1’s increase in transmission power in the above example is to compensate for the increased intercell interference power.

2.5 System Performance

The performance of the system is quite hard to measure. First of all one needs to define what we expect from the system. This expectation clearly depends on what kind of user you are: subscriber or operator.

A subscriber typically expects to have a high probability of being allowed to use the system at any given time. This defines a quantity called Grade of Service (GoS). Once connected, the subscriber expects to experience some sort of Quality of Service (QoS) which is a truly subjective measure. For example, in the case of a speech service, the subscriber expects to hear the other person reasonably well, or in the case of a data-service, that the connection is fast enough without a too long delay. A connected subscriber expects to be able to finish his business without being disconnected, something that is considered far worse than not getting a connection at all. These different measures, just from the subscribers point of view, makes defining the system performance a hard task.

The operator, who owns and manages the system, also puts expectations on the system. These can for example be total throughput. However, maximizing the total throughput often comes with a degrade in individual
subscriber satisfaction, which then, in the long run, means less customers for the operator.

This leads to a discussion of which policy to use, i.e., how to prioritize subscribers and what service to give them. The result is of course a trade off between individual subscriber satisfaction and maximum utilization of the resources.

2.6 Radio Resource Management Algorithms

Obviously the ability of a system to provide service is limited. However, it is not as obvious in what way it is limited. An always present trade off is the one between capacity, quality and coverage. We want to provide service to as many subscribers as possible at any given time. Allocating one subscriber a higher data rate or lower bit error probability means that his received signal power will be higher and thus higher interference to other subscribers is introduced. This means that less users can be accommodated into the system. Furthermore, since the mobiles’ transmission powers are limited, higher interference power in the base stations means less coverage.

The choice of which subscribers to admit and what service they will be provided is a matter of policy. The choices which repeatedly have to be made in order to follow this policy are done by Radio Resource Management (RRM) algorithms. There is a wide range of RRM algorithms, starting from algorithms making decisions several times each millisecond such as fast power control to more long term decisions such as admission control which considers time periods of whole sessions (e.g., expected length of a phone call).

2.6.1 Power control

Power control is used to set the users’ transmission power such that they experience the block error rate (BLER) they have been guaranteed by other RRM algorithms, while not inducing unnecessary interference power into the system. For uplink capacity reasons among others, users’ transmission powers have to be kept to a minimum while still providing satisfactory user individual SIR. Power control is often implemented as two loops, inner and outer loop.

The inner loop, also known as fast power control, adjusts each user’s transmission power such that the SIR equals a user individual target SIR,
$\gamma^{tgt}$. An example of the actual algorithm is one based on decision feedback

Receiver: $s_i(t) = \text{sign}(\gamma_i^{tgt}(t) - \gamma_i(t))$

Transmitter: $p_i(t) = p_i(t) + \Delta s_i(t - n)$,

where $\Delta$ is a fixed value by which the transmission power is changed every update and $n$ is a delay caused by measurement times and discrete transmission instants. A fast update rate enables tracking of the fast fading at low velocities. In for example WCDMA the fast power control updates each user’s transmission power 1500 times a second. However, a fast update rate also induces a few problems. Perhaps most obvious is the amount of signaling required between each user and the base stations, but most severe is the ability to rapidly raise the transmission powers to levels which may jeopardize the system stability.

The outer power control loop, adjusts each user’s target SIR. Here, quality is simply converted into a certain target BLER. The outer loop is necessary because an average SIR can not be directly mapped to a BLER, i.e., if SIR fluctuates too much the received BLER increases. The outer loop adjusts each user’s target SIR at a rate of somewhere around 10 to 100 Hz.

### 2.6.2 Load Controlling Resource Management Algorithms

If the system has too many users, power control can not find transmission powers such that all users achieve their target SIR, not even if they had access to infinite transmission power. Too many users in the system simultaneously results in what is usually referred to as party effect, which can be understood by studying Equation (2.2). An increase in the interference power, results in power control demanding an increase in transmission power. Increased transmission power means higher interference power, and so on. If a stable solution, providing required carrier-to-interference ratio to all users in the network, is possible given all constrains such as limited transmission power, the power control problem is said to be feasible, otherwise it is infeasible (Zander, 1993). To avoid too high transmission powers caused by too many users in the system, we need some way of controlling the load of the system. Therefore the system has a number of load controlling resource management algorithms which operate on different time scales, all much slower than the fast power control. Below is a brief explanation of some load controlling radio resource management algorithms.

- **Admission Control**, which is basically a door watchman. In order to send information, a user needs a session. A new user is given a session only if the system is expected to cope with that user’s request...
for system resources throughout the entire session. Note that, being granted a session does not always imply continuous access to the network resources, see Packet Scheduling and Channel Switching below. Admission control can for example study uplink received power as load measure, see (Choi and Bhak, 2001; Dimitriou et al., 2000; Holma and Laakso, 1999; Huang and Yates, 1996; Ishikawa and Umeda, 1997; Kuri and Mermelstein, 1999; Lei et al., 1999; Outes et al., 2001; Timus and Pettersson, 2001).

- **Packet Scheduling**, is the task of selecting which packet users that will be granted to send. This is typically done on a time scale of about ten to twenty milliseconds. This algorithm only applies to packet users, i.e., not for speech users. Laakso et al. (1998) formulate the packet scheduling problem as an optimization problem. A special mode of WCDMA in which higher data rates can be accomplished at the price of less fairness is discussed by Kolding et al. (2002).

- **Channel Switching** handles users who already have a session. During the session, a user’s channel conditions and the load of the base stations change. To adapt to these changes, channel switching increases and decreases the users’ maximum transmission rate. A lower transmission rate means a lower target SIR. Channel switching in WCDMA is studied by Gyung-Ho and Dong-Ho (2000).

- **Congestion Control** can be compared to a security guard. In more extreme cases, when neither Channel Switching or Packet Scheduling can take the system from an overload situation back to the target load, congestion control uses actions which are stronger than the previous algorithms. Examples of possible actions are
  - denying users’ requests for increased transmission power in the downlink
  - move some users to an alternative network, e.g., GSM
  - down switching speech users to a worse speech quality or even drop some real time users.

Being disconnected is usually consider far more annoying for a user than never being connected at all. Hence disconnection is strongly avoided by the system.

- **Link Admission** is the algorithm considering users’ requests for augmenting their active set with another cell. Important to note here is
that the link admission decision depends on the downlink only. The reason being that an extra link means higher transmission power for the base station serving the new cell. In the uplink, on the other hand, an additional link does not imply increased transmission power. Therefore, the uplink can only gain from more soft handover links.

2.7 WCDMA

2.7.1 WCDMA Architecture

The architecture of a WCDMA system contains a number of levels. Starting from cell level, the service in each cell is provided by a base station, see Figure 2.5. A base station hosts all lower level (link level) algorithms such as fast power control. The figure also shows how signals from different base stations are combined further up in the hierarchy in a radio network controller, RNC. A radio network controller receives information gathered in several cells, even from cells controlled by other RNCs. This, more complete, information about the system as a whole can be used by radio resource management algorithms in general and relative load estimates in particular. All load controlling algorithms in the previous section are located in a RNC.

2.7.2 WCDMA Standard

A common standard is a requirement for product interaction over manufacture borders. The committee specifying the WCDMA standard is called Third Generation Partnership Project (3GPP). They specify parameters regarding measurement accuracies, possible data-rate configurations and power control frequency to mention a few. In this section we will only mention the small part of the standardization process concerning this work.

To enable reasonable well tracking of the fast fading while not using too much bandwidth, the power control feedback has a bandwidth of 1500 bps. The standard states that the system should contain RNCs. Each RNC controls a number of base stations. Since the RNC has knowledge of the situation in a far larger area than a single base station does, it is here the load controlling algorithms reside.

\[^{2}\text{http://www.3gpp.org}\]
Figure 2.5 The architecture of a WCDMA system. The outside world is connected to the network at Radio Network Controllers (RNC). Each RNC is connected to a number of base stations (BS). Lower down in the architecture, each base station is connected to several mobiles.
3

Uplink Load

Introduction

This chapter will address uplink capacity related theory within the area of CDMA cellular radio systems. A term which is central in this theory, the *uplink relative load*, will be introduced and explained in Section 3.1. Since the uplink is in fact interference power limited, and not limited by the amount of hardware in the system, one separates between load measures based on *hard* resources and those based on *soft* resources. These two terms will be explained and compared in the next section as well.

Since the amount of used capacity is related to the received interference power there is not a one-to-one relation between how loaded the system is and how many channels that are occupied. In other words, used capacity should no longer be put in terms of how many channels that are currently occupied, but in amount of received interference power or a transformation thereof such as the uplink relative load. Both of the two latter quantities are hard to accurately measure or estimate. It is therefore much more complicated to determine how loaded a CDMA cellular system is, compared to systems where the capacity is limited by the amount of hardware.

Some resource management algorithms reside in a base station. An advantage is of course that a minimum of signaling is required if the decisions can be made on this intermediate level. Relative load estimates designed
to reside in base stations are referred to as decentralized estimates. Theory related to such estimates is presented in Section 3.2.

A base station has no access to information regarding the situation in cells supported by other base stations. Thus, a decentralized estimate cannot predict how resource management decisions will affect cells supported by other base stations. However, nodes at a higher level in the system’s architecture have access to information gathered in several cells. This more complete information about the system as a whole can be used by load estimates at this level. Section 3.3 discusses load from a more theoretical point of view.

The theoretical total capacity, the pole capacity, is interesting to study for comparative purposes, even though it is not achievable in practice. Along the way, the pole capacity will be discussed from the different angles presented here.

Finally, Section 3.4 discusses different aspects of the presented approaches to estimating the uplink load. All inequalities between vectors in this chapter should be interpreted component-wise.

### 3.1 System Load and Capacity

A basic requirement for providing service to users is that there is sufficient power available to maintain an acceptable Quality of Service. In the uplink this, among other things, means that the total received interference power in the base station must not be too high. The total received uplink interference power is

$$ I_{\text{tot}} = N + \sum_{i=1}^{M} C_i, \quad (3.1) $$

where $N$ is the background noise power, $C_i$ is user $i$’s received carrier power and $M$ is the number of users in the entire network. In the literature, interference power is often related to background noise power through the uplink noise rise.

**Definition 3.1 (Uplink Noise Rise)**

Uplink noise rise, $\Lambda$, is defined as the total uplink received interference power, $I_{\text{tot}}$, divided by the background noise power, $N$, i.e.,

$$ \Lambda \triangleq \frac{I_{\text{tot}}}{N}. $$
3.1 System Load and Capacity

Since the uplink noise rise is the constraining resource, and increasing the number of active users or the active users’ quality results in a noise rise increase, there is a natural trade off between the number of users and quality. Furthermore, as the users have limited transmission power, a higher noise rise means reduced coverage. An always present trade off is therefore one between the number of users, quality and coverage. Perhaps not useful in practice, but still an educational model is

\[
\text{quality + number of users + coverage} = \text{utilized resources.}
\]

The amount of available soft resources is unknown and time varying. An alternative is therefore to estimate the uplink relative load or uplink fractional load, \(L\), i.e., the amount of currently utilized resources relative to the total amount of resources

\[
\text{quality + number of users + coverage} = L \cdot \text{total resources.}
\]

Since performance of the system is related to the currently received interference power, a more formal definition of \(L\) should incorporate this property of the system. One way is to relate the useful received interference power to the total received interference power. Mathematically this can be expressed as

\[
L = \frac{\sum_{i=1}^{M} C_i}{N + \sum_{i=1}^{M} C_i} = \frac{I_{\text{tot}} - N}{I_{\text{tot}}} = 1 - \frac{1}{\Lambda}.
\]

We will use the above as a definition of \(L\).

**Definition 3.2 (Uplink Relative Load)**

The uplink relative load of a CDMA cellular system is defined as

\[
L \overset{\Delta}{=} 1 - \frac{1}{\Lambda}.
\]  

(3.2)

A rearrangement of the expression in the above definition yields the pole equation (Holma and Laakso, 1999)

\[
\Lambda = \frac{I_{\text{tot}}}{N} = \frac{1}{1 - L}.
\]  

(3.3)

The equation clearly shows that \(L = 0\) implies \(I_{\text{tot}} = N\), i.e., an empty system with only background noise. As \(L\) approaches one the system is operated close to the system’s theoretical capacity, the pole capacity, and the interference power goes to infinity, see Figure 3.1.
Example 3.1 (Uplink Relative Load, Single Cell)
Consider a single cell scenario. The total received interference power can be expressed as in Equation (3.1), and the QoS of each user is related to the carrier-to-total-interference ratio, $\beta_i$

$$\beta_i = \frac{C_i}{I_{tot}}.$$  (3.4)

Solving for $C_i$ in (3.4) and inserting it into (3.1) yield

$$I_{tot} = N + \sum_{i=1}^{M} \beta_i I_{tot} \iff \Lambda = \frac{I_{tot}}{N} = \frac{1}{1 - \sum_{i=1}^{M} \beta_i}.$$  (3.5)

The equation has the same form as Equation (3.3) and therefore according to Definition 3.2

$$L = \sum_{i=1}^{M} \beta_i.$$  (3.6)

The relative load is clearly a function of the received interference power. In the multi cell case, the relative load is therefore not purely a function of how many users there are in the system, but also for example where the users are located in the radio environment. This means that the pole capacity in terms of e.g., number of users is in general both unknown and time varying. From the inherent requirement of a positive noise rise and Equation (3.3) we can conclude that a feasible total resource allocation is associated with a relative load between zero and one. The above example shows that the capacity of a CDMA system with conventional receivers (i.e., not utilizing multiuser detection) is in fact interference power limited because the interference from other connections limits the capacity. The opposite is a noise limited system.

Example 3.2 (Noise Limited System)
Consider a system with one user in an isolated cell. The carrier-to-total-interference ratio is

$$\beta = \frac{C}{I_{tot}} = \frac{C}{N + C} < 1,$$
3.1 System Load and Capacity

eliminating $C$ in this expression gives

$$I_{tot}^\beta = \frac{1}{1-\beta} N < \infty.$$  

The total interference power, $I_{tot}^\beta$ is finite for all possible $\beta$ since $\beta < 1$. In a noise limited system, quality (data rate) is therefore limited by the amount of available transmission power.

Equation (3.5) implies that the number of users in an interference power limited system is, even with unlimited transmission power, limited by the mutual interference power between the connections. Furthermore, the amount of additional interference power caused by admitting a new user depends on the current interference power in the system. Figure 3.1 illustrates the higher interference power contribution of an admitted user at high load, compared to that at low load. In Figure 3.1, $\Delta L$ is associated with the amount of relative load a user contributes with. For example in the isolated cell case, $\Delta L = \beta_{tot}$ for the admitted user according to (3.6).

A traditional definition of relative load, which is usually used in a FDMA or TDMA system, is the number of currently used channels over total number of channels, i.e.,

$$L_{trad}^L \triangleq \frac{M}{M_{max}}.$$  

A capacity defined as a fixed maximum number of channels is an example of a hard capacity. The hard capacity of a system is fixed and known, unlike the soft capacity. The soft capacity can only be achieved when a soft resource is studied in the resource management algorithms. Uplink noise rise is an example of a soft resource since it depends on time varying variables such as the path gains the users experience. As the uplink of a CDMA cellular system is limited by this spatial resource, the uplink’s capacity depends on the situation in several cells. If the load is low in surrounding cells, little interference power is received from these cells. This results in an increased capacity in the own cell compared to when the surrounding cells are more loaded. A centralized resource management algorithm based on a soft resource can, unlike a decentralized algorithm studying a hard resource, utilize this additional capacity. An informative example will be given towards the end of the chapter, see Example 3.4.

In the cases where the soft capacity equals the hard capacity, the traditional definition of relative load coincides with Definition 3.2. Consider a
single cell situation in which the only service available is characterized by a target carrier-to-total-interference ratio of $\beta_0$. The pole capacity can then be calculated using Equation (3.6) as ($L = 1$ and assuming perfect power control, $\beta_i = \beta_0$, give the maximum number of users)

$$1 = \sum_{i=1}^{M^{pole}} \beta_0 = M^{pole} \beta_0 \Leftrightarrow M^{max} = M^{pole} = \frac{1}{\beta_0}.$$

According to Equation (3.6), the relative load in this scenario is merely the number of users times $\beta^tst$. The relative load can thus be expressed as

$$L = M \beta_0 = \frac{M}{1/\beta_0} = \frac{M}{M^{max}} = L^{trad}. \quad (3.7)$$
Huang and Yates (1996) studies a single service scenario with several cells. The total interference is therein divided into three parts; background noise, intracell interference, \( I_{\text{own}} \), which is interference from users within the cell and intercell interference, \( I_{\text{other}} \), which is interference from users served by another cell

\[
I_{\text{tot}} = N + I_{\text{own}} + I_{\text{other}}. 
\]  

(3.8)

Since a single service scenario is studied, \( I_{\text{other}} \) can be converted into a corresponding number of users, \( M_{\text{other}} = I_{\text{other}} / C \), where \( C \) is the received power required to maintain \( \beta_0 \). This once again enables an expression corresponding to Equation (3.7), only with \( M \) substituted by \( M_{\text{own}} + M_{\text{other}} \).

### 3.2 Decentralized Load

Some resource management algorithms act in a decentralized node. Therefore, an attractive property of an estimate is that it uses only information locally available. For example, the part of the interference power caused by users connected to another cell, \( I_{\text{other}} \), depends on variables not known in the own cell. Therefore a decentralized estimate has to either measure it or somehow estimate it using local variables only.

#### 3.2.1 Interference Expansion Factor

One way of eliminating the inter-cell interference from Equation (3.8) is to simply state that it is a nominal fraction, \( f \), of the intra-cell interference, i.e.,

\[
I_{\text{other}} = f I_{\text{own}}.
\]

This is a natural assumption since an increase in interference power in one cell leaks to surrounding cells. Higher interference power, in turn, causes users in these cells to use higher transmission power. This effect would not be captured if \( I_{\text{other}} \) would have been assumed constant.

Combining the above expression with Equation (3.8) results in an expression for \( I_{\text{tot}} \) that contains only local variables

\[
I_{\text{tot}} = N + (1 + f) I_{\text{own}}.
\]

According to Equation (3.4), the received power of user \( i \) is \( C_i = \beta_i I_{\text{tot}} \). The interference power from the own cell, \( I_{\text{own}} \), is simply the sum of these user
individual carrier powers. An expression for the total interference power is therefore

\[ I_{\text{tot}} = N + (1 + f) \sum_{i=1}^{M_{\text{own}}} C_i = N + (1 + f) \sum_{i=1}^{M_{\text{own}}} \beta_i I_{\text{tot}}. \]

Solving for \( I_{\text{tot}} \) yields

\[ I_{\text{tot}} = \frac{N}{1 - (1 + f) \sum_{i=1}^{M_{\text{own}}} \beta_i}. \]

Once again comparing with Equation (3.3) we see that an estimate of the uplink relative load in a multi cell scenario is

\[ L = (1 + f) \sum_{i=1}^{M_{\text{own}}} \beta_i. \tag{3.9} \]

A comparison with Equation (3.6) shows that one should not consider cells as isolated, since this gives an underestimation of the relative load. This is perhaps obvious since considering cells as isolated corresponds to completely ignoring the intercell interference, \( I_{\text{other}} \) in Equation (3.8). Using this technique, the requirement for pole capacity that the relative load should equal one corresponds to

\[ \sum_{i=1}^{M_{\text{own}}} \beta_i = \frac{1}{1 + f}. \]

Hence, an estimate of the pole capacity is put in terms of combined carrier-to-total-interference ratio. In case of single service scenario, an estimate of the maximum number of users would be

\[ M^{\text{max}} = \left\lfloor \frac{1}{(1 + f) \beta_0} \right\rfloor. \]

The intercell-to-intracell factor, \( f \), is widely used throughout the literature. Boyer et al. (2001); Hiltunen and Binucci (2002); Holma and Laakso (1999); Ying et al. (2002); Zhang and Yue (2001) and Sanchez et al. (2002) uses it in relative load expressions. A range in which \( f \) is usually chosen is between 0.5 and 0.6 if uniformly distributed traffic is expected.
### 3.2.2 Interference Power Measurement Based Approximations

Another way of considering the entire interference power using only local information is to simply assume that it is measurable. As concluded in Section 3.1, the increase in interference power due to an admitted user depends on the interference level. Holma and Laakso (1999) uses measurements of $I_{tot}$ and approximates the additional interference power a new user would cause through derivatives of Equation (3.2)

$$\frac{\delta I_{tot}}{\delta L} = \frac{N}{(1 - L)^2} = I_{tot} \frac{1}{1 - L}$$

An approximative expression for the interference power increase due to an additional load of $\Delta L$ is then

$$\Delta I_{tot} = \frac{\delta I_{tot}}{\delta L} \Delta L = I_{tot} \frac{\Delta L}{1 - L_0},$$

where $L_0$ is the relative load before admitting the new user. An alternative expression for the interference power increase is derived as the integrated difference in $I_{tot}$,

$$\Delta I_{tot} = \int_{L_0}^{L_0 + \Delta L} \frac{\delta I_{tot}}{\delta L} dL = I_{tot} \frac{\Delta L}{1 - L_0 - \Delta L}.$$

Motivated by the calculations leading to Equation (3.6), $\Delta L$ can be estimated as the new user’s target CTIR. Using measurements of the total interference power inherently catches the variations in intercell interference. However, it relies heavily on somewhat accurate measurements of the current uplink interference power.

### 3.3 Centralized Load

The estimates in Section 3.2 can not predict the effects a decision made in one cell will have on other cells. Especially important are users located close to the cell border. These users can occasionally introduce considerable interference power in other cells as the following example shows.

---

**Example 3.3**

Study a prospective user in cell $k$. Power control will force him to use a
transmission power $p_i$ which satisfies

$$C_{i,k} = p_i g_{i,k} = \beta_i^{\text{tgt}} I_k^{\text{tot}} \Rightarrow p_i = \beta_i^{\text{tgt}} I_k^{\text{tot}} \frac{1}{g_{i,k}},$$

where $g_{i,k} (< 1)$ is the path gain between user $i$ and cell $k$ and $I_k^{\text{tot}}$ is the total received interference power in cell $k$. User $i$’s signal will be received in cell $j$ with a power of

$$C_{ij} = p_i g_{i,j} = \beta_i^{\text{tgt}} I_k^{\text{tot}} \frac{g_{i,j}}{g_{i,k}}.$$

$C_{ij}$ will in cell $j$ be a part of the intercell interference, $I_{\text{other}}$. How large $C_{ij}$ will be depends on where the user is located. If user $i$ is close to the cell border, $g_{i,j}$ and $g_{i,k}$ will be of the same order and $C_{ij}$ will thus to a greater extent contribute to the uplink load in cell $j$, compared to the case where user $i$ is close to base station $k$.

From a resource management point of view it is quite interesting to have an idea of how much load a user will actually induce in the own and surrounding cells. This, however, requires information which is gathered in several cells and an estimate of the uplink load using this information would therefore have to reside in a more centralized site than a base station serving just a few cells.

### 3.3.1 Feasibility Relative Load

Earlier relative load has been related to received interference power. It can equally well be related to the users’ transmission powers. All work in this section are done under an assumption that a user is power controlled in exactly one cell, i.e., soft handover is not utilized here. Zander (1993) studies solvability of the power control equation in the downlink. The power control equation is derived below. Let user $i$ be power controlled solely from cell $j_i$. The $i$:th position of the vector $j$ thus contains the number of the cell user $i$ is connected to. User $i$’s downlink carrier-to-interference ratio can be expressed as

$$\gamma_i = \frac{C_i}{I_i^{\text{tot}} - C_i} = \frac{p_i g_{i,j_i}}{N_i + \sum_{t \neq i} p_t g_{t,j_t}} = \frac{N_i}{\frac{N_i}{\gamma_i} + \sum_{t \neq i} \frac{p_t}{\gamma_i}}.$$

Since this is the downlink, $p_i$ is the transmission power which information to user $i$ is sent with. $N_i$ is the background noise user $i$ experiences. Introduce
3.3 Centralized Load

the following matrix and vectors

\[ P(t) \triangleq [p_i(t)], \quad \eta = [\eta_i] \triangleq \left[ \frac{N_i}{g_{i,j_i}} \right], \quad Z = [z_{ik}] \triangleq \left[ \frac{g_{i,j_k}}{g_{i,j_i}} \right]. \]

The matrix \( Z \) has size \( M \times M \). In a single service system, i.e., all \( \gamma_i = \gamma_0 \), the above equation for all users can be put in matrix form

\[ P(t) = \frac{\gamma_0}{1 + \gamma_0}(\eta + ZP(t)). \quad (3.10) \]

Clearly, it is interesting to study for which \( \gamma_0 \) this equation is solvable (with all \( p_i > 0 \)). Zander (1993) shows that, given knowledge of all path gains in the system, it is possible to determine the maximum achievable \( \gamma_0 \) in the noise free case, \( \gamma_0^* \).

**Theorem 3.1 (Zander, 1993)**

Whenever the receiver noise is negligible, there exists, with probability one, a unique maximum achievable C/I-level

\[ \gamma_0^* = \sup\{\gamma_0 | \exists P \geq 0 : \gamma_i \geq \gamma_0 \forall i\}. \]

The maximum is given by

\[ \gamma_0^* = \frac{1}{\lambda^* - 1}, \]

where \( \lambda^* \) is the greatest real eigenvalue of \( Z \). The power vector \( P^* \) achieving this maximum is the eigenvector corresponding to \( \lambda^* \).

According to Theorem 3.1, simply choosing all users’ \( \gamma_i^{tg} \) less than or equal to \( \gamma_0^* \) guarantees existence of a positive solution to Equation (3.10) in the single service case and in absence of noise, i.e., \( N_i = 0 \). However, Zander (1993) also states that even in the noisy case, where \( N_i > 0 \), the influence of background noise can be made arbitrarily small by scaling all users’ transmission powers with a factor which is large enough.

The above theorem can be used to determine whether or not there is a base station transmission power vector \( P \) such that all user achieves the maximum possible C/I-requirement. In a cellular system, it can sometimes be interesting to use a smaller \( \gamma_0 \) than \( \gamma_0^* \). By formulating and solving a linear programming problem Zander (1993) shows the following theorem
Theorem 3.2 (Zander, 1993)
Whenever $\gamma_0 < \gamma_0^*$, the power vector $P$ of least total (sum) power achieving the C/I-level $\gamma_0$ will be the solution to the following system of linear equations:

$$\left( \frac{1 + \gamma_0}{\gamma_0} E - Z \right) P = \tilde{N},$$

where $E$ denotes the identity matrix.

Thus, by combining Theorem 3.1 and 3.2, we can conclude that all possible $\gamma_0 < \gamma_0^*$ are possible to achieve. As pointed out by Gunnarsson (2000), requiring all users’ $\gamma_i^{tgt} < \gamma_0^*$ is a sufficient but not necessary condition in a multi service scenario. Thus a load measure based on e.g., the ratio between the maximum current $\gamma_i^{tgt}$ and $\gamma_0^*$ would be quite conservative. To handle multiple service, introduce the matrix $\Gamma^{tgt}$ which is a diagonal matrix with the users’ target CIR in the diagonal,

$$\Gamma^{tgt} \triangleq \text{diag}(\gamma_1^{tgt}, \gamma_2^{tgt}, \ldots, \gamma_M^{tgt}).$$

Gunnarsson (2000) uses the above ideas to define feasibility relative load for multi service systems.

Definition 3.3 (Gunnarsson, 2000)
The feasibility relative load is defined as

$$L_r = \inf \{ \mu \in \mathbb{R} : \frac{1}{\mu} \Gamma^{tgt} \text{ is feasible} \}.$$

A set of target CIR:s are said to be feasible if there is a solution to the resulting power control problem. Gunnarsson (2000) also presents the following theorem to find the feasibility relative load in a system without soft handover

Theorem 3.3 (Gunnarsson, 2003)
The relative load, $L_r$, of a system is

$$L_r = \max \{ \text{eig} \{ \Gamma^{tgt}(Z - E) \} \}. \quad (3.11)$$

In case of a feasible system, i.e., $L_r < 1$, the users’ transmission powers are

$$P = (E - \Gamma^{tgt}(Z - E))^{-1} \Gamma^{tgt} \eta,$$

where $\eta$ is a M-dimensional vector with the $i$:th element equal to $N_i/g_{i,i}$. 
This definition of relative load actually coincides with the definition of relative load as the ratio between current target CIR and $\gamma_0$ in the single service scenario,

$$L_r = \max \text{eig}(\Gamma^{tgt}(E - Z)) = \max \text{eig}(\gamma_0 E(Z - E)) = \gamma_0 (\lambda^* - 1) = \frac{\gamma_0}{\gamma_0},$$

(3.12)

We thus have a definition of relative load which is natural given Theorem 3.1.

### 3.3.2 Convergence of Power Control Algorithm

Inspired by the presentation done by Hanly and Tse (1999) we study the uplink interference power in a multi-cell system. Therein, user $i$ is power controlled solely from one cell, here denoted cell $j_i$. Consider a power control algorithm that at each time instant $t$ sets user $i$’s transmission power according to

$$p_i(t) = \gamma_i^{tgt} \left( \frac{N_{ji}}{g_{i,ji}} + \sum_{\ell \neq i} p_{\ell}(t-1) \frac{g_{\ell,j_i}}{g_{i,ji}} \right),$$

where $N_{ji}$ is the background noise power user $i$ experiences (which is assumed constant) and $p_i(t)$ is user $i$’s transmission power at time $t$. The sum in the above expression considers all users in the entire network. A matrix expression for all users update at time $t$ is therefore

$$P(t) = \Gamma^{tgt}(\tilde{N} + (\tilde{Z}^T - E) P(t-1)),$$

(3.13)

where

$$P(t) \triangleq [p_i(t)], \quad \tilde{N} = [N_i] \triangleq [\frac{N_{ji}}{g_{i,ji}}], \quad \tilde{Z} = [\tilde{z}_{ik}] \triangleq [\frac{g_{\ell,j_i}}{g_{i,ji}}].$$

The matrix $\tilde{Z}$ will, just as $\Gamma^{tgt}$, have size $M \times M$. $\tilde{Z}$ is similar to $Z$ but not quite the same, see Appendix A.2 It is well known from theory of linear systems (see e.g., Kailath (1980)) that the recursion in (3.13) will converge if and only if all eigenvalues of $\Gamma^{tgt}(\tilde{Z}^T - E)$ are within the unit circle. Thus, a measure of the convergence rate is

$$L_c = \max | \text{eig} (\Gamma^{tgt}(\tilde{Z}^T - E)) | = \max \text{eig}(\Gamma^{tgt}(\tilde{Z}^T - E)),$$

(3.14)

where the last equality is a result of theory for positive matrices, see Appendix A.1. Note the resemblance with Equation (3.11) which is related to existence of a solution to the power control problem while the above expression is related to convergence of a power control algorithm.
Example 3.4 (Multi cell Load Estimation)

Study a system consisting of three users and two cells in which power control is implemented according to Equation (3.13). The $G$-matrix is

$$G = \begin{pmatrix} 0.53 & 0.15 \\ 0.4 & 0.1 \\ 0.14 & 0.8 \end{pmatrix}.$$

Denote the cell represented in the first column $I$ and the second column $II$. Users 1 and 2 are connected to cell $I$, see Figure 3.2, and the third user applies for a connection to cell $II$ yielding a $j$-vector of $j = [1 \ 1 \ 2]$. The corresponding $Z$ and $\tilde{Z}$ matrices are thus

$$Z = \begin{pmatrix} 1 & 1 & 0.28 \\ 1 & 1 & 0.25 \\ 0.18 & 0.18 & 1 \end{pmatrix} \quad \text{and} \quad \tilde{Z} = \begin{pmatrix} 1 & 1.33 & 0.19 \\ 0.75 & 1 & 0.13 \\ 0.26 & 0.35 & 1 \end{pmatrix}.$$

respectively. $Z$ has been given here simply to exemplify the calculation of it, it will not be used in the rest of the example. Note that the upper left two-by-two submatrix of $Z$ is an all one matrix. This part of $Z$ corresponds to a single cell system. At first, consider $\beta^{tgt}_i$ values according to

$$\beta^{tgt}_1 = 0.5, \quad \beta^{tgt}_2 = 0.3 \quad \text{and} \quad \beta^{tgt}_3 = 0$$

which corresponds to a $\Gamma$ matrix according to

$$\Gamma^{tgt} = \text{diag} \begin{pmatrix} 1 & 0.43 & 0 \end{pmatrix}.$$

This yields $L_c \approx 0.65$ according to Equation (3.14). The top plot of Figure 3.3 shows how the users’ transmission powers converge. After 100 iterations, user three is connected to cell $II$ with $\beta^{tgt}_3 = 0.35$. Naturally this affects the transmission powers for all users in the network, as indicated in Figure 3.3. The bottom plot of Figure 3.3 shows what the different estimates provide in this specific example. When the new user is admitted $L_c$ increases up to approximately 0.70. Thus, the fact that admitting a user in one cell requires additional transmission powers for users connected to other cells is reflected in an increased $L_c$. The estimate defined by Equation (3.9) is in the first half, of course, in compliance with the load according to Definition 3.2. However, the estimate is stationary completely insensitive to the situation in other cells, it eventually settles to the same relative load as before User 3 was admitted. We have not used any intercell-to-intracell
3.3 Centralized Load

Figure 3.2 Admission of a new user. User 3 applies for a channel in cell II. Users connected to other cells will then experience increased intercell interference power.

factor here. In this scenario the true intercell-to-intracell factor when the power control has settled is

\[ f = \frac{I_{\text{other}}}{I_{\text{own}}} = \frac{g_{3,1}p_3}{g_{1,1}p_1 + g_{2,1}p_2} \approx 0.04. \]

As can be seen in the table below, \( L_r \) equals \( L_c \). This and other comparisons between different load definitions will be discussed in Chapter 5. The table also shows how a decentralized estimate such as Equation (3.9) is completely insensitive to what happens in other cells as well as to fluctuations in soft capacity.

<table>
<thead>
<tr>
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<th>1,2</th>
<th>1,2,3</th>
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<tbody>
<tr>
<td>( L = 1 - \frac{1}{\lambda} )</td>
<td>Eq. (3.2)</td>
<td>0.8</td>
</tr>
<tr>
<td>( (1 + f) \sum \beta_i )</td>
<td>Eq. (3.9)</td>
<td>0.8</td>
</tr>
<tr>
<td>max eig(( \Gamma^{\text{tgt}}(\bar{Z} - E) ))</td>
<td>Eq. (3.11)</td>
<td>0.65</td>
</tr>
<tr>
<td>max eig(( \Gamma^{\text{tgt}}(\bar{Z}^t - E) ))</td>
<td>Eq. (3.14)</td>
<td>0.65</td>
</tr>
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</table>

The example also indicates the trade off between coverage and throughput. When a new user is admitted in cell II, the throughput increases but the higher transmission power of the users contributes to a decreased coverage.
Figure 3.3  Intercell effects. Top: Ratio between users’ transmission power and the background noise power. Bottom: Relative load in cell I according to Definition 3.2 and two different estimates of it. A new user is admitted in cell II at iteration number 100. $f = 0$ has been used in Equation (3.9).

due to limited transmission powers.

3.3.3 Link Based Estimates

An estimate which considers the number of links each user has can to some extent capture the soft handover gain. If we assume that maximum ratio combining is used in the combining of the locally received information, a simple way of utilizing the information regarding number of links is to assume that each user’s contribution to the relative load is inversely proportional to the number of soft handover links. The relative load estimate
would then become
\[ L_j = (1 + f) \sum_i^{M_j} \frac{\beta_i}{n_i}, \]

where \( M_j \) is the number of users power controlled in cell \( j \) and \( n_i \) is user \( i \)'s number of soft handover links. There has been some studies where link based admission control has been compared with interference power based admission control algorithms. For example Ishikawa and Umeda (1997) and Gunnarsson et al. (2002) show that using a link based resource compared to using an interference power based resource in the call admission control yields approximately the same performance, but the link based algorithm is far more sensitive to changes in the radio environment. Since these types of estimates indirectly considers variations in the radio environment the users experience, estimated pole capacity will change over time.

3.4 Discussion

In this chapter theory regarding uplink load in cellular CDMA systems is presented. It has been shown that, despite unlimited transmission power, the capacity of an interference power limited system is in fact bounded by a finite time varying capacity, the pole capacity. Since the maximum capacity of the system is generally both unknown and time varying a quantity called uplink relative load has been introduced. The uplink relative load relates the current amount of used capacity to the current maximum capacity, even though both are unknown. Properties of the uplink relative load has been explored through a survey of different approaches to estimating it.

Decentralized Load  This type of estimates use information locally available in each cell. An advantage with these is of course that they can directly be used in local resource management algorithms. However, these estimates have no real knowledge of the effects that resource management decisions have on the surrounding cells. Furthermore, a decentralized estimate which makes the assumption that the other cells are equally loaded as the own cell can not fully utilize the soft capacity of a CDMA cellular system. In fact, it can be argued that some of these estimates are purely related to a hard capacity of the system. Measuring the current total interference power is one way of locally estimate how loaded the surrounding cells are. This, however, requires accurate interference power estimates – something which should not be taken for granted.
Centralized Load  If a centralized estimate is used, information from several cells can be considered. The type of information may be simply the number of soft handover links each user has or, more advanced, it may be the path gains each user experiences. Some simplifying assumptions still have to be made also in the centralized cases. In the estimate where the number of links is used for example, the assumption is that the base stations received signals are combined using maximum ratio combining, which is not true in general.

A more theoretical approach has also been handled in the chapter. One can show that there is a strong connection between uplink relative load and feasibility of the power control problem. By using techniques from system theory, solvability of the power control problem has been associated with a relative load below one. As the power control problem is concerned with the entire network, this relative load applies to the entire network as opposed to just one cell.
4

Uplink Load Expressions

This chapter contains the derivation of a number of expressions for the uplink load. Section 4.1 contains a derivation of a set of nonlinear equations for the uplink interference power. The uplink load expressions are all approximative solutions to this system of equations. Different ways of solving the equations lead to two fundamentally different types of expressions which are derived in two different subsections of Section 4.2. Because of details in the current WCDMA standard, some of the information required by these expressions cannot be made available. A separate subsection derives expressions that explicitly deal with this problem. The section also contains examples on how the required information can be made available. A small comparison of the expressions is made in Section 4.3 before Section 4.4 addresses the sources of estimation errors.

4.1 Uplink Interference Power Expressions

As seen in the previous chapter, uplink relative load is closely related to the uplink noise rise. Therefore, just as the uplink noise rise, the uplink relative load is a truly spatial quantity. A relative load expression using information gathered in several cells can be made sensitive to changes in the load caused by changed circumstances in other cells. Below are a number of expressions derived. These expressions assume path gain values, i.e., all $g_{i,j}$, and target
carrier-to-total-interference ratios, $\beta_{i}^{tgt}$, to be readily available in a central node.

As a starting point for our derivation of the expressions, consider the following expression for the total uplink interference power in cell $j$

$$I_{j}^{tot} = N_{j} + \sum_{i=1}^{M} C_{i,j}^{i}$$

(4.1)

where $M$ is the number of users in the entire network and $C_{i,j}^{i}$ is the power of the signal from user $i$ received in cell $j$. $N_{j}$ represents the part of the interference power in cell $j$ which is not power controlled by our system. All contributions to the total interference power which originate from terrestrial non-power controlled sources, such as cellular systems operating at neighbouring frequencies and electronic equipment in for example cars, are thus embedded into $N_{j}$. Neubauer et al. (2001) concludes through measurements that this is a reasonable assumption to make, at least without cellular systems operating at neighbouring frequencies. The signal power from user $i$ in cell $j$ is

$$C_{i,j} = p_{i}g_{i,j}$$

(4.2)

where $g_{i,j}$ is the path gain between user $i$ and the base station in cell $j$ and $p_{i}$ is the transmission power of user $i$. Since we want to combine (4.1) and (4.2) we need an expression for $p_{i}$, user $i$’s transmission power. Due to the fast update rate of the transmission powers in WCDMA, a RNC does not know the users’ transmission powers. However, power control tries to adjust the users’ transmission powers such that a certain user individual target-carrier-to-total-interference ratio, $\beta_{i}^{tgt}$, is maintained. By assuming perfect power control, i.e., $\beta_{i} = \beta_{i}^{tgt}$, we can solve for the transmission powers.

**Maximum ratio combining** If we approximate the combination of selection combining and maximum ratio combining with just maximum ratio combining, the total combined carrier-to-total-interference ratio is approximately the sum of the individually received carrier-to-total-interference ratios in the separate cells

$$\beta_{i}^{tgt}(t) \approx \sum_{k \in K_{i}} \frac{g_{i,k}(t)p_{i}(t)}{I_{k}^{tot}(t)}$$

(4.3)

where $K_{i}$ is the set of cells user $i$ is connected to. The approximation in the above equation is due to a nonlinear relation between CIR and CTIR, see
4.1 Uplink Interference Power Expressions

Section 4.4.1. Solving for $p_i(t)$ above yields an expression of the transmission power at time $t$

$$p_i(t) \approx \frac{\beta_i^{gt}(t)}{\sum_{k \in K_i} \frac{g_{i,k}(t)}{I_k(t)}}.$$  \hfill (4.4)

Inserting this expression of $p_i$ into equation (4.1) yields

$$I_j(t) = N_j(t) + \sum_{i=1}^{M} p_i(t) g_{i,j}(t) \approx N_j(t) + \sum_{i=1}^{M} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} \frac{g_{i,k}(t)}{I_k(t)}}.$$  

**Selection combining** The above expression for the interference power in cell $j$ was derived with an assumption that softer handover is used everywhere. If soft handover is assumed instead, the signals from the different cells are combined using *selection combining*, i.e., the signal with the highest reliability tag is chosen. Consequently, the total combined carrier-to-total-interference ratio is

$$\beta_i^{gt}(t) = \max_{k \in K_i} \frac{g_{i,k}(t)p_i(t)}{I_k(t)}.$$  \hfill (4.5)

**Maximum ratio and selection combining** Another, more accurate, assumption is to use the correct combination of soft and softer handover. Let user $i$’s configuration of soft and softer handover be defined by $K_i$. The function $c(K_i, G(t), I(t))$ represents the, to the denominator of Equation (4.4), corresponding expression of maximum and sums.

Divide the set of cells user $i$ is connected to, i.e., $K_i$, into subsets, $K_{i,r}$ where each subset contains cells which are all served by the same base station. The subsets $K_{i,r}$ are disjunct and $K_i = \bigcup_r K_{i,r}$. The function $c$ may then be defined as

$$c(K_i, G, I) = \max(\sum_{k \in K_{i,1}} g_{i,k} I_k, \sum_{k \in K_{i,2}} g_{i,k} I_k, \ldots).$$

Thus, maximum ratio combining is used on the signals received in each separate set $K_{i,r}$ and selection combining is then applied on the results from the maximum ratio combining.

The three different ways of approximating $p_i(t)$ yield three different expressions for the uplink interference power in cell $j$.  

Chapter 4  Uplink Load Expressions

Applying any one of these three equations on all cells in a radio network, i.e., for \( j = 1, 2 \ldots B \), defines a system of nonlinear equations.

### 4.2 Uplink Load Expressions

Three different expressions for the uplink interference power are derived in the previous section. This section is devoted to solving for the uplink noise rise from these expressions. As discussed in Chapter 3, there is a one-to-one relation between uplink noise rise and uplink relative load. Thus, any of the expressions for uplink noise rise derived here can be converted to an expression for uplink relative load.

We will first give some general methods for solving nonlinear equations and then present two fundamentally different ways of solving the system of nonlinear equations defined by Equations (4.6). In order to solve a problem of distributing required information another set of expressions are given in a separate subsection. Finally, a discussion of how required information can be made available is included in this section.

#### 4.2.1 Methods for Solving Nonlinear Equations

Each one of the Equations (4.6a) to (4.6c) define a system of nonlinear equations. If an approximative solution to these equations can be found, it can be used as to approximate the true load.

There are numerous ways of numerically solving such systems in the literature. In this section we will mention a few of the methods after a framework for using them has been defined.

Generally the methods look for a solution to \( f(x, c) = 0 \), where \( x \) is a vector of variables which will be altered to find a solution to the equation

\[
I_j(t) \approx N_j(t) + \sum_{i=1}^{M} \beta_{lq}^{i}(t) \frac{g_{t,j}(t)}{\sum_{k \in K} g_{t,k}(t)} (4.6a)
\]

\[
I_j(t) \approx N_j(t) + \sum_{i=1}^{M} \beta_{tlq}^{i}(t) \frac{g_{t,j}(t)}{\max_{k \in K} g_{t,k}(t)} (4.6b)
\]

\[
I_j(t) \approx N_j(t) + \sum_{i=1}^{M} \beta_{lq}^{i}(t) \frac{g_{t,j}(t)}{c(K, G(t), I(t))} (4.6c)
\]
while $c$ is considered constant. In our case this corresponds to

$$f(I, N, \beta, G, K) = 0,$$

where the vector $I$ corresponds to $x$ in the general case. One way of solving the equation is to linearize it or through some other approximation make the nonlinear equations algebraically solvable. Another way to go is through numerical algorithms. The perhaps most famous algorithm is the \textit{Newton-Raphson-method}, where the approximative solution is, in each step, updated according to

$$I^{(i+1)} = I^{(i)} - \left( \frac{df}{dI} \right)_{I=I^{(i)}}^{-1} f(I^{(i)}).$$

A problem with this is of course that the derivative of $f(x)$ has to be available in algebraic form. If this is not the case, an approximation of the derivative can be used instead. This leads to a group of methods commonly referred to as \textit{Secant-methods} (Dennis and Schnabel, 1983).

If the nonlinear equations can be written on the form $x = f(x, c)$, fix point iterations can be applied, i.e.,

$$x^{i+1} = f(x^i, c).$$

Equations (4.6) are on this form, where $x$ would represent the interference power, $I$.

In this work, Equations (4.6) have been solved using linear approximations of the equations as well as fix point iterations of the nonlinear equations.

### 4.2.2 Approximation I: Equal Interference In All Cells

For a moment, assume that the interference powers in all cells are the same as in the cell we want to express the interference power in, i.e., $I_k = I_j$. Equation (4.6a) then becomes

$$I_j(t) \approx N_j(t) + I_j(t) \sum_{i=1}^M \beta_i^{i, j}(t) \frac{g_{i, j}(t)}{\sum_{k \in K_i} g_{i, k}(t)}.$$

The system of coupled nonlinear equations defined by (4.6a) is then simplified into a number of linear decoupled equations. Solving for $I_j(t)$ yields an
explicit expression of the interference power

\[ I_j(t) = \frac{N_j}{1 - \sum_{i=1}^{M} \beta_j^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}}. \]  

(4.9)

When comparing the above expression with (3.3) we conclude that it is natural to associate the uplink relative load with

\[ L_{j}^{IMRC}(t) = \sum_{i=1}^{M} \beta_j^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}, \]  

(4.10)

where index IMRC has been added to indicate that maximum ratio combining has been used and that the interference powers in all cells were temporarily assumed equal during the derivation. This is a practically tractable expression of the uplink relative load in cell \( j \) based on readily available information. The corresponding expression for uplink noise rise is easily found by dividing Equation (4.9) with \( N_j \),

\[ \Lambda_{j}^{IMRC}(t) = \frac{1}{1 - \sum_{i=1}^{M} \beta_j^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}}. \]

Starting from equation (4.6b) instead yields a different approximative expression for \( L_j(t) \), namely

\[ L_{j}^{ISEL}(t) = \sum_{i=1}^{M} \beta_j^{gt}(t) \frac{g_{i,j}(t)}{\max_{k \in K_i} g_{i,k}(t)}, \]  

(4.11)

where the index ISEL indicates the assumption of soft handover and equal interference power in all cells. The corresponding expression of the uplink noise rise is

\[ \Lambda_{j}^{ISEL}(t) = \frac{1}{1 - \sum_{i=1}^{M} \beta_j^{gt}(t) \frac{g_{i,j}(t)}{\max_{k \in K_i} g_{i,k}(t)}}. \]

Finally the most accurate way of combining the information used in Equation (4.6c) results in yet another approximative expression for the uplink load

\[ I_{j}^{IBOTH}(t) = \sum_{i=1}^{M} \beta_j^{gt}(t) \frac{g_{i,j}(t)}{b(K,G(t))}. \]  

(4.12)
and the uplink noise rise

\[ \Lambda_{j}^{NBOTH}(t) = \frac{1}{1 - \sum_{i=1}^{M} \beta_{i}^{gt}(t) \frac{g_{i,j}(t)}{b(K_{i},G(t))}}, \]

where \( b(K_{i},G) \) is a function combining elements of the \( G \)-matrix according to the configuration implied by \( K_{i} \).

To summarize we have the following three approximative expressions for the uplink noise rise

\[
\begin{align*}
\Lambda_{j}^{IMRC}(t) &= \frac{1}{1 - \sum_{i=1}^{M} \beta_{i}^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_{i}} g_{i,k}(t)}} \quad (4.13a) \\
\Lambda_{j}^{ISEL}(t) &= \frac{1}{1 - \sum_{i=1}^{M} \beta_{i}^{gt}(t) \frac{g_{i,j}(t)}{\max_{k \in K_{i}} g_{i,k}(t)}} \quad (4.13b) \\
\Lambda_{j}^{IBOTH}(t) &= \frac{1}{1 - \sum_{i=1}^{M} \beta_{i}^{gt}(t) \frac{g_{i,j}(t)}{b(K_{i},G(t))}} \quad (4.13c)
\end{align*}
\]

The above expressions can be rearranged into expressing the uplink relative load instead, see Equation (4.10). The relative expression would then equal Equation 3.6 in a single cell system. Note that, when not utilizing soft handover, all expressions in Equations 4.13 will be the same.

### 4.2.3 Approximation II: Equal Background Noise

The other approach to solving the equations defined by (4.6) is to assume equal background noise power in all cells, i.e., \( N_{j} = N \). Dividing Equation (4.6a) with \( N \) yields

\[
\frac{I_{j}(t)}{N} \approx 1 + \sum_{i=1}^{M} \beta_{i}^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_{i}} g_{i,k}(t) \frac{g_{i,k}(t)}{I_{k}(t)}}.
\]

By using Definition 3.1, \( \frac{I_{j}}{N} \) can be substituted to \( \Lambda_{j} \).

\[
\Lambda_{j}(t) \approx 1 + \sum_{i=1}^{M} \beta_{i}^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_{i}} \frac{g_{i,k}(t)}{\Lambda_{k}(t)}}.
\]

This yields a new nonlinear system of equations defined by variables we either know or try to estimate. This system can be solved through any of
the numerical methods previously discussed in Section 4.2.1. Most simple of these is to use the previous estimate of $k$ in the right hand side of the above equation

$$
\Lambda_j^{NMRC}(t) = 1 + \sum_{i=1}^{M} \beta_i(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} \lambda_k^{NMRC}(t-1)},
$$

(4.14)

where $NMRC$ indicates that maximum ratio combining and equal background noise in all cells were assumed during the derivation. If $\beta_i^{gt}(t)$ and $g_{i,j}$ are constant or slowly varying for all $i$ and $j$, this is a fix point iteration as in Equation (4.7). This expression of the uplink noise rise can be converted into an expression of the uplink relative load according to

$$
L_j^{NMRC}(t) = 1 - \frac{1}{\lambda_j^{NMRC}}.
$$

(4.15)

Once again, assuming soft handover instead of softer handover provides yet another expression of the uplink noise rise,

$$
\Lambda_j^{NSEL}(t) = 1 + \sum_{i=1}^{M} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} \lambda_k^{NSEL}(t-1)}.
$$

Also in this case the correct combination of soft and softer handover can be used

$$
\Lambda_j^{NBOTH}(t) = 1 + \sum_{i=1}^{M} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{c(K_i, g(t), \Lambda_j^{NBOTH}(t-1))},
$$

where $c(K, g, \Lambda)$ is a function combining the noise rise vector with the $G$-matrix according to the configuration implied by $K$.

We thus have three more expressions for the uplink noise rise

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<thead>
<tr>
<th>Expression</th>
<th>Equation</th>
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<tbody>
<tr>
<td>$\Lambda_j^{NMRC}$</td>
<td>(4.16a)</td>
</tr>
<tr>
<td>$\Lambda_j^{NSEL}$</td>
<td>(4.16b)</td>
</tr>
<tr>
<td>$\Lambda_j^{NBOTH}$</td>
<td>(4.16c)</td>
</tr>
</tbody>
</table>
4.2 Uplink Load Expressions

Just as with Equations 4.13, all of these expressions are equal when soft handover is not utilized. These expressions can be rearranged into expressing the uplink relative load instead. In a single cell system, the corresponding expression for relative load once again coincides with Equation (3.6).

In Equations (4.16), one iteration at each time update is used. This can be generalized into using several iterations before each update, and the assumption of slowly varying $\beta_i^{tgt}$ and $g_{i,j}$ can be relaxed. Denote by $B^{tgt}$ a diagonal vector with the users’ target carrier-to-total-interference ratios in the main diagonal, i.e., $B^{tgt} = \text{diag}(\beta_1^{tgt}, \beta_2^{tgt}, \ldots, \beta_M^{tgt})$.

Algorithm 4.1

Let $\Lambda(t, 0) = \Lambda(t - 1)$
Assign one of the right hand sides of Equations (4.16) to $f(\Lambda, B^{tgt}, G)$
For $n = 1$ to $NI$
  For $j = 1$ to $B$
    Let $\Lambda_j(t, n) = f(\Lambda(t, n - 1), B^{tgt}, G)$
  \end{algorithm}

By using fix point iterations like these, the expressions catch frequent updates in path gain measurements better. Convergence of this algorithm when using Equation (4.16a) is analyzed in Section 5.3.

4.2.4 Approximation III: Distributed Information

In reality there will be several RNC:s serving an area over which a user may move during a session. According to the 3GPP standard, a user initiating its session in one RNC, call it $RNC_2$, and during the session moves to an area supported by another RNC, $RNC_1$, will not report path gain measurements to $RNC_1$ but to $RNC_2$. A user located inside $RNC_1$’s service area may thus introduce considerable interference power to the base stations without delivering any path gain reports to $RNC_1$. All the above expressions for uplink noise rise can only consider users reporting path gain measurements to the RNC the estimate resides in. Besides the obvious solution to neglect users not reporting their path gain, we propose a set of noise rise approximations which are a combination of the techniques used in the two previous sections. Below is a derivation of the alternative expression which approximates the noise rise in base stations belonging to $RNC_1$.

Split the sum over users in the Equations (4.6) into two sums, one being over the users reporting their path gain measurements to $RNC_1$ and the
other over users reporting to other RNCs (here represented by $RNC_2$). Exemplifying with Equation (4.6a)

\[
I_j(t) = N_j + \sum_{i \in RNC_1} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}
+ \sum_{i \in RNC_2} \beta_i^{gt}(t) \frac{g_{i,k}(t)}{\sum_{k \in K_i} g_{i,k}(t)}.
\] (4.17)

Here cell $j$ belongs to $RNC_1$. Consider just the sum over users in $RNC_2$ and assume that $I_k(t)$ therein approximately equals $I_j(t)$. This allows us to approximate the sum as

\[
\sum_{i \in RNC_2} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)} \approx I_j(t) \sum_{i \in RNC_2} \beta_i^{gt}(t) \sum_{k \in K_i} g_{i,k}(t).
\]

Substituting the second sum in Equation (4.17) by the above expression and solve for $I_j(t)$ yield

\[
I_j(t) = \frac{N_j + \sum_{i \in RNC_1} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}}{1 - \sum_{i \in RNC_2} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}}.
\]

Assume that $N_j = N_k = N$ and that $\Lambda_k(t) = \Lambda_k(t-1)$. Dividing the above equation by $N$ then results in an expression of the uplink noise rise which is a combination of equation (4.13a) and (4.16a)

\[
\Lambda_j^{RNCMRC}(t) = \frac{1 + \sum_{i \in RNC_1} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}}{1 - \sum_{i \in RNC_2} \beta_i^{gt}(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}}.
\]

Note that the sum in the denominator represents the load that users reporting to $RNC_2$ introduce to cell $j$. Neglecting these users results in a larger denominator, which yields a smaller estimate.

This estimate is interesting since $RNC_2$ can now send a message to $RNC_1$ containing the additional load that users belonging to $RNC_2$ introduce in each cell controlled by $RNC_1$. This message would then be far smaller than one containing the complete path gain information regarding these users. Obviously we can choose to assume soft handover is used everywhere here as well, or we can use the actual combination of soft and softer handover. This gives us three types of expressions that use the technique with sending relative load information between different RNCs.
4.2 Uplink Load Expressions

\[
\Lambda_j^{RNCMRC}(t) = \frac{1 + \sum_{i \in RNC_1} \beta_i^j(t) \frac{g_{i,j}(t)}{\sum_{k \in K_1} \Lambda_k^{RNCMRC}(t-1)}}{1 - \sum_{i \in RNC_2} \beta_i^j(t) \frac{g_{i,j}(t)}{\sum_{k \in K_2} g_{i,k}(t)}}
\]

(4.18a)

\[
\Lambda_j^{RNCSEL}(t) = \frac{1 + \sum_{i \in RNC_1} \beta_i^j(t) \frac{g_{i,j}(t)}{\max_{k \in K_1} \Lambda_k^{RNCSEL}(t-1)}}{1 - \sum_{i \in RNC_2} \beta_i^j(t) \frac{g_{i,j}(t)}{\max_{k \in K_2} g_{i,k}(t)}}
\]

(4.18b)

\[
\Lambda_j^{RNCBOTH}(t) = \frac{1 + \sum_{i \in RNC_1} \beta_i^j(t) \frac{g_{i,j}(t)}{c(K_i,G(t)) \Lambda_i^{RNCBOTH}(t-1)}}{1 - \sum_{i \in RNC_2} \beta_i^j(t) \frac{g_{i,j}(t)}{b(K_i,g_{i,k}(t))}}
\]

(4.18c)

4.2.5 Required Information

All the above derived expressions for the uplink load rely on knowledge of users’ target carrier-to-total-interference-ratio, \(\beta^{tgt}\), which are assumed known by the system. They also need path gain measurements, which can be made available in two different ways: (3GPP, 1999, 2000c)

**M1:** The mobile stations are requested to periodically (but not necessarily synchronously) report pilot power measurements. As an example, pilot power from the six strongest base stations can be reported at a rate of 0.5 Hz.

**M2:** For handover purposes, the mobile typically reports similar measurements in an event-driven fashion. It measures the pilot powers from the neighboring cells and reports up to the six strongest path gains at handover events.

In both cases, the channel is assumed reciprocal, i.e., the uplink path gain is assumed approximately equal to the corresponding downlink path gain with respect to distance dependent path loss and shadow fading. Fast fading is assumed filtered out in the lower layer filtering. Furthermore, these different strategies only provide data from a limited set of mobile-to-base path gains. The remaining path gains, however, are considered small and set equal to
zero. According to the 3GPP standard the individual measurement errors in relative path gain should not be greater than 1.5 dB with a probability of 90% (3GPP, 2000c).

4.3 Comparison of the uplink load expressions

In the previous section we have presented two fundamentally different ways of solving the nonlinear system of equations implied by Equation (4.6a). The approximate expressions derived in Section 4.2.2 which assume the interference power to be equal in all cells, may at first seem redundant since we have expressions which are derived under far more reasonable assumptions, the ones assuming equal background noise. $\Lambda^{IMRC}$, $\Lambda^{ISEL}$ and $\Lambda^{IBOTH}$ have, however, one major advantage compared to the iterations. When trying to estimate the uplink noise rise in cell $j$, we assume the interference power in cell $k$ to equal that in cell $j$, i.e., $I_k = I_j$. In the case cell $j$ is the cell with the highest interference power in the area, this implies (starting with Equation (4.6a))

$$I_j \approx N_j + \sum_{i=1}^{M} \beta^{tgt}_i \frac{g_{i,j}}{\sum_{k \in K_i} g_{i,k}} \leq N_j + \sum_{i=1}^{M} \beta^{tgt}_i \frac{g_{i,j}}{\sum_{k \in K_i} g_{i,k}}.$$

This property of the estimate can be useful when using the estimate in radio resource management algorithms. The algorithms will receive an earlier warning of a potential future congestion.

The main strength of the above expressions for uplink load is that they, like the true interference power, depend on the actual situation in several cells and do not rely on an intercell-to-intracell-interference factor, $f$. Furthermore, since the expressions involve the users’ carrier-to-total-interference target, users who require high transmission power will inherently be given less coverage if an expression like this is used in the resource management algorithms. To exemplify, study Equation (4.10)

$$L_j^{IMRC}(t) = \sum_{i=1}^{M} \beta^{tgt}_i(t) \frac{g_{i,j}(t)}{\sum_{k \in K_i} g_{i,k}(t)}.$$

Each users’ contribution is a product of the target carrier-to-total-interference ratio and a ratio between path gains. A user far from the own base station
but close to another, will have a small gain to its own base station \((g_{i,k})\) and a relatively large gain to the other base station \((g_{i,j})\). Therefore, the ratio of the path gains will be relatively large which can be compensated for with a small target carrier-to-total-interference ratio.

### 4.4 Sources of Estimation Errors

A number of different effects and approximations degrade the performance of the load approximations. These include:

- The nonlinear relation between \(\gamma_i\) and \(\beta_i\) (see section 4.4.1)
- TX rise (see section 4.4.2)
- the soft(er) handover assumption
- imperfect power control
- the assumption regarding relation between the uplink interference in all cells, alternatively the background noise and the dynamics of the uplink interference
- the assumption that unknown path gains are equal to zero
- path gain measurement errors according to 3GPP (2000c) and sparsely sampled path gain measurement reports

How these affect the expressions is investigated through simulations in Chapter 7.

#### 4.4.1 Nonlinear relation between \(\gamma\) and \(\beta\)

In Equation (4.3) we assume that the sum over the \(\beta_{i,k}\) received in different cells equals the \(\beta_i\) which is required by the user’s service, i.e.,

\[
\beta_i = \sum_{k \in K_{i}} \beta_{i,k}.
\]

This is not true in general where we may have more than one term in the sum, even though \(\sum_{k \in K_{i}} \gamma_{i,k} = \gamma_{i}\). A simple proof is

\[
\sum_{k \in K_{i}} \beta_{i,k} = \frac{\gamma_{i,1}}{1 + \gamma_{i}} + \frac{\gamma_{i,2}}{1 + \gamma_{i,2}} + \frac{\gamma_{i,3}}{1 + \gamma_{i,3}} \geq \frac{\gamma_{i,1}}{1 + \gamma_{i}} + \frac{\gamma_{i,2}}{1 + \gamma_{i}} + \frac{\gamma_{i,3}}{1 + \gamma_{i}} = \frac{\gamma_{i}}{1 + \gamma_{i}} = \beta_{i}.
\]
where equality holds if and only if two of the $\gamma_{i,k}$'s are zero (which corresponds to no soft handover). Thus, the sum over the separately received $\beta_{i,k}$ is larger than $\beta_i$. Therefore this is a reason to why the users’ transmission powers are underestimated.

How big a difference it is depends on the relation between the separately received $\gamma_{i,k}$: if a user is almost solely connected to one cell, the difference is small. The total $\gamma_i$ also influences how big the difference is. The bigger $\gamma_i$ is, the bigger the difference may be.

### 4.4.2 TX Rise

In this section we define and discuss TX rise. For simplicity, assume that each user is allowed to connect to no more than one base station at a time in this section. TX rise is a result of fast power control adjusting each user’s transmission power to compensate for fast fading dips in the path gain between mobile and connected base station. The mobile uses a high power during deep fades. This may yield a considerable interference increase to neighboring cells since a dip to the connected base station does not always imply an, as deep, dip to the neighboring base stations. B. Hashem, E.S. Sousa (1999) provides theoretical expressions for the average power increase due to TX rise. Moreover, Sipilä et al. (1999) gives a short theoretical derivation of the average power increase under an assumption of perfect power control and conclude through simulations that the expressions are reasonable even with imperfect power control.

To explain what TX rise is, consider mobile 1, which is connected to base station 2, and study its interference contribution to base station 3. Due to power control, each user’s transmission power is approximately inversely proportional to the momentary path gain between the user and the controlling base station, i.e.,

$$p_1 \propto \frac{1}{g_{1,2}}.$$  

Base station 3 will receive the signal $p_1 g_{1,3}$ which is proportional to the ratio between the momentarily path gains, i.e.,

$$C_{1,3} \propto \frac{g_{1,3}}{g_{1,2}}.$$  

The path gain reports provided by the users do not include fast fading (the actual reported value is low pass filtered; $F\{g_{i,j}\}$). Since the uplink load expressions, unlike fast power control, use users’ path gain reports there will
be a part of the uplink interference power that will not be counted for in the expressions. Essentially, we can observe $\frac{F\{g_{i,j}\}}{F\{g_{i,k}\}}$ but not $\frac{g_{i,j}}{g_{i,k}}$, which is related to the actual interference contribution in base station 3. In the top graph of Figure 4.1 we have plotted the ratio of low-pass filtered versions of the path gain measurements (solid). Of course this will not detect the high peaks in the ratio of the momentarily path gains (dashed). Perhaps momentarily sharp peaks are tolerable by base station 3. A better model could therefore be to consider the low-pass filtered load contribution which is essentially proportional to $F\{\frac{g_{i,j}}{g_{i,k}}\}$. However, as indicated by the bottom graph of Figure 4.1, the load contribution is still underestimated. Essentially, this exemplifies Jensen’s inequality (Billingsley, 1995):

$$\frac{E\{g_{i,j}\}}{E\{g_{i,k}\}} \leq E\left\{\frac{g_{i,j}}{g_{i,k}}\right\}.$$ 

The solid line in the bottom graph of Figure 4.1 thus represents the quantity used in our expressions of the average noise rise while the dashed line represents the quantity that corresponds to user $i$’s actual contribution to the average uplink noise rise in base station $j$.

Aside from the expressions, there are ideas of how to schedule transmission in such a way that transmission during deep fades are avoided. This will make the variance of the momentary path gain values less and hence decrease the TX rise effect. An example of such a work is Törnqvist (2003). TX rise has been studied by e.g., Ariyavisitakul and Chang (1991); B. Hashem, E.S. Sousa (1999) and Sipilä et al. (1999).

### 4.5 Discussion and Open Issues

In this chapter a number of uplink load expressions were derived. The expressions originates from a system of nonlinear equations which relate the interference power in different cells to each other. Different methods of solving these equations results in different approximations. All the derived expressions use only information regarding $\gamma^{tgt}$, path gains $g_{i,j}$ and the current link configuration of the system, i.e., information readily available. Unfortunately the approximations are impaired by a number of approximations and absence of complete path gain information. Some of the reasons, such as TX rise, have been discussed in a bit more detail.
Figure 4.1  Top: Ratio between filtered path gain measurements $F_{i,j}$ (solid) and momentary measurements $g_{i,j}$ (dashed). The momentary high peaks in the ratio between the momentary measurements are not present in the ratio between the filtered versions. Bottom: Ratio between filtered measurements (solid) and filtered ratio, $F\frac{g_{i,j}}{g_{i,j}}$ (dashed). The average of the filtered ratio between momentary measurements (dashed) is higher than the ratio between the filtered measurements (solid).

4.5.1 TX Rise Compensation: Additional Transmission Power Estimation

Sipilä et al. (1999) provides expressions for the average transmission power increase when utilizing the diversity gain of a RAKE receiver. The amount of additional average interference power induced by TX rise is a function of how many fast fading rays the receiver considers and the relative strength these have. A first result is that in case there are $N$ equally strong rays the interference power is increased a factor $\frac{N}{N-1}$. We study a more general
and practically interesting case, where the rays have different strength. If the strength of each received multi path ray, \( X_k, k = 1, 2 \ldots, N \) can be assumed exponentially distributed, the weighted sum, \( X = \sum a_k x_k \), will have a probability density function according to

\[
f_X(x) \approx \sum_{k=1}^{N} \frac{\pi_k e^{-\frac{x}{a_k}}}{a_k},
\]

where \( \pi_k \) is

\[
\pi_k = \prod_{i=1, i \neq k}^{N} \frac{a_k}{a_k - a_i}.
\]

This is an approximative expression because perfect utilization of each received multi path ray was assumed during the derivation. Let the coefficients \( a_k \) be normalized so that their sum equals 1. The expected average transmission power increase can now be expressed as

\[
\tau = \sum_{k=1}^{N} \frac{a_k^{N-2} \log(a_k)}{\prod_{i=1, i \neq k}^{N} (a_k - a_i)} \cdot \sum_{k=1}^{N} a_k = 1,
\]

where \( \tau \) is the ratio between expected transmission power with fast fading and the transmission power without. Note, that the above expression requires all the relative strengths, \( a_k \), to be unique. In a practical scenario, however, this is satisfied with probability 1.

### 4.5.2 TX Rise Compensation: Taylor Expansion

One possible way of estimating the increase in transmission power that TX rise causes is by considering more terms in a Taylor expansion of the path gain ratios. If we assume the total path gain as consisting of two factors, \( g^s_i \) and \( g^f_i \), where the latter represents the fast fading and the former the rest of the total path gain. The fast fading part can then be assumed independent between two different users, i.e., \( g^f_1 \) is independent of \( g^f_2 \), as well as of all the slow fading parts. An expression for the expected value of the ratio is therefore

\[
E \left\{ \frac{g_1}{g_2} \right\} = E \left\{ \frac{g^s_1 g^f_1}{g^s_2 g^f_2} \right\} = E \left\{ \frac{g^s_1}{g^s_2} \right\} E \left\{ \frac{g^f_1}{g^f_2} \right\} E \left\{ \frac{1}{g^f_2} \right\}.
\]
The second factor in the last expression, \( E\{g_1^f\} \), equals 1 by definition. A Taylor expansion of the last factor is 
\[
\left( E\left\{ \frac{1}{g_2^f} \right\} = 1 \right) = 1 - \frac{1}{1^2} E \left\{ g_2^f - 1 \right\} + \frac{2}{2} \frac{1}{1^3} E \left\{ (g_2^f - 1)^2 \right\} + \mathcal{O}(g_2^f)^3) = 1 + E \left\{ (g_2^f - 1)^2 \right\} + \mathcal{O}(g_2^f)^3) .
\]

Therefore, an estimate of \( g_2^f \)'s variance can be used to compensate for a part of the TX rise contribution.

### 4.5.3 Utilizing Estimation Error Statistics

Chapter 7 contains an extensive simulation study showing how well the expressions derived in this chapter approximates the true uplink load. Figure 4.2 shows a histogram of the difference between true noise rise and the noise rise given by Equation (4.16c). Clearly, the errors in noise rise can be assumed Gaussian distributed. Thus, given an assumption of Gaussian distributed errors the distribution, we can calculate confidence intervals. The bounds of this interval would than be given by average and standard deviations as arrived at in Chapter 7.
Figure 4.2  Histogram of difference between true noise rise and the noise rise given by $N_{BOTH}$. A Gaussian probability distribution function has also been fitted to the data.
This chapter studies uplink load from a theoretical perspective. Alternative definitions of uplink load related to the entire system as opposed to just one cell are introduced. Some of the definitions also explicitly relate uplink load to existence of a solution to the power control problem of finding appropriate transmission powers for all users. Section 5.1 introduces alternative definitions of uplink load and discusses how the load of a system according to these definitions can be determined. In Section 5.2 relations between different definitions of uplink relative load as well as some of the expressions in the previous chapter are established. All expressions for the uplink load in the previous chapter are derived through approximations of a system of nonlinear equations. Some of the expressions use iterations. Sufficient conditions for convergence of one of the iterations are developed in Section 5.3. Divergence of the iterations will also be related to a lower bound of the uplink load.

All inequalities between vectors in this chapter should be interpreted component wise. Furthermore, since this is a theoretical study, it is assumed that the system has complete knowledge of all path gains.
5.1 System Load

Even though a relative load per cell is perhaps interesting from a practical point of view since it is with advantage used in the resource management, a load related to the entire system is equally interesting from a theoretical point of view. From what has been said in for example Chapter 3 it is natural to define the system load as the maximum cell load in the system.

**Definition 5.1 (System Load)**
The system load is defined as

\[ L_s \triangleq \max_j L_j, \]

where \( L_j \) is the uplink relative load in cell \( j \) according to Definition 3.2.

The expressions for uplink relative load in Chapter 4 can thus be used to approximate the system load, e.g.,

\[ L_s^{IMRC} \triangleq \max_j L_j^{IMRC} \]

\[ L_s^{ISEL} \triangleq \max_j L_j^{ISEL}. \]

Yet another definition of load is one based on the system as a whole, i.e., not as consisting of a number of small systems (cells). The load of a system can then be explicitly related to existence of a solution to the system’s power control problem of finding appropriate transmission powers to satisfy all users’ requirements on carrier-to-interference ratio. To justify the alternative definition of load it is first shown that there is a maximum achievable carrier-to-total-interference ratio at least in a single service system and that we can hence calculate how much all users’ carrier-to-total-interference ratio can be scaled with before the system becomes unstable.

As a starting point, consider the following expression for the interference power user \( i \) experiences, \( I_i^{tot} \)

\[ I_i^{tot} = \sum_{\ell=1}^{M} g_{\ell,j_i} p_{\ell} + N_i, \]

where \( N_i \) is a user individual background noise power and the \( i \):th element of the vector \( j \) is the number of the cell user \( i \) is solely connected to. In a
tractable situation, power control maintains each user’s transmission power such that

\[ \beta_\ell = \frac{P_\ell g_{\ell,j}^\ell j}{I_{\ell}^{\text{tot}}} \geq \beta_0, \]

where \( \beta_0 \) is the carrier-to-total-interference power required by the only service supported. The above requirement on the transmission powers yields

\[ I_{\ell}^{\text{tot}} \geq \beta_0 \sum_{\ell=1}^{M} \frac{g_{\ell,j}^\ell j}{g_{\ell,j}^\ell j} I_{\ell}^{\text{tot}} + N_i. \]

Let us use the vector \( N = [N_i] \) and the matrix \( Z \) which is defined in Section 3.3 as \( Z = [z_{\ell,i}] \overset{\Delta}{=} \frac{g_{\ell,j}^\ell j}{g_{\ell,j}^\ell j} \). Properties of \( Z \) is discussed in Appendix A.2. The above expression for the interference power can then be expressed in matrix form as

\[ I_{\ell}^{\text{tot}} \geq \beta_0 Z^T I_{\ell}^{\text{tot}} + N, \tag{5.1} \]

where \( I_{\ell}^{\text{tot}} \) is a \( M \times 1 \) vector containing the interference power each user experiences. The following is a version of Theorem 3.1 which uses a requirement of positive interference powers instead of positive transmission powers.

**Theorem 5.1**

Assume that the \( G \)-matrix is a strictly positive matrix, i.e., all its elements are strictly positive. Then there exists a maximum achievable carrier-to-total-interference ratio in the noiseless case

\[ \beta_0^* = \max \{ \beta_0 | I_{\ell}^{\text{tot}} > 0 : \beta_\ell \geq \beta_0 \forall i \}. \]

Furthermore, the maximum is given by

\[ \beta_0^* = \frac{1}{\lambda^*}, \]

where \( \lambda^* \) is the greatest eigenvalue of \( Z \).

**Proof** The maximum achievable carrier-to-total-interference ratio, \( \beta_0^* \) is found by studying the following version of Equation (5.1)

\[ Z^T I_{\ell}^{\text{tot}} = \frac{1}{\beta_0^*} I_{\ell}^{\text{tot}} = \lambda I_{\ell}^{\text{tot}}. \]
The smallest real \( \lambda \) which gives a positive real solution \( I^{\text{tot}} \) is the greatest eigenvalue of \( Z \), \( \lambda^* \), according to theory for positive matrices, see Appendix A.1. Since a larger \( \beta_0^* \) corresponds to the a smaller eigenvalue of \( Z \), \( \frac{1}{\beta_0^*} = \lambda^* \).

In order to properly relate this to uplink relative load we need to define system feasibility.

**Definition 5.2 (System Feasibility)**

Given an uplink path gain matrix, \( G \), and user individual target carrier-to-total-interference ratios, \( \beta_i^{\text{tgt}} \), a system is feasible if there are user individual finite positive transmission powers such that

\[
\beta_i \geq \beta_i^{\text{tgt}} \quad \forall i.
\]

Otherwise, the system is infeasible.

Thus, in a single service scenario it is easy to determine whether a system is feasible or not; the system is feasible if \( \beta_0 < \beta_0^* \). However, in a multiple service scenario requiring all users' \( \beta_i^{\text{tgt}} < \beta^* \) would result in poor utilization of available resources. In order to incorporate user individual carrier-to-total-interference ratios, we use the following matrix (which was first introduced in Chapter 4)

\[
B^{\text{tgt}} = \text{diag}(\beta_1^{\text{tgt}}, \beta_2^{\text{tgt}}, \ldots, \beta_M^{\text{tgt}}).
\]

Denote by \((B^{\text{tgt}}, G, N)\) a system with background noise powers in the different cells according to the vector \( N \) and in which the users have target carrier-to-total interference ratios according to the matrix \( B^{\text{tgt}} \) and path gain values according to the matrix \( G \).

An alternative definition of uplink load related to the distance to infeasibility is

**Definition 5.3 (Feasibility Uplink Relative Load)**

The feasibility uplink relative load, \( L_f \), of a system \((B^{\text{tgt}}, G, N)\) is defined as

\[
L_f \overset{\Delta}{=} \inf \{ \mu \in \mathbb{R} : \left( \frac{1}{\mu} B^{\text{tgt}}, G, N \right) \text{ is feasible} \}.
\]

This definition is similar to Definition 3.3. They are, however, not equal since this one uses carrier-to-total-interference ratio instead of carrier-to-interference ratio. Note that \( L_f \) has the desirable properties that \( L_f \geq 0 \).
and that a system is feasible if and only if $L_f < 1$. $L_f$ can be found in some systems by using Theorem 5.2 below. We first need to generalize Equation (5.1) to handle multiple services as well

$$I^{tot} \geq (B^{tgt}Z)^T I^{tot} + N \iff (E - (B^{tgt}Z)^T) I^{tot} \geq N. \quad (5.2)$$

**Theorem 5.2**

In a system $(B^{tgt}, G, N)$ where each user is connected to exactly one base station, the feasibility uplink relative load is

$$L_f = \max \text{eig} (B^{tgt}Z) = \max | \text{eig} (B^{tgt}Z)|.$$

**Proof** A solution, $I^{tot}$, to the right hand side of Equation (5.2) exists if either of the following two requirements is satisfied

- $E - (B^{tgt}Z)^T$ is non-singular or
- $N \in \text{span}(E - (B^{tgt}Z)^T)$.

Since $Z$ consists of random numbers, the second requirement is satisfied with zero probability if $E - (B^{tgt}Z)^T$ is singular. Thus, the equation has a solution if and only if the matrix $E - (B^{tgt}Z)^T$ is non-singular. Study a system where the users have target carrier-to-total-interference ratios according to the matrix $\frac{1}{\mu} B^{tgt}$. Singularity of the matrix $E - \frac{1}{\mu}(B^{tgt}Z)^T$ yields

$$0 = \text{det}(E - \frac{1}{\mu}(B^{tgt}Z)^T) = \frac{1}{\mu^M} \text{det}(\mu E - (B^{tgt}Z)^T).$$

Hence, $\mu$ is an eigenvalue of $B^{tgt}Z$. Since all users’ transmission powers are positive if and only if all the interference powers are positive, we require a positive solution to Equation (5.2). According to theory for positive matrices, the smallest $\mu$ yielding a positive real solution, $I^{tot}$, to

$$(E - \frac{1}{\mu}(B^{tgt}Z)^T) I^{tot} = N$$

is the maximum eigenvalue, see Appendix A.1. This proves the first equality. The second equality in the theorem, that $\max \text{eig} (B^{tgt}Z) = \max | \text{eig} (B^{tgt}Z)|$, is a result from theory for positive matrices, see Appendix A.1.

**Remark 5.1**

Since soft handover can only decrease the uplink load, a $L_f$ found using Theorem 5.2 in a system without soft handover will be an overestimation of the load in the corresponding system with soft handover.
An application of the theorem is to show that $L_f$ equals the natural definition of feasibility relative load in a single service scenario

$$L_f = \max \mathrm{eig}(B^{tgt} Z) = \max \mathrm{eig}(\beta_0^{tgt} Z) = \beta_0^{tgt} \max \mathrm{eig}(Z) = \beta_0^* \lambda^* = \frac{\beta_0^{tgt}}{\beta_0^*}.$$

### 5.2 Relative Load Comparisons

In a single cell system the relative load according to Definition 3.2, Definition 5.1 and Definition 5.3 are the same according to the following theorem.

**Theorem 5.3**

In a single cell system with multiple services, the relative load according to Definition 3.2, Definition 5.1 and Definition 5.3 equals

$$L_f = \sum_{i=1}^M \beta_i^{tgt} = L = L_s.$$ 

**Proof** In a single cell system $Z$ will consists of just ones (see Appendix A.2) and will therefore have rank 1. Thus, all but one of the eigenvalues to $B^{tgt} Z$ will be zero. The only nonzero eigenvalue is $\lambda = \sum_{i=1}^M \beta_i^{tgt}$. Furthermore, this will trivially be the maximum eigenvalue since it is a sum of positive target carrier-to-total-interference ratios. Hence,

$$L_f = \max \mathrm{eig}(B^{tgt} Z) = \sum_{i=1}^M \beta_i^{tgt} = L,$$

where the first equality is true according to Theorem 5.2 and the last equality can be concluded from calculations similar to those in Example 3.1. Finally, obviously $L_s = \max_j L_j = L$ in a single cell system. $\square$

Other results, which are perhaps more useful in practice, are the following two theorems.

**Theorem 5.4**

Consider a system in which each user is power controlled in exactly one cell. Given complete knowledge of all users’ path gain, $L_s^{IMRC} \geq L_f$. 


5.2 Relative Load Comparisons

Proof Since all users’ path gains are assumed known, the expression will consider all contributions to the uplink relative load. In case each user is power controlled from just one cell, the expression given in Equation (4.10) can be expressed in vector form as \((B^{tgt}Z)^T 1\). Thus,

\[ L_s^{IMRC} = \| (B^{tgt}Z)^T 1 \|_\infty. \]

There is a theorem, see for example (Skogestad and Postlethwaite, 1996), which states that for any matrix norm

\[ \max | \text{eig}(A) | \leq \| A \|. \]

Choosing \( A = (B^{tgt}Z)^T \) and using Theorem 5.2 give the expression in the theorem.

Remark 5.2

This result shows that, in case of no soft handover, a \( L_s^{IMRC} < 1 \) is associated with a feasible uplink power control problem. Allowing soft handover can only yield lower uplink load and therefore a \( L_s^{IMRC} < 1 \) found without soft handover also proves stability of a system with soft handover.

Remark 5.3

In the above calculations we used expressions derived under an assumption of maximum ratio combining, any of the other two assumptions, soft handover or correct combination, might just as well have been made since these three are exactly the same when each user is allowed no more than one link.

Theorem 5.5

Given complete knowledge of all users’ path gain,

\[ L_s^{ISEL} \geq L_s. \]

Proof A system in which only selection combining is used will have higher noise rise levels than one using maximum ratio combing as well. By studying a system without maximum ratio combining, the results below hence applies for a system with maximum ratio combining as well.

Assume that cell 1 is the cell experiencing highest interference power, i.e., the cell defining the true system load \((L_s)\), then

\[ I_1 = N_1 + \sum_{i=1}^{M} \beta_i^{tgt} \frac{g_{i,1}}{\max_{k \in K_i} g_{i,k} T_k} \leq \]

\[ N_1 + \sum_{i=1}^{M} \beta_i^{tgt} \frac{g_{i,1}}{\max_{k \in K_i} \frac{g_{i,k}}{T_1}} = N_1 + I_1 \sum_{i=1}^{M} \beta_i^{tgt} \frac{g_{i,1}}{\max_{k \in K_i} g_{i,k}}. \]
Solving for $L_1^{SEL} = 1 - \frac{N_1}{L_1}$ yields an expression equal to Equation (4.11) expressing the uplink relative load in cell 1. The above calculations show that $L_1^{SEL}$ does not underestimate the relative load in the cell with highest load. Therefore,

$$L_s = \max_j L_j = L_1 \leq L_1^{SEL} \leq \max_j L_j^{SEL} = L_s^{SEL}.$$ 

\hspace{1cm} \square

### 5.3 Convergence of Fix Point Iterations

The stationary points of the iterations in Section 4.2.3, if they exist, are approximations of the uplink noise rise. This section provides sufficient conditions for convergence of iterations applied to Equation (4.16a) as well as a criteria for making system infeasibility probable. The iterations may be fix point iterations or single updates using Equation (4.16a) with the assumption of constant path gain values and target carrier-to-total-interference ratios during the time it takes the iterations to converge. Fix point iterations may be implemented as the algorithm below.

**Algorithm 5.1**

Let $\Lambda(t, 0) = \Lambda(t - 1)$

For $n = 1 \rightarrow NI$

\hspace{1cm} For $j = 1 \rightarrow B$

\hspace{2cm} Let $\Lambda_j(t, n) = 1 + \sum_{i=1}^{M} \beta_i^{gt}(t) \frac{g_{r,i}(t)}{\sum_{k \in K_i} g_{r,k}(t)}$

\hspace{2cm} $\Lambda(t) = \Lambda(t, NI)$

First, however, some theory regarding Lipschitz theory and existence of a unique stationary point for certain iteration functions is provided.

#### 5.3.1 Lipschitz theory

Lipschitz theory can be used to determine convergence of nonlinear iterations. Dennis and Schnabel (1983) provides a good basis for using Lipschitz theory in practice. The following theorem, which is here given without proof, can be used to prove existence of a stationary point of the iterations.
5.3 Convergence of Fix Point Iterations

**Theorem 5.6 (Contractive Mapping Theorem)**
Let \( f : \mathbb{R}^n \to \mathbb{R}^n \), be an iteration function, and \( S_r = \{ x \mid ||x - x(0)|| < r \} \) be a ball of radius \( r \) around a given point \( x(0) \in \mathbb{R}^n \). Assume that \( f \) is a contraction mapping in \( S_r \), i.e.,

\[
\forall u, v \in S_r \Rightarrow ||f(u) - f(v)|| \leq \alpha ||u - v||, \tag{5.3}
\]

where \( \alpha < 1 \). Then if

\[
||x(0) - f(x(0))|| \leq (1 - \alpha)r
\]

the equation \( x = f(x) \) has a unique solution \( x_* \) in the closure \( \bar{S}_r = \{ x \mid x - x(0)|| \leq r \} \). This solution can be obtained by the convergent iteration process \( x(k + 1) = f(x(k)), \ k = 0, 1, \ldots \)

The requirement on \( \alpha \) in Equation (5.3) suggests that \( \alpha \) is a Lipschitz constant to \( f \). A comfortable way of finding a Lipschitz constant for a function \( f \) is through the following lemma

**Lemma 5.1**
Let the function \( f(x), \mathbb{R}^n \to \mathbb{R}^n \), be differentiable in a convex set \( D \subset \mathbb{R}^n \). Then \( \alpha = \max_{y \in D} ||f'(y)|| \) is a Lipschitz constant for \( f \).

\( f'(y) \) is the Jacobian matrix of \( f \) at \( y \).

5.3.2 Global Convergence

Yates (1995) shows that, given the three conditions in the theorem below, the equilibrium point \( x_* \) is unique in the entire \( \mathbb{R}^n \). Furthermore, we will have convergence independently of the initialization point.

**Theorem 5.7 (Global Convergence)**
Assume that there exist a \( x_* \) such that \( f(x_*) = x_* \) and \( f(x) \) satisfies the following properties

- **Positivity**: \( f(x) > 0 \)
- **Monotonicity**: If \( x \geq x' \) then \( f(x) \geq f(x') \)
- **Scalability**: For all \( \mu > 1 \), \( \mu f(x) > f(\mu x) \).

Then the iteration \( x(t + 1) = f(x(t)) \) converges to \( x_* \) for all \( x(0) \in \mathbb{R}^n \).

The above inequalities, just as all inequalities between vectors in this chapter, should be interpreted component wise.

**Proof** See Yates (1995).
5.3.3 Conditions for Convergence

The theory previously given in this section can be applied to the iterations Algorithm 5.1. As the uplink noise rise expression based on Equation (4.16a) requires convergence of the algorithm it is interesting to establish conditions for its convergence.

Let $x$ in the previous two subsections be the approximative uplink noise rise $\Lambda^{NMRC}$, $n$ equal the number of cells, $B$ and $f(x) = f(\Lambda^{NMRC})$ be defined by

$$f_j(\Lambda^{NMRC}) = 1 + \sum_{i=1}^{M} \beta_i^{gt} \frac{g_{i,j}}{\sum_{k \in K_i} \Lambda_k^{NMRC}}, \quad j = 1, 2 \ldots, B. \quad (5.4)$$

We also need the Jacobian matrix of $f(\Lambda^{NMRC})$. The element on row $j$ and column $\ell$ of the Jacobian matrix is given by

$$\frac{df_j(\Lambda^{NMRC})}{d\Lambda^{NMRC}_\ell} = \sum_{i=1}^{M} \frac{\beta_i^{gt} g_{i,j}}{(\sum_{k \in K_i} \Lambda_k^{NMRC})^2} \Lambda^{NMRC}_\ell. \quad (5.5)$$

Note that this equation is linear in $\beta_i^{gt}$.

$f(\Lambda^{NMRC})$ satisfies the conditions in Theorem 5.7. Positivity and monotonicity are satisfied trivially because of the nature of the functions origin; obviously the noise rise is positive and obviously if the noise rise is increased in one cell users in other cells compensates for the increased intercell interference with an increase in transmission power. Mathematically, positivity is realized by observing that $f(\Lambda^{NMRC})$ is a sum of positive terms and monotonicity follows from the fact that also the derivative of $f(\Lambda^{NMRC})$ in Equation (5.5) is non-negative for all $\Lambda^{NMRC}$. Scalability is proven by (remember, $\mu > 1$)

$$f_j(\mu \Lambda^{NMRC}) = 1 + \sum_{i} \beta_i^{gt} \frac{g_{i,j}}{\sum_{k \in K_i} \frac{g_{i,k}}{\mu \Lambda_k^{NMRC}}} = 1 + \mu \sum_{i} \beta_i^{gt} \frac{g_{i,j}}{\sum_{k \in K_i} \frac{g_{i,k}}{\Lambda_k^{NMRC}}} \quad < \mu \left(1 + \sum_{i} \beta_i^{gt} \frac{g_{i,j}}{\sum_{k \in K_i} \frac{g_{i,k}}{\Lambda_k^{NMRC}}}ight) = \mu f_j(\Lambda^{NMRC}).$$

Let us first study a system without soft handover. The following lemma shows that there exists a solution to Equation (5.2) if the system considered is feasible. Denote by $\Lambda^{(user)} \in \mathbb{R}^M$ a vector in which position $i$ contains the uplink noise rise user $i$ experiences.
5.3 Convergence of Fix Point Iterations

**Lemma 5.2**
Consider a system \((B^{tgt}, G, \Upsilon \ 1^B)\), where \(\Upsilon\) is the background noise power common to all cells. Assume that soft handover is not utilized in the system. There exists a time independent finite solution to Equations (4.16) if the system is feasible.

**Proof** All of the expressions in (4.16) are equal since soft handover is not utilized. The to Equation (4.16a) corresponding expression for \(\Lambda^{(user)}\) can be written if matrix form as

\[
\Lambda^{(user)} = 1^M + (B^{tgt} Z)^T \Lambda^{(user)}. \tag{5.6}
\]

\(L_f < 1\) is less than one since the system is feasible. According to Theorem 5.2 and theory for positive matrices, the matrix \((E - (B^{tgt} Z)^T)^{-1}\) exists and is positive. Hence, as \(N > 0\), \(I^{tot} = (E - (B^{tgt} Z)^T)^{-1} N\) must be positive. Dividing the equation with the common background noise, \(N_i = \Upsilon\), and substituting \(I^{tot}\) for \(\Lambda^{(user)}\) yield

\[
\Lambda^{(user)} = (E - (B^{tgt} Z)^T)^{-1} 1^M \iff \Lambda^{(user)} = 1^M + (B^{tgt} Z)^T \Lambda^{(user)}.
\]

The right hand expression is the same as in Equation (5.6) and hence that equation has a positive, finite solution.

Applying Theorem 5.7 to Algorithm 5.1 gives the following theorem.

**Theorem 5.8**
Algorithm 5.1 will converge if applied to a feasible system without soft handover.

**Proof** In case of no soft handover, \(f(\Lambda^{NMRC})\) will be an affine function in the elements of \(\Lambda^{NMRC}\). The corresponding expression for \(f\) when using vector \(\Lambda^{(user)}\) is

\[
f_i(\Lambda^{(user)}) = 1 + \sum_{\ell=1}^{M} \beta^{tgt}_\ell \frac{g_{\ell,j_i}}{\Lambda^{(user)}_{\ell,j_i}} = 1 + \sum_{\ell=1}^{M} \Lambda^{(user)}_{\ell,j_i} \beta^{tgt}_\ell z_{\ell,i}.
\]

This can now be written in matrix form as

\[
f(\Lambda^{(user)}) = 1^M + (B^{tgt} Z)^T \Lambda^{(user)}.
\]

Substituting \(f(\Lambda^{(user)})\) for \(\Lambda^{(user)}\) in yields an equation in \(\Lambda^{(user)}\). This equation is the same as Equation (5.6). Since that equation is solvable in
case of feasibility, there is a $\Lambda^{(\text{user})}$ such that $f(\Lambda^{(\text{user})}) = \Lambda^{(\text{user})}$. Furthermore, since $f(\Lambda^{(\text{user})})$ satisfies the conditions in Theorem 5.7 the algorithm will converge.

We can therefore be certain that the algorithm converges when applied to a feasible system without soft handover. In the case where soft handover is utilized we need to apply Lipschitz theory in order to relate feasibility to convergence of the iterations.

The Lipschitz theory given earlier in this section can be summarized into the following theorem

**Theorem 5.9**

Let $f(\Lambda^{\text{NMRC}})$ be defined according to (5.4) and $\alpha$ be defined as

$$\alpha \triangleq \max_{\Lambda^{\text{NMRC}} \in S_r} \|f'(\Lambda^{\text{NMRC}})\|,$$

where $S_r$ is a ball with radius $r$ around the point $\Lambda^{\text{NMRC}}(0)$. If

$$\|\Lambda^{\text{NMRC}}(0) - f(\Lambda^{\text{NMRC}}(0))\| \leq (1 - \alpha)r$$

(5.7)

the iteration $\Lambda^{\text{NMRC}}(t + 1) = f(\Lambda^{\text{NMRC}}(t))$ converges to $\Lambda^{\text{NMRC}} \in S_r$ if initialized with $\Lambda^{\text{NMRC}}(0)$.

**Proof** According to Lemma 5.1, choosing $\alpha$ as in the theorem makes it a Lipschitz constant in $S_r$. If we can find a $\Lambda^{\text{NMRC}}(0)$ and a $r$ such that this $\alpha$ is less than 1 and the inequality (5.7) is satisfied than Theorem 5.6 states that the iteration converges to a point in $S_r$.

The above theory is now applied to a system consisting of two cells and two users.

**Example 5.1 (Convergence of Fix Point Iterations)**

Let the path gain and link matrix be

$$G = \begin{pmatrix} 0.53 & 0.15 \\ 0.14 & 0.8 \end{pmatrix}, K = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

respectively. The users have target carrier-to-total-interference ratios according to

$$B^{\text{tgt}} = \text{diag}(0.75, 0.45).$$
Since there are only two cells, the gradient’s dependence on the noise rise vector, $\Lambda^{NMR}$, is through the ratio of the noise rise in the two cells,

$$
\frac{df_j}{d\Lambda^{NMR}_i} = \sum_{i=1}^{M} \frac{\beta_i^{gt} g_{i,j}}{\left(\sum_{k \in K_i} g_{i,k}^{NMR}\right)^2} g_{i,j} \Lambda^{NMR}_i \sum_{i=1}^{M} \frac{\beta_i^{gt} g_{i,j}^{NMR}}{\left(\sum_{k \in K_i} g_{i,k}^{NMR}\right)^2} g_{i,j}.
$$

Therefore the dependence on $\Lambda^{NMR}$ is easily visualized. Figure 5.1 shows the norm of the gradient according to Equation (5.8) for different $\eta = \frac{\Lambda^{NMR}}{\Lambda^{NMR}}$. From the figure we can conclude that, by choosing $\alpha = 0.82$, we get a lower bound of $\eta_{\text{min}} = 0.4$ and no upper bound on $\eta$. Choosing $\Lambda^{NMR}(0) = (3 \ 3)^T$, $\eta_{\text{min}}$ gives us a ball with radius $r = 1.67$. Since

$$
||\Lambda^{NMR}(0) - f(\Lambda^{NMR}(0))|| = 0.15 \leq 0.3 = (1 - \alpha)r
$$

the iterations will converge according to Theorem 5.9.

Since $f(\Lambda^{NMR})$ satisfies the conditions of Theorem 5.7, the choice of $S_r$ in Theorem 5.9 is not critical, i.e., the iteration will converge for any initialization point, $\Lambda^{NMR}(0)$, if convergence has been proven for some initialization point.

The following theorem can be used to make infeasibility of a general system probable.

**Theorem 5.10**

If the background noise power in all cells of a feasible system can be assumed equal, then

$$
1 \leq f(\Lambda^{NMR}) \leq \Lambda \text{ if } 1 \leq \Lambda^{NMR} \leq \Lambda,
$$

where $f(\Lambda^{NMR})$ is defined by Equation (5.4) and $\Lambda$ is the true noise rise vector.

**Proof** If the system is feasible, there is a finite true noise rise vector, $\Lambda$ since $p_i$ is finite for all $i$ and

$$
\Lambda_j = \frac{I_j}{N} = 1 + \sum_{i=1}^{M} \frac{p_i g_{i,j}}{N},
$$
Figure 5.1 Lipschitz constant estimate in Example 5.1. The dashed line defines the limit for which \(|f'(\eta T)| \leq 0.82\).

where \(N\) is the background noise power common to all cells. \(f(1) \geq 1\) trivially since all terms in the sum of Equation (5.4) are non-negative. Furthermore, as \(f(\Lambda^{NMRC})\) is strictly increasing

\[ f_j(\Lambda) \leq \Lambda_j \Rightarrow f_j(\Lambda^{NMRC}) \leq \Lambda_j, \forall ji if \\Lambda^{NMRC} \leq \Lambda. \]

Thus, we only need to show that \(f(\Lambda) \leq \Lambda\). This can be done by using the inequality proved in Section 4.4.1

\[ \sum_{k \in K_i} \beta_{i,k} = \sum_{k \in K_i} \frac{p_i g_{i,k}}{I_k} \geq \beta_i \Rightarrow \frac{p_i}{N} \geq \frac{\beta_i}{\sum_{k \in K_i} \frac{g_{i,k}}{I_k}}. \]

Therefore,

\[ f_j(\Lambda) = 1 + \sum_i \beta_i g_{i,j} \frac{g_{i,j}}{\sum_{k \in K_i} \frac{g_{i,k}}{I_k}} \leq 1 + \sum_i \frac{p_i}{N} g_{i,j} = \Lambda_j. \]
Hence, if Algorithm 5.1 is initialized with a $\Lambda^{NMRC}(0) = 1$ in a feasible system it will terminate in a $\Lambda^{NMRC}(NI) \leq \Lambda$, where $NI$ the number of iterations performed in Algorithm 5.1. As a result of Theorem 5.10, we can conclude that if Algorithm 5.1 diverges the system is infeasible. An application of this is shown in the following example.

**Example 5.2**
Algorithm 5.1 applied to the system in Example 5.1 converges. If we choose $\Lambda^{NMRC}(0)$ equal to the convergence point, $||\Lambda^{NMRC}(0) - f(\Lambda^{NMRC}(0))|| = 0$. We can thus choose $S_r$ to an infinitely small ball with center in $\Lambda^{NMRC}(0) = (2.85 \ 2.66)^T$. This yields $\alpha = 0.64$ according to Figure 5.1.

Recall that $\alpha$ scales with the users’ target carrier-to-total interference ratio for a given link configuration and path gain matrix. Lipschitz theory can thus give us an hint on how much to scale $B^{tgt}$ with since the system only with target carrier-to-total-interference ratios according to $\frac{1}{\alpha}B^{tgt}$ will have a Lipschitz constant estimate equal to 1. Figure 5.2 shows that scaling $B^{tgt}$ with $\frac{1}{\alpha = 0.64}$ has no effect on the convergence, while scaling $B^{tgt}$ with $\frac{1}{\alpha = 0.63}$ seems to make it diverge. According to Theorem 5.10 and Definition 5.3, $L_f$ is thus probably greater than 0.63.

![Figure 5.2](attachment:image.png) **Figure 5.2** Convergence and divergence of Algorithm 5.1 for different target carrier-to-interference ratios in Example 5.2.
Remark 5.4
The definitions of load given in this chapter have been concerned with the entire system. The expressions in Chapter 4, however, provide one relative load measure per cell. From a theoretical point of view, a measure of the load related to the entire system is equally interesting since nowhere is the load allowed to exceed 1 if the system is to be stable.

5.4 Summary

In this chapter a number of alternative definitions of uplink load has been given. One of them directly relates uplink load to existence of a solution to the power control problem. It has been shown that, in a single cell system, this definition coincides with the more traditional definition of load related to noise rise. However, in a general system they are different. Despite this, it has been shown that expressions for uplink load derived in Chapter 4 can under somewhat idealistic circumstances be used to provide bounds on the alternative definitions of load given herein.

All expressions for uplink noise rise in Chapter 4 originate from a system of nonlinear equations. One way of solving these equations is to use iterations. The iterations are obviously useful only if they converge to a stationary point. This chapter has given sufficient conditions for existence of such a point. Furthermore, it has been proven that, for a given system, there exists at most one stationary point of the iterations.

Finally, a result stating that the iterations will not diverge if a solution to the power control problem exists is provided. This can be used to make infeasibility probable.
Filtering and Prediction

In this chapter, advanced filtering is applied to a readily available load measurement given by any of the expressions in Chapter 4. As signal model, a biased auto-regressive model has been developed and used. Using the model together with a change detection technique results in a stable estimate which quickly adapts to new load levels. In the first section some motivation for applying filtering is given. In Section 6.2 a signal model is proposed. Section 6.3 presents some theory related to adaptive filtering and discusses design choices made. Adaptive filtering is then applied to the model in Section 6.4 before a summary is made in Section 6.5.

6.1 Motivation

As can be seen in Figure 6.1, the raw measurement of the uplink relative load has, just as the true load, a trend and low frequency oscillations on top of this trend. The oscillations are due to the constant user movement; as the users move around in the environment their number of soft handover links regularly changes. As can be seen in Figure 6.2, oscillations in a cell’s uplink noise rise are strongly correlated with oscillations in the number of soft handover links. These oscillations can easily be canceled by low pass filtering the signal. However, applying a simple low pass filter could be risky since it is important to be alert on sudden changes in the load. Therefore we
have applied a signal model together with Kalman filtering and change detection to the relative load estimate in order to reduce noise and oscillations while keeping track of sudden changes in the signal. A more stable signal enables operation at a higher load level since the resource management algorithms would not need as large margin to secure stability of the system. The time-varying model also provides an indication of towards where the load is currently heading, something which enables more aggressive resource management algorithms, e.g., some form of PD-control.

6.2 Signal Model

When dealing with signals whose behaviour is expected to change, continuously or abruptly, a fixed model is obviously not attractive. Using a parameterized signal model and estimating not only the actual signal value but also the parameters of the model enables a more accurate signal estimate as well as an earlier detection of an abrupt change, in the parameters and/or the signal level. A signal $y(t)$, originating from an unbiased $n$:th order Auto Regressive (AR)-model, may be described as (see e.g., (Ljung, 1999))

$$y(t) + a_1y(t-1) + a_2y(t-2) + ... + a_ny(t-n) = e(t),$$

where $e$ is white Gaussian noise and $a_1, a_2, ..., a_n$ are the parameters of the model. The signal $y(t)$ in Equation (6.1) will have $Ey(t) = 0$, but the
signals we study have $E y(t) = L(t)$. Therefore we need a slightly more complex model where the time-varying bias is subtracted from each $y(t)$ giving $E(y(t) - L(t)) = 0$. Replacing $y(t)$ by $y(t) - L(t)$ in (6.1) yields

$$
(y(t) - L(t)) + \sum_{i=1}^{n} a_i(y(t - i) - \bar{L}(t - i)) = e(t).
$$

(6.2)

This equation defines what can be described as a biased AR model, the oscillations are no longer restricted to be around zero. There are other alternatives when modeling a signal with oscillations around a non zero mean. Alasti and Farvardin (2000) uses non-zero-mean Gaussian noise to derive an expression for a biased AR model which is used to model the amount of requested bandwidth in a wireless network.

We use a fourth order biased AR model to describe $y$, i.e., $n = 4$ in (6.2). It is natural to assume different time scales of the bias variation and the dynamics, otherwise the AR-model is ambiguous. That is, the AR-model takes care of short term oscillations and $L(t)$ models the long term drifts.
Then Equation (6.2) may be rearranged into
\[ y(t) = \hat{L}(t)(1 + \sum_{i=1}^{4} a_i) - \sum_{i=1}^{4} a_i y(t - i) + e(t). \] (6.3)

A state space representation of Equation (6.3) in discrete time is
\[
\begin{aligned}
x(t) &= [\hat{L}(t) \dot{\hat{L}}(t) a_1 a_2 a_3 a_4]^T \\
x(t + 1) &= \begin{pmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_4 \end{pmatrix} x(t) + \begin{pmatrix} \frac{I_2}{T} & 0 \\ \frac{T}{T} & 0 \\ 0 & T I_4 \end{pmatrix} w(t) \\
y(t) &= (1 + \sum a_i) x + e(t),
\end{aligned}
\]
where \( w(t) \) is a 5-dimensional vector containing the process noise. Note that the measurement \( y(t) \) has a non-linear relation to the parameters. This makes the parameter estimation more difficult. By introducing two new variables, \( \tilde{L}(t) = (1 + \sum a_i) \hat{L} \) and \( \dot{\tilde{L}}(t) \), we can convert the above nonlinear state space model and obtain a linear state space model according to
\[
\begin{aligned}
\tilde{x}(t) &= [\tilde{L}(t) \dot{\tilde{L}}(t) a_1 a_2 a_3 a_4]^T \\
\tilde{x}(t + 1) &= \begin{pmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_4 \end{pmatrix} \tilde{x}(t) + \begin{pmatrix} \frac{I_2}{T} & 0 \\ \frac{T}{T} & 0 \\ 0 & T I_4 \end{pmatrix} w(t) \\
y(t) &= (1 + \sum a_i) \tilde{x}(t) + e(t),
\end{aligned}
\] (6.4)

Even though \( \dot{\hat{L}}(t) \) includes time derivatives of \( a_i \), the variation in the bias can perhaps be assumed far bigger than that in the AR-parameters and thus \( \dot{\tilde{L}}(t) \approx (1 + \sum a_i) \dot{\hat{L}}. \) Denote by \( Q \) the covariance matrix for the vector \( w(t) \). We assume the parameters to be independent of each other. This results in a diagonal \( Q \) with elements \( q_1, q_2 \cdots q_5 \) in the diagonal. The values \( q_2 \) to \( q_5 \) are all the same and much less than \( q_1 \). Hence we encourage changes in \( \tilde{L} \) and \( \dot{\tilde{L}} \) rather than in the AR-parameters. The innovations, \( e(t) \) are assumed to be zero mean Gaussian with constant variance \( R \).

As we are interested in \( \hat{L} \) the following conversions are necessary to obtain the correct final result,
\[
\begin{aligned}
\hat{L}(t) &= \frac{\dot{\tilde{L}}}{1 + \sum_{i=1}^{4} \tilde{a}_i} = \frac{\dot{\tilde{x}}_1}{1 + \sum_{k=3}^{6} \tilde{x}_k} = f(\dot{\tilde{x}}) \\
\text{Var } \tilde{L} &\approx f'_x(\dot{\tilde{x}})^T \text{ Cov } \tilde{x} \ f'_x(\dot{\tilde{x}}),
\end{aligned}
\] (6.5)
6.3 Adaptive Filtering

The expression for the variance is motivated by studying a first order Taylor expansion of \( f(\hat{x}) \).

6.3 Adaptive Filtering

In this section we will explain the techniques used in the filtering. Figure 6.3 shows the different part of the adaptive filter applied in this chapter. Kalman filtering is explained in Subsection 6.3.1. Some of the output of this algorithm is fed to a CUSUM detector. The idea and implementation of a CUSUM algorithm is explained in Subsection 6.3.2. The figure also shows how the behaviour of these algorithms are decided by a number of design parameters. A brief discussion on the design choices made when setting the values of these parameters is also included in Subsection 6.3.3. A good basis for using Kalman filtering, adaptive filtering and change detection in practice is given by Gustafsson (2000).

6.3.1 Kalman Filtering

Equation (6.4) is of the form

\[
\begin{align*}
x(t + 1) &= Ax(t) + B_v w(t) \\
y(t) &= C(t)x(t) + e(t).
\end{align*}
\]

We want to estimate the state vector \( x(t) \) in the above state space model such that the covariance of the state error, \( E(\hat{x}(t) - x(t))(\hat{x}(t) - x(t))^T \), is minimized given measurements \( y(t) \). Introduce the notation \( \hat{x}(t|\tau) \) for detected change.

\[\text{Figure 6.3 The different parts and parameters of an adaptive filter with change detection.}\]
the estimate of $x(t)$ given measurements up until time $\tau$. The notation is justified by the fact that at each time instant there is a measurement update, where the new measurement is considered, and a time update, where the predictions are updated. A natural way of updating the state estimate with a new measurement is

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K(t)\epsilon(t),$$

where $\epsilon(t) = y(t) - C(t)\hat{x}(t|t-1)$ is called the residual at time $t$. Out of all possible ways of choosing $K(t)$, the Kalman filter (Kalman, 1960) chooses $K(t)$ such that the covariance of the residual, $\epsilon(t)$, is minimized. This optimality holds if the noise vectors are Gaussian, otherwise the Kalman filter is the best possible linear filter. $K(t)$ is chosen based on knowledge of the measurement errors’ variance, $R = E(\epsilon(t)\epsilon(t)^T)$, the process noise covariance, $Q = E(w(t)w(t)^T)$, and an estimate of the covariance of the current estimation error, $P(t|\tau) = E((x - \hat{x}(t|\tau))(x - \hat{x}(t|\tau))^T)$. Since $P$ is time-varying it, too, has to be updated continuously. The update of $P$, $K$ and $\hat{x}$ is done according to the following algorithm.

**Algorithm 6.1 (Kalman filter)**

\[
\begin{align*}
K(t) &= P(t|t-1)C(t)^T(C(t)P(t|t-1)C(t)^T + R)^{-1} \\
\hat{x}(t|t) &= \hat{x}(t|t-1) + K(t)\epsilon(t) \quad (6.6a) \\
P(t|t) &= P(t|t-1) - K(t)C(t)P(t|t-1) \quad (6.6b) \\
\hat{x}(t+1|t) &= A\hat{x}(t|t) \quad (6.6c) \\
P(t+1|t) &= AP(t|t)A^T + B_vQB_v^T \quad (6.6d) \\
\hat{x}(0|-1) &= \hat{x}_0, P(0|-1) = P_0 \quad (6.6e)
\end{align*}
\]

Equations (6.6a) and (6.6b) are the measurement update, whereas (6.6c) and (6.6d) are the time update. $Q$ as well as $R$ are design parameters of the filter. Filtering $y(t)$, as oppose to predicting $y(t+1)$, corresponds to studying $\hat{x}(t|t)$, as oppose to $\hat{x}(t+1|t)$.

Consequently, the Kalman filter delivers an estimate of the state vector $x(t)$, an estimated covariance matrix for this estimate (indicating the accuracy of the estimate), $P(t)$, and a residual $\epsilon(t)$ at each time instant $t$. 
6.3 Adaptive Filtering

6.3.2 Change Detection

An always present problem with traditional estimation is the trade off between tracking and noise suppression. A way of getting around this problem is by studying the residuals, $e(t)$, of a process estimating $\tilde{x}$. In the model we use, the residuals are expected to be zero mean Gaussian, i.e., $e(t) \in N(0, R)$. If the estimates of $e(t)$, $\epsilon(t)$, are not zero-mean Gaussian we have reason to believe there has been an abrupt change in one or several of the parameters.

In order to detect a bias in the residuals, the squared normalized residuals (normalized with their estimated standard deviation), $\hat{e}^2$, are fed to a distance measurement algorithm. At every time instant this algorithm adds the normalized residual minus a drift term $\nu$ to the previous accumulated distance. Hence, several consecutive residuals with considerable large magnitude will result in a growing distance $g(t)$, and eventually a change detection once $g$ has exceeded a fix threshold, $h$. The cumulative sum (CUSUM) algorithm is explained by the following pseudo code which is run through each time step.

**Algorithm 6.2 (CUSUM)**

\[
g = g + \hat{e}^2 - \nu \\
\text{if } g < 0 \\
\quad g = 0 \\
\text{end} \\
\text{if } g > h \\
\quad g = 0 \\
\quad \text{flag for detected change} \\
\text{end}
\]

Once a change is detected, the filter characteristics is somehow temporarily changed in order to quickly adapt to the new situation.

6.3.3 Design Choices

Figure 6.4 shows an example of a raw signal produced by the load estimation (i.e., Equations (4.16c) and (3.3)). The period of the slowest oscillations is about 20 seconds. The sampling time of this signal is $T_{\text{frame}} = 0.01$ seconds. Thus the slowest oscillations have a period of about 2000 samples. To describe the correlation in a signal having oscillations with such a long period would require thousands of parameters. Another way of looking at
it is that when using far too few AR parameters, the difference between the samples considered by the AR model is simply noise. A result of using too short time horizon is an estimated model which only integrates noise. To put it mathematically, the model will have a pole close to +1. A pole close to one means that \( q = 1 \) satisfies the characteristic equation of the AR model

\[
1 + a_1q^{-1} + a_2q^{-2} \cdots a_nq^{-n} = 0.
\]

Thus, if the sum of the estimated AR parameters is close to minus one, it is likely that the time horizon is too short.

A rule of thumb is that the fundamental period of the oscillations to be modeled should be approximately 10 samples (Ljung, 1999). Therefore we use every two hundredth sample when constructing the regression vector, \( C(t) \). This is a kind of down sampling which requires an anti alias filter to be applied. The chosen low pass filter is a second order filter with both its poles on the real axis.

\[
y(t) = \frac{4 \cdot 10^{-4}}{1 - 1.96q^{-1} + 0.9604q^{-2}} L(t)
\]

This results in a filter which does not introduce any additional oscillations but on the other hand does not have a linear phase shift. Since we are interested in the low frequencies of the signal, a linear phase shift is not a requirement here. The choice of filter bandwidth is a trade off between low alias effects and small time delay between input and output of the anti-alias filter.
The state space representation in Equation (6.4) has to be slightly modified to adapt to the resampling. Thus the final state space model used in the simulations is
\[
\dot{x}(t) = [\dot{L}(t) \dot{\dot{L}}(t) a_1 a_2 a_3 a_4]^T
\]
\[
\dot{x}(t+1) = \begin{pmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_4 \end{pmatrix} \dot{x}(t) + \begin{pmatrix} \frac{T^2}{2} & 0 \\ 0 & T I_4 \end{pmatrix} w
\]
\[
y(t) = \begin{pmatrix} 1 & 0 & -y(t-200) & \cdots & -y(t-800) \end{pmatrix} \dot{x}(t) + e(t).
\]

Note that \(y(t)\) is the output from the anti alias filter, and \(T\) is the update rate of the filter times the length of a frame, \(T_{frame} = 0.01\) s. The Kalman filter is parameterized by the constants in the matrix \(Q\) and the scalar \(R\), which thus can be seen as filter design variables. A larger value in one of \(Q\)'s components means that the corresponding state variable(s) are more willing to change during the time between two time instants, whereas a larger \(R\) corresponds to less measurement accuracy. Hence, larger \(Q\) results in larger \(K\) and larger \(R\) results in a filter which puts less trust in the measurements and \(K\) is chosen smaller yielding a "slower" filter. The choice of the size of \(Q\) versus the size of \(R\) is thus a trade-off between tracking and noise suppression. \(R\) should be chosen such that it approximately equals the variance of the residuals.

Another design choice concerns the update rate of the filter. A reason for not choosing a faster update rate, which of course provides a possibility for earlier detection of large changes in the relative load, is the increased computational burden a higher update rate induces. So perhaps updating every sample is not an obvious choice. We have studied filter that has an update rate of every tenth or every hundredth estimate sample. Simulations shows that the performance in steady state when filtering is the same regardless of the update rate.

The behaviour of the CUSUM detector is primarily decided by the parameters \(h\) and \(\nu\). These have been chosen in such a way that we detect sudden changes of considerable amplitude within a reasonable time without having too many false alarms. The choice of \(h\) is dependent on the update rate of the filter. We have chosen to scale \(h\) with the update rate, i.e.,
\[
h = \frac{T}{T_{frame}} h_0,
\]
where \(h_0\) is the threshold used when updating every sample. In case of a change detection, the values of \(P\) corresponding to \(\dot{x}_1\) and \(\dot{x}_2\) are increased...
a factor five, which results in making filtered value of these states more sensitive to the current input to the filter, \( y(t) \). The motivation for increasing only these two states is that the change is believed to be in the load level and not in the AR parameters.

6.4 Simulations

The estimate used as input to this work is from a simulation scenario consisting of 21 cells where multiple services are provided. We have used a Kalman filter to extract the load level \( L \) from the signal \( y \) in Equation (6.3). By using the change detection technique described in Section 6.3 we can easily follow any sudden jumps in the load estimate.

The top plot of Figure 6.5 shows the approximation of \( L \) in Equation (3.3) where \( \lambda \) is approximated using Equation (4.16c). Also shown in the figure is the Kalman filter estimate. The filter estimate is updated every frame, so \( T = T_{frame} = 0.01 \text{ s} \). The filtered estimate is free of oscillations and follows an underlying trend in the original estimate. Notice, how the filtered estimate jumps when there is a distinct jump in the raw estimate. The middle plot of Figure 6.5 shows a close up version of the top plot together with a rough low pass filtered version of the original estimate. Both filters suppress noise equally well, but notice the low pass filtered signal’s slow adaptation to the new load level. In case of a sudden jump upwards, the slow acclimatization of the low pass filtered signal can be risky. In case of a jump downwards, we can improve the resource utilization by admitting more load earlier if we use the output from the Kalman filter as input to a resource management algorithm compared to using the low pass filtered version. As a comparison, the bottom plot of Figure 6.5 shows the performance of the filter when using three different update rates; once every frame as above, once every tenth frame and once every hundred frame. In steady state, the three versions of the filtering provides almost identical outputs. But, due to its superior update rate, the version which is updated every frame detects the jump earlier. However, when updating every hundredth frame detects the jump even before the every tenth frame update version in this particular instant. Thus, a faster update rate is perhaps not crucial, especially when comparing with the low pass filtered version which is also shown in the same plot. Considering the increased computational burden a faster update rate introduces, the faster update rate is probably not worth its price. The estimation process also provides us with an estimate of the derivative of the trend in the relative load which enables a more aggressive
Figure 6.5  Performance of the kalman filtering and change detection when applied to a signal with a rather big jump in it. The different plots are magnified versions of the same simulation. Solid: original estimate. Solid, thick: updated each frame. Solid, ring: updated every tenth frame. Solid, plus: updated every hundredth frame. Dashed: low pass filtered
Due to the anti alias filter applied before the Kalman filter, the measurement noise $e(t)$ is no longer white. Therefore the residuals from the estimation process are not white either. This indeed indicates the need for a more complex model, for example an ARMA model. However, as the simulations show, we still manage to provide a good estimate of the underlying trend in the relative load estimate.

### 6.5 Summary

Due to user movement, the uplink relative load is subject to slow oscillations. Earlier work has provided an estimate of the uplink relative load which to a great extent captures these oscillations. In this paper we have proposed a signal model, which describes the relative load as a signal consisting of a bias with a trend together with a fourth order auto regressive model. A Kalman filter together with a change detection algorithm has been applied to the model. The result is a stable signal representing a time average of the uplink load. Unlike what an ordinary low pass filter would provide, we are still alert on sudden changes in the load level due to the change detection algorithm used. Furthermore, the filter provides us with an indication towards where the relative load is heading which enables a more aggressive resource management.
In this chapter we have gathered simulations showing the performance of the expressions for uplink noise rise derived in Chapter 4. The chapter is divided into five sections. In Section 7.1 information about the simulator is given. In the following three sections simulation results are presented and discussed. Section 7.2 investigates the expressions’ sensitivity to in which manner the path gains are made available. Section 7.3 and Section 7.4 study the performance of the expressions when one or several radio network controllers are used, respectively. Finally a summary is provided in Section 7.5.

In this chapter relative load refers to the load according to Definition 3.2.

7.1 Simulator

In this section properties of the simulator will be described. Algorithms and parameters of the radio environment as well as user creation, movement and transmission pattern are described in two subsections. Later, properties of the system such as base station and cell configuration are given together with resource management algorithms and parameters.

A wide range of algorithms of a WCDMA system is simulated, from algorithms operating at a short time scale such as power control and fast fading to mechanisms/algorithms naturally operating on a far longer time scale such as user creation and link configuration. For the purpose of evalu-
ating the performance of the expressions for noise rise derived in Chapter 4, momentary true uplink noise rise and all the approximative expressions for it is sampled every frame, i.e., 100 times a second. The true noise rise is calculated using Definition 3.1, in which the interference power is found using Equation (3.1). The total path gain is calculated using two factors; $g^s$ and $g^f$. Antenna gain, distance attenuation and shadow fading are all represented by $g^s$ while $g^f$ represents the variations caused by the fast fading. The true noise rise is thus calculated according to

$$\Lambda_j(t) = \frac{N + \sum_{i}^{M} g^s_{i,j}(t) g^f_{i,j}(t) p_i(t)}{N},$$

where $N$ is a constant representing the thermal noise only.

Each approximative expression in Chapter 4 is then compared to the true noise rise, time instant by time instant. In the following sections of this chapter, statistics of the resulting differences in noise rise are reported for different true noise rise levels.

### 7.1.1 Models

**Attenuation** The simulator models the total path gain as a product of four parts; antenna gain, distance attenuation, shadow fading and fast fading. In the factor representing the antenna gain, a direction-dependent attenuation is modeled. The three first parts, represented by $g^s$ above, are considered fixed for a certain position in the simulator area throughout the entire simulations. The propagation loss is parameterized through the path loss coefficient, $\alpha = 3.5$ and the shadow fading through $d_0 = 100$ m and $\sigma = 6$ dB. Fast fading, $g^f$ above, is updated every slot with a function of traveled distance. This is used to model the stochastics in the fast fading even for a given point in space. On top of this an extra component is used to model the performance of the link estimation algorithms of a real system. This factor is a function of the user’s speed. Characteristics of the fast fading model used is given in (3GPP, 2000a) under the name of ”3GPP Typical Urban”. A RAKE receiver with three fingers is incorporated in the model.

**Users** The users move around in the simulation area with a constant absolute velocity of 5 km/h but with angular variations. No traffic model is used, instead admitted users are continuously transmitting, i.e., the users’ buffers are full for the entire session. To maintain simulation times to a moderate level, rather demanding services have been chosen; continuous
transmission with 64 kbps or even 192 kbps. All users are created during the initialization of the simulation. At the time of admission the user’s session time is decided according to an exponential distribution.

To isolate the sources of estimation error considered here, the users have no upper transmission power limit.

### 7.1.2 Other Features

The simulator updates the transmission power of all users at every slot, i.e., at a rate of 1500 Hz. Feedback delay is implemented and power control errors are modeled. The update rate is in agreement with what is done in the true system according to (3GPP, 2000b). The simulation area is a system of 7 base stations each serving 3 cells. In order to avoid border effects in the outermost cells, a wrapping technique has been used. The wrapping applies to both interference and user movement.

Measurements of path gains are subject to errors according to (3GPP, 2000c), i.e. a 90% confidence interval of 1.5 dB. The users’ inability to report path gain to all base stations is modeled through limiting each report to the six strongest base stations. This is also the number of cells that is reported in the true system.

### 7.2 Measurement Report Frequency

The approximative expressions in Chapter 4 rely on measurements of the path gains between user and base station. In reality, we will not have complete knowledge of the entire path gain matrix since path gain reports are sparsely spread in time and they do not cover all base stations. Therefore this section is devoted to studying the expressions’ sensitivity to limited path gain knowledge. As a comparison, Figure 7.1 shows average error of $\Lambda^{(N\text{BOTH})}$ and $\Lambda^{(I\text{BOTH})}$ (which both can be found in Equation (4.16)) when knowing the whole uplink path gain matrix and actual received $\beta_i$. Already under these relatively idealistic circumstances we can see the effect of assumption of equal interference power in all cells; the average error when using $\Lambda^{(I\text{BOTH})}$ deviates from the trend for high loads. The samples representing this part of the curve have probably been sampled in the cells which experience the highest load. Thus, assuming all other cells to have the same high noise rise contributes to an over estimation, which makes the average error smaller since it is an underestimation. The path gain measurements which all the expressions evaluated herein depend on can be made available in many different ways. Two different ways, which are both
in the current 3GPP standard, are described in Section 4.1. Basically the one referred to as M1 schedules the path gain reports in a periodic manner with a periodicity of 0.5 Hz, while when using M2 users report only in conjunction with soft hand over requests, i.e., in an event driven manner. In this section we will investigate the expressions sensitivity to how the measurements are scheduled. Not surprisingly, as can be seen in Figure 7.2, using event driven path gain reports (i.e., M2) results in far less reports per user and hence less signaling overhead for the system. More interesting is the fact that using M2 does not necessarily imply a worse approximation, see Figure 7.3, 7.4 and 7.5 which show the average estimation error of $\Lambda^{(N_{MRC})}$, $\Lambda^{(N_{SEL})}$ and $\Lambda^{(N_{BOTH})}$ when using users traveling at an average speed of 10, 20 and 70 km/h, respectively. A comparison between expressions assuming $I_k = I_j$ as opposed to $N_k = N_j$ shows that there is no statistical difference in average error. However, using M1 results in considerably larger standard deviation when assuming $I_k = I_j$, i.e., $\Lambda^{IMRC}$, $\Lambda^{ISEL}$ or $\Lambda^{IBOTH}$ (all from Equation (4.13)), see Figure 7.6 and 7.7.

Only when traveling at a rather high speed, 70 km/h, using M1 or M2 results in different error statistics for $\Lambda^{(N_{MRC})}$, $\Lambda^{(N_{SEL})}$ and $\Lambda^{(N_{BOTH})}$. At this speed, M1 provides a better average error but as can be seen in Figure 7.6, the standard deviation is much higher compared with using M2. The average error may be canceled by an error correction method, but we can do almost nothing to combat a high standard deviation. Average error

\[ \text{Figure 7.1 Mean error in noise rise when knowing the entire path gain matrix. \text{"\*\"} represents } \Lambda^{(N_{BOTH})} \text{ and \text{"\'-\"} represents } \Lambda^{(I_{BOTH})}. \]
cancellation also requires a low standard deviation, which is why M2 can be considered providing an even better approximation than M1.

It is the users that are close to the cell border that are the most important ones to have somewhat accurate path gain knowledge of since these are the users that cause most of the inter-cell-interference. Since these users are more likely to change their soft handover setup compared to users within the cell, it is also more likely that we get a report from these users when using M2. From simulations shown in this section we can conclude that scheduling the users’ path gain reports in a event driven manner, such as M2, provides an at least equally good approximation of the uplink noise rise while requiring less signaling overhead.

Figure 7.2  Cumulative sum over number of reports per user for M1 (solid) and M2 (dashed)

7.3 One Radio Network Controller

In the previous section it is shown that the expressions are practically independent of the type of path gain reporting, periodic or event-based, and therefore only event-based reports are considered in this section.

7.3.1 Uniform Traffic With Single Service

Figure 7.8 shows the performance of four different expressions when using 64 kbps streaming users. As each user has no more than one link, assuming soft or softer handover does not make any difference in this scenario. The
Figure 7.3 Mean error in noise rise. User speed: 10 km/h.
Solid:M1, dashed:M2.

major gap between the lines is due to TX rise (see Section 4.4.2), since fast fading is not simulated in the runs corresponding to the dashed lines. From this figure we can conclude that TX rise does give a considerable contribution to the total error. Also, all four expressions produce an underestimate of the uplink noise rise, mainly due to that measurements of all path gains are not available. In Figure 7.9 a more realistic scenario is shown where each user is allowed up to three handover links at a time. First of all, assuming softer handover ($\Lambda_{IMRC}$ and $\Lambda_{NMRC}$) gives larger average error compared to assuming soft handover (i.e., $\Lambda_{ISEL}$ and $\Lambda_{NSEL}$) or both soft and softer handover (i.e., $\Lambda_{IBOTH}$ and $\Lambda_{NBOTH}$). This is natural since assuming softer handover everywhere means overestimating the system’s ability to correctly receive signals, which in turn means underestimating the required transmission powers. Under these circumstances, i.e., almost equal load in all cells, the errors from all six expressions are small in average and has comfortably small standard deviation. Assuming $I_{k}^{\text{tot}}(t) = I_{j}^{\text{tot}}(t-1)$ and equal background noise power (i.e., $\Lambda_{NMRC}$, $\Lambda_{NSEL}$ and $\Lambda_{NBOTH}$) provide approximations with equal average error but with lower standard deviation compared to assuming $I_{k}^{\text{tot}} = I_{j}^{\text{tot}}$. This is explained by the fact that assuming equal background noise power makes the performance of the expression independent of different load in different cells. In $\Lambda_{NMRC}$, $\Lambda_{NSEL}$ and $\Lambda_{NBOTH}$, we use the previous approximation to approximate each user’s contribution to the noise rise. By looking at Equation (4.16a), which is repeated below, we realize that underestimating
the noise rise at time $t - 1$ results in underestimating every term in the sum that constitutes the noise rise at time $t$

$$
\Lambda_j^{NMRC}(t) = 1 + \sum_{i=1}^{M} \beta_i^{gt} \frac{g_{i,j}}{\sum_{k\in K_i} \Lambda_k^{NMRC}(t-1)}.
$$

As can be expected, the performance of $\Lambda^{NBOTH}$ and $\Lambda^{IBOTH}$ can be described as an average of the two other assumptions regarding soft or softer
handover. The two middle lines in the upper graph of Figure 7.9 represent $\Lambda^{IBOTH}$ and $\Lambda^{NBOTH}$. We can see that they approximate the noise rise with an average error less than 0.5 dB while maintaining a low standard deviation even for high true noise rise levels, see Figure 7.9. The slight increase in error as the noise rise increases, may be explained by looking in the load domain. As mentioned in the introduction, even a small difference in load gives a substantial increase in noise rise at high load levels. This also means that a small load error results in a large noise rise error when operating at high load levels. In Figure 7.10 statistics for load expressions are plotted versus true load computed according to Equation (3.2),

$$L = 1 - \frac{1}{\Lambda},$$

where $\Lambda$ is given by the simulator. In this domain we see no dramatic increase in error as the load of the system increases. All the lines in the figure are quite straight but do not have the correct incline. This can best be explained by studying Equation (4.10). Ideally, the expression is a sum over all users in the network. The fact that for example the line representing $L^{IMRC}$ in Figure 7.10 does not have the correct incline is thus an effect of each user’s contribution being underestimated and that not all users are considered due to limited path gain reports. All the expressions have a sum over, ideally, all the users, shown in (4.16a), and hence the same reasoning applies. To summarize, there are two reasons why the lines in Figure 7.9 deviates from a straight line for high load levels:
Figure 7.7 Standard deviation of single noise rise errors. 'o': $\Lambda^{IMRC}$, '+': $\Lambda^{ISEL}$, '*': $\Lambda^{IBOTH}$. All is for 70 km/h. solid:M1, dashed:M2

Figure 7.8 Error in noise rise with 64 kbps users. One link per user. Dashed: No fast fading. $\Lambda^{IMRC}$: 'o', $\Lambda^{NIMRC}$ 'x', $\Lambda^{ISEL}$: '+', $\Lambda^{NSEL}$: '*
• even a minor error at high loads produces a non-negligible error in the noise rise

• the sum representing the load approximation has a larger error due to many users, all with a minor underestimated contribution.

7.3.2 Hot Spot

When we derived $\Lambda^{IMRC}$, $\Lambda^{ISEL}$ and $\Lambda^{IBOTH}$ in Chapter 4 we assumed the interference power to be approximately equal in all cells. This is an assumption that is unsuitable for scenarios with hot spots, i.e., when one or several base stations have considerably higher load than the others. As
can be seen in the upper graph of Figure 7.11 all expressions still provide a pretty good average noise rise approximation for moderate noise rise levels. However, under these circumstances $\Lambda^{ISEL}$ is useless due to its errors have a high standard deviation at heavily loaded cells. In order to explain the severe over estimation done by $\Lambda^{ISEL}$ at high loads, we study a scenario where user 1 is connected, by soft handover, to base stations $j$ and $\ell$. Since both base stations are in the active set, the path gains from the user to the base stations, i.e., $g_{1,j}$ and $g_{1,\ell}$, cannot differ too much. If $I^\text{tot}_j$ is much greater than $I^\text{tot}_\ell$, than the contribution from user 1 is according to (4.6b)

$$g_{1,j} \frac{\beta^{\text{tot}}_1}{\max\{g_{1,j}, g_{1,\ell}\}} = g_{1,j} \frac{\beta^{\text{tot}}_1}{g_{1,\ell}} I^\text{tot}_\ell$$

(7.1)

while it is reflected in $\Lambda^{ISEL}$ by

$$g_{1,j} \frac{\beta^{\text{tot}}_1}{p_{ij}} \max\{g_{1,j}, g_{1,\ell}\} = g_{1,j} \frac{\beta^{\text{tot}}_1}{\max\{g_{1,j}, g_{1,\ell}\}} I^\text{tot}_j.$$ 

(7.2)

If $I^\text{tot}_j >> I^\text{tot}_\ell$, the load contribution (7.1) is likely highly overestimated using (7.2). This explains $\Lambda^{ISEL}$’s overestimation of the load in highly loaded cells, as illustrated in Figure 7.11. The high standard deviation is a result of the expressions alternating between being an over estimation in the case of different interference levels in neighboring base stations and under estimation in the case of almost equal interference levels. $\Lambda^{NBOTH}$ provides approximations with both low average error and low standard deviation.
7.4 Several Radio Network Controllers

The knowledge on users’ path gains can be even further limited by considering reports from only a subset of the users. As argued in Section 4.2.4, some users will not report their path gain to the RNC they are currently controlled by if they started their session when located in an area served by another RNC. In this section we have used path gain reports according to the scheduling technique referred to as M2 and compared the performance of the expressions defined by Equations (4.18) for different fractions of external users (i.e., users not reporting their path gain to the RNC referred to as $RNC_1$ in Chapter 4). As a comparison we have also computed statistics for the expression that completely ignores the external users. Figure 7.12, 7.13 and 7.14 show the average estimation error when 10, 20 and 30%
of the users report to another RNC than the one they are closest to, respectively. Clearly, just ignoring that some users are not considered in the expression can be quite drastic. However, when using an expression that incorporates an estimate of the load contribution from the external users (i.e., $\Lambda^{RNC\text{MRC}}$, $\Lambda^{RNC\text{SEL}}$ or $\Lambda^{RNC\text{BOTH}}$), the performance is almost equal to the case where measurement reports from all users are available. A comparison between the lower plot of Figure 7.9 and Figure 7.15 shows that the expressions studied in this section have approximately the same standard deviation as the ones assuming equal background noise power in all cells.

The expressions assuming $I_k = I_j$ are not shown here. The problem of not receiving path gain reports from some users can be solved in the same way, i.e., with small messages between different RNCs, without any additional changes to the expressions.

![Figure 7.12](image)

**Figure 7.12** Mean errors in noise rise when using two RNCs. Solid: All reports, dashed: Reports in one RNC, dashdotted: $\Lambda^{(RNC)}$. 10% of the users report to another RNC.

### 7.5 Summary

In this chapter simulations have been used to evaluate the statistical properties of the expressions of uplink noise rise derived in Chapter 4. The expressions are build on path gain reports. It was shown that they are fairly insensitive to in which manner these reports are scheduled, periodically or only in conjunction with soft handover requests. A rather extensive study on the reasons for the estimation error was conducted. The main reasons...
Figure 7.13  Mean errors in noise rise when using two RNCs. Solid: All reports, dashed: Reports in one RNC, dashdotted: $\Lambda^{(RNC)}$. 20 % of the users report to another RNC.

Figure 7.14  Mean errors. Solid:All reports, dashed:Reports in one RNC, dashdotted: $\Lambda^{(RNC)}$. 30 % of the users report to another RNC.
for under estimating the uplink relative load is the unavailability of some path gain measurements and that, in the cases where iterations are used, an underestimation is amplified. The unavailability of path gain measurements is further aggravated when a simulation area consisting of several RNCs is considered. However, using for the purpose specialized expressions, this problem can be dealt with fairly smoothly without too much loss in performance. Overall, we can approximate the uplink relative load with an average error of less than 1 dB and comfortably low standard deviation for reasonable load levels.

Figure 7.15  Standard deviation of single errors. Dashdotted: $\Lambda^{(RNC)}$. 30 % of the users report to another RNC.
Conclusions

The uplink load of a CDMA cellular system is hard to characterize since it depends on many unknown variables, some of them being properties of the current radio environment. This thesis proposes different ways of looking at uplink load and establishes relations between them. Furthermore, instruments for finding the current load according to the different views are provided. Accurate knowledge of both current and possible future uplink load is crucial for good utilization of available resources without jeopardizing the system stability.

One possible definition of uplink load, herein referred to as feasibility uplink relative load, relates load directly to the power control problem of finding transmission powers such that all users are satisfied with their current quality of service. It is shown how to find the feasibility relative load from the users’ required carrier-to-interference ratios and the system’s path gain matrix when all users can be assumed to be power controlled from exactly one cell. In the more general case, a lower bound on the feasibility uplink relative load is provided by using Lipschitz theory.

Another definition of uplink load relates it to the uplink noise rise instead. The literature survey included in the thesis suggests that this is a much more common view and that there has been a lot of work on how to approximate it. Load expressions which have the rather unique property of utilizing information gathered in several cells are proposed. Compared
to expressions using only local information, this property makes the expressions more sensitive to the soft capacity inherent in all CDMA cellular systems.

A thorough simulation study is used to evaluate the performance of the uplink load expressions when operating in an environment carefully made to imitate that of a WCDMA system. It shows that, without additional signaling between the nodes of the network, it is possible to approximate the uplink noise rise with an average error of less than 1 dB for practical load levels. Furthermore, the performance of the expressions is completely insensitive to non-uniform distributed traffic load, variations in the radio environment and inherently incorporates different user requirements on data rate.

A theoretical comparison between the different views on uplink load yields, first of all, that the two views are in fact the same in a single cell scenario. In a multi cell system not utilizing soft handover, the approximative expressions can be used to provide bounds on the feasibility uplink relative load. This is useful even in a system utilizing soft handover since soft handover can only decrease the uplink load.

Advanced signal processing is also applied to one of the expressions. The expressions, just like the true load, consist of oscillations superimposed on a time-varying bias. An unbiased auto-regressive model is extended to a biased auto-regressive model, i.e., a model describing a signal oscillating around some time-varying bias. Kalman filtering is combined with change detection to extract a stable estimate and its time-derivative from the load expression. This enables more aggressive resource management algorithms since a smaller back-off is required and the estimate of the time derivative can be used to forecast a possible future high load situation.
A

Appendix

A.1 Theory of Positive Matrices

The characterizing property of positive matrices is that they have only positive elements. The following theorem can be found in (Gantmacher, 1974). All inequalities in this Appendix should be interpreted component wise.

**Theorem A.1**

A non-negative matrix $A$ always has a non-negative eigenvalue $r$ such that the moduli of all the eigenvalues of $A$ do no exceed $r$. To this 'maximal' eigenvalue $r$ there corresponds a non-negative eigenvector

$$Ay = ry \ (y \geq 0, \ y \neq 0).$$

The adjoint matrix $B(\lambda) = (\lambda E - A)^{-1} \Delta(\lambda)$ satisfies the inequalities

$$B(\lambda) \geq 0, \ \frac{d}{d\lambda} B(\lambda) \geq 0 \ for \ \lambda \geq r.$$  

Another word for moduli is absolute value and $\Delta(\lambda)$ is the characteristic polynomial of $A$, $\Delta(\lambda) = |\lambda E - A|$.

**Proof** See (Gantmacher, 1974). 

Some, for this work, interesting consequences of the theorem are
The maximum eigenvalue, i.e., the eigenvalue with greatest absolute value, is real

\[ \max \text{eig}(A) = \max |\text{eig}(A)| \]

The smallest real eigenvalue such that the inequality

\[ Ay \leq ry \]

has positive real solutions \( y \) is the maximum eigenvalue.

Choosing \( \lambda = 1 \) in the theorem yields that

\[ (E - A)^{-1} = \frac{B(\lambda)}{\Delta(\lambda)} > 0 \]

if the greatest eigenvalue, \( r \), is less than one and \( A \) is an irreducible matrix.

A strictly positive matrix is always irreducible.

### A.2 Normalized Path Gain Matrices

In Chapter 3 and Chapter 5, the matrices \( Z \) and \( \tilde{Z} \) appear. These matrices are defined as

\[ Z = [z_{\ell,j}] \Delta g_{\ell,j} \text{ and } \tilde{Z} = [\tilde{z}_{tk}] \Delta g_{tk,jk} \]

respectively. \( j \) is here a vector with its \( k \):th component equal to the number of the cell user \( k \) is solely power controlled by. The matrices can be expressed using two new matrices. First an extended path gain matrix, \( G_E \), is introduced. \( G_E \) is a square matrix with as many rows and columns as there are users in the network. The \( k \):th column of \( G_E \) is a copy of the \( j_k \):th column of \( G \). Secondly, the diagonal matrix \( G_{ED} \) which also is a square matrix with all elements equal to zero except the ones in the diagonal which are equal to the corresponding elements in the diagonal of \( G_E \).

\[ G_E = [g_{E_{\ell,i}}] \Delta g_{\ell,i}, \quad G_{ED} = \text{diag}\((g_{1,j_1}, g_{2,j_2}, \ldots, g_{M,j_M})\)) \]

\( Z \) and \( \tilde{Z} \) may now be expressed as

\[ Z = G_{ED}^{-1}G_E \text{ and } \tilde{Z} = G_EG_{ED}^{-1} \]
$Z^T$ is thus, in general, not equal to $\tilde{Z}$ for example. The two will, however, have the same eigenvalues. Furthermore, if the path gain matrix is reciprocal and $Z$ can be used to express the downlink transmission powers, $\tilde{Z}$ can be used to express the uplink transmission powers. The reverse case is true when expressing the user individual received interference powers; if $Z$ is used in uplink, $\tilde{Z}$ can be used in downlink.

$$|\lambda E - G_{ED}^{-1}G_E| = |G_{ED}^{-1}| |\lambda G_{ED} - G_E| =$$

$$|\lambda G_{ED} - G_E| |G_{ED}^{-1}| = |\lambda E - G_E G_{ED}^{-1}|$$

In case of a single cell system, $Z$ and $\tilde{Z}$ will be all-one matrices

$$Z = \text{diag} \begin{pmatrix} g_{1,1} & g_{2,1} & \cdots & g_{M,1} \end{pmatrix}^{-1} \begin{pmatrix} g_{1,1} & g_{1,1} & \cdots & g_{1,1} \\ g_{2,1} & g_{2,1} & \cdots & g_{2,1} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M,1} & \cdots & \cdots & g_{M,1} \end{pmatrix} = 1^{M \times M}$$

$$\tilde{Z} = \begin{pmatrix} g_{1,1} & g_{1,1} & \cdots & g_{1,1} \\ g_{2,1} & g_{2,1} & \cdots & g_{2,1} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M,1} & \cdots & \cdots & g_{M,1} \end{pmatrix} \text{diag} \begin{pmatrix} g_{1,1} & g_{2,1} & \cdots & g_{M,1} \end{pmatrix}^{-1} = 1^{M \times M}$$
Bibliography


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