Implementation of the GIW-PHD filter

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Abstract

This report contains pseudo-code for, and a computational complexity analysis of, the Gaussian inverse Wishart Probability Hypothesis Density filter.

Keywords: Extended target, probability hypothesis density, random matrix, Gaussian, inverse Wishart, computational complexity.

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Abstract

This report contains pseudo-code for, and a computational complexity analysis of, the Gaussian inverse Wishart Probability Hypothesis Density filter.

1 Introduction

This technical report contains information regarding implementation of the Gaussian inverse Wishart probability hypothesis density (GIW-PHD) filter [1]. Pseudo-code is given for the main filter recursion. The complexity is analyzed in terms of the number of multiplications and summations that are needed, and a *worst case* approximate of the complexity is given. Please refer to [1] for further details.

2 Pseudo-code

The main filter recursion is given in Table 1, prediction and construction of correction components is given in Table 2, and correction is given in Table 3. The pruning and merging scheme is given in Table 4, and target extraction is given in Table 5. Note that pruning and merging, and target extraction, is performed similarly to [2].

Table 1: Pseudo-code for the Gaussian inverse Wishart PHD filter

input: Sequence of measurement sets $\{\mathbf{Z}_k\}_{k=1}^{K}$.
initialize: Set $J_{0|0} = 0$.
for $k = 1, \ldots, K$ **do**
  Compute measurement set partition $\{\mathrm{p}_p\}_{p=1}^{P}$, see [1, 3].
  Predict and construct correction components, Table 2.
  Correct, Table 3.
  Prune and merge, Table 4.
  Extract estimated target set, Table 5.
end for
output: Sequence of estimated target sets $\left\{\hat{\mathbf{X}}_k\right\}_{k=1}^{K}$

Table 2: Pseudo-code for GIW-PHD filter prediction and correction components

input: GIW components $\left\{w_{k-1|k-1}^{(j)},\ \xi_{k-1|k-1}^{(j)}\right\}_{j=1}^{J_{k-1|k-1}}$, and set of measurement set partitions $\{\mathrm{p}_p\}_{p=1}^{P}$.
$i = 0$
for $j = 1, \ldots, J_{b,k}$ **do**
  $i \leftarrow i + 1$
  $w_{k|k-1}^{(i)} \leftarrow w_{b,k}^{(j)}, \qquad \xi_{k|k-1}^{(i)} \leftarrow \xi_{b,k}^{(j)}$
end for
for $j = 1, \ldots, J_{k-1|k-1}$ **do**
  $i \leftarrow i + 1$
  $w_{k|k-1}^{(i)} \leftarrow p_{\mathrm{S}} w_{k-1|k-1}^{(j)}$
  $m_{k|k-1}^{(i)} \leftarrow \left(F_{k|k-1} \otimes \mathbf{I}_d\right) m_{k-1|k-1}^{(j)}$
  $P_{k|k-1}^{(i)} \leftarrow F_{k|k-1} P_{k|k}^{(j)} F_{k|k-1}^{\mathrm{T}} + \mathbf{Q}_{k|k-1}$
  $\nu_{k|k-1}^{(i)} \leftarrow e^{-T_s/\tau} \nu_{k-1|k-1}^{(j)}$
  $V_{k|k-1}^{(i)} \leftarrow \frac{\nu_{k|k-1}^{(i)} - d - 1}{\nu_{k-1|k-1}^{(j)} - d - 1} V_{k-1|k-1}^{(j)}$
end for
$J_{k|k-1} \leftarrow i$
for $p = 1, \ldots, P$ **do**
  for $w = 1, \ldots, |\mathrm{p}_p|$ **do**
    $\bar{\mathbf{z}}_k^{W_w^p} \leftarrow \frac{1}{|W_w^p|} \sum_{\mathbf{z}_k^{(i)} \in W_w^p} \mathbf{z}_k^{(i)}$
    $Z_k^{W_w^p} \leftarrow \sum_{\mathbf{z}_k^{(i)} \in W_w^p} \left(\mathbf{z}_k^{(i)} - \bar{\mathbf{z}}_k^{W_w^p}\right) \left(\mathbf{z}_k^{(i)} - \bar{\mathbf{z}}_k^{W_w^p}\right)^{\mathrm{T}}$
  end for
end for
for $j = 1, \ldots, J_{k|k-1}$ **do**
  $\hat{K}_{k|k-1}^{(j)} \leftarrow P_{k|k-1}^{(j)} H_k^{\mathrm{T}}$
  $\hat{S}_{k|k-1}^{(j)} \leftarrow H_k \hat{K}_{k|k-1}^{(j)}$
  $\hat{\mathbf{z}}_{k|k-1}^{(j)} \leftarrow (H_k \otimes \mathbf{I}_d)\, m_{k|k-1}^{(j)}$
end for
output: GIW components $\left\{w_{k|k-1}^{(j)},\ \xi_{k|k-1}^{(j)}\right\}_{j=1}^{J_{k|k-1}}$, and correction components $\left\{\left\{\bar{\mathbf{z}}_k^{W_w^p}, Z_k^{W_w^p}\right\}_{w=1}^{|\mathrm{p}_p|}\right\}_{p=1}^{P}$ and $\left\{\hat{K}_{k|k-1}^{(j)}, \hat{S}_{k|k-1}^{(j)}, \hat{\mathbf{z}}_{k|k-1}^{(j)}\right\}_{j=1}^{J_{k|k-1}}$

Table 3: Pseudo-code for GIW-PHD filter correction

input: GIW components $\left\{w_{k|k-1}^{(j)},\ \xi_{k|k-1}^{(j)}\right\}_{j=1}^{J_{k|k-1}}$, partitions $\{\mathrm{p}_p\}_{p=1}^{P}$, and correction components $\left\{\left\{\bar{\mathbf{z}}_k^{W_w^p}, Z_k^{W_w^p}\right\}_{w=1}^{|\mathrm{p}_p|}\right\}_{p=1}^{P}$ and $\left\{\hat{K}_{k|k-1}^{(j)}, \hat{S}_{k|k-1}^{(j)}, \hat{\mathbf{z}}_{k|k-1}^{(j)}\right\}_{j=1}^{J_{k|k-1}}$.

for $j = 1, \ldots, J_{k|k-1}$ **do**

    $w_{k|k}^{(j)} \leftarrow \left(1 - \left(1 - e^{-\gamma^{(j)}}\right) p_{\mathrm{D}}^{(j)}\right) w_{k|k-1}^{(j)}$

    $\xi_{k|k}^{(j)} \leftarrow \xi_{k|k-1}^{(j)}$.

end for

$\ell = 0$

for $p = 1, \ldots, P$ **do**

    for $w = 1, \ldots, |\mathrm{p}_p|$ **do**

        $\ell \leftarrow \ell + 1$

        for $j = 1, \ldots, J_{k|k-1}$ **do**

            $S \leftarrow \hat{S}_{k|k-1}^{(j)} + \frac{1}{|W_w^p|}$, $\quad K \leftarrow \hat{K}_{k|k-1}^{(j)} S^{-1}$

            $\varepsilon \leftarrow \bar{\mathbf{z}}_k^{W_w^p} - \hat{\mathbf{z}}_{k|k-1}^{(j)}$, $\quad N \leftarrow S^{-1} \varepsilon \varepsilon^{\mathrm{T}}$

            $m_{k|k}^{(j+J_{k|k-1}\ell)} \leftarrow m_{k|k-1}^{(j)} + (K \otimes \mathbf{I}_d)\, \varepsilon$

            $P_{k|k}^{(j+J_{k|k-1}\ell)} \leftarrow P_{k|k-1}^{(j)} - KSK^{\mathrm{T}}$

            $\nu_{k|k}^{(j+J_{k|k-1}\ell)} \leftarrow \nu_{k|k-1}^{(j)} + |W_w^p|$

            $V_{k|k}^{(j+J_{k|k-1}\ell)} \leftarrow V_{k|k-1}^{(j)} + N + Z_k^{W_w^p}$

            $w_{k|k}^{(j+J_{k|k-1}\ell)} \leftarrow \frac{e^{-\gamma^{(j)}} \left(\gamma^{(j)}\right)^{|W_w^p|} p_{\mathrm{D}}^{(j)}}{\beta_{FA,k}^{|W_w^p|} \left(\pi^{|W_w^p|} |W_w^p| S\right)^{d/2}} \frac{\left|V_{k|k-1}^{(j)}\right|^{\nu_{k|k-1}^{(j)}/2}}{\left|V_{k|k}^{(j+J_{k|k}\ell)}\right|^{\nu_{k|k}^{(j+J_{k|k}\ell)}/2}} \frac{\Gamma_d\left(\nu_{k|k}^{(j+J_{k|k}\ell)}/2\right)}{\Gamma_d\left(\nu_{k|k-1}^{(j)}/2\right)} w_{k|k-1}^{(j)}$

        end for

        $d_{W_w^p} \leftarrow \delta_{|W_w^p|,1} + \sum_{j=1}^{J_{k|k-1}} w_{k|k}^{(j+J_{k|k-1}\ell)}$

        $w_{k|k}^{(j+J_{k|k-1}\ell)} \leftarrow \frac{w_{k|k}^{(j+J_{k|k-1}\ell)}}{d_{W_w^p}}$ for $j = 1, \ldots, J_{k|k-1}$

    end for

    $\omega_{\mathrm{p}_p} \leftarrow \prod_{w=1}^{|\mathrm{p}_p|} d_{W_w^p}$

end for

$J_{k|k} \leftarrow J_{k|k-1}\,(\ell + 1)$

$J_{\mathrm{aux}} \leftarrow J_{k|k-1}$

$\omega_{\mathrm{p}_p} \leftarrow \frac{\omega_{\mathrm{p}_p}}{\sum_{p'=1}^{P} \omega_{\mathrm{p}_{p'}}}$ for $p = 1, \ldots, P$

for $p = 1, \ldots, P$ **do**

    $w_{k|k}^{(j+J_{\mathrm{aux}})} \leftarrow w_{k|k}^{(j+J_{\mathrm{aux}})} \omega_{\mathrm{p}_p}$, for $j = 1, \ldots, J_{k|k-1}\,|\mathrm{p}_p|$

    $J_{\mathrm{aux}} \leftarrow J_{\mathrm{aux}} + J_{k|k-1}\,|\mathrm{p}_p|$

end for

output: GIW components $\left\{w_{k|k}^{(j)},\ \xi_{k|k}^{(j)}\right\}_{j=1}^{J_{k|k}}$

Table 4: Pseudo-code for GIW-PHD filter pruning and merging

input: GIW components $\left\{w_{k|k}^{(j)}, \xi_{k|k}^{(j)}\right\}_{j=1}^{J_{k|k}}$, a truncation threshold T, a merging threshold U and a maximum allowable number of GIW components $J_{\max}$.

initialize: Set $\ell \leftarrow 0$ and $I \leftarrow \left\{i = 1, \ldots, J_{k|k} \middle| w_{k|k}^{(i)} > T\right\}$.

repeat

$\quad \ell \leftarrow \ell + 1$

$\quad j \leftarrow \arg\max_{i \in I} w_{k|k}^{(i)}$

$\quad$ Compute $\hat{P}_{k|k}^{(j)}$, see [1].

$\quad L \leftarrow \left\{ i \in I \middle| \left(m_{k|k}^{(i)} - m_{k|k}^{(j)}\right)^{\mathrm{T}} \left(\hat{P}_{k|k}^{(j)}\right)^{-1} \left(m_{k|k}^{(i)} - m_{k|k}^{(j)}\right) \leq U \right\}$

$\quad \tilde{w}_{k|k}^{(\ell)} \leftarrow \sum_{i \in L} w_{k|k}^{(i)}$

$\quad \tilde{m}_{k|k}^{(\ell)} \leftarrow \frac{1}{\tilde{w}_{k|k}^{(\ell)}} \sum_{i \in L} w_{k|k}^{(i)} m_{k|k}^{(i)}$

$\quad \tilde{P}_{k|k}^{(\ell)} \leftarrow \frac{1}{\tilde{w}_{k|k}^{(\ell)}} \sum_{i \in L} w_{k|k}^{(i)} P_{k|k}^{(i)}$

$\quad \tilde{\nu}_{k|k}^{(\ell)} \leftarrow \frac{1}{\tilde{w}_{k|k}^{(\ell)}} \sum_{i \in L} w_{k|k}^{(i)} \nu_{k|k}^{(i)}$

$\quad \tilde{V}_{k|k}^{(\ell)} \leftarrow \frac{1}{\tilde{w}_{k|k}^{(\ell)}} \sum_{i \in L} w_{k|k}^{(i)} V_{k|k}^{(i)}$

$\quad I \leftarrow I \setminus L$

until $I = \emptyset$

If $\ell > J_{\max}$ then replace $\left\{\tilde{w}_{k|k}^{(j)}, \tilde{m}_{k|k}^{(j)}, \tilde{P}_{k|k}^{(j)}, \tilde{\nu}_{k|k}^{(j)}, \tilde{V}_{k|k}^{(j)}\right\}_{j=1}^{\ell}$ by those of the $J_{\max}$ GIW components with largest weights.

output: $\left\{\tilde{w}_{k|k}^{(j)}, \tilde{\xi}_{k|k}^{(j)}\right\}_{j=1}^{\ell}$, $\tilde{\xi}_{k|k}^{(j)} = \left(\tilde{m}_{k|k}^{(j)}, \tilde{P}_{k|k}^{(j)}, \tilde{\nu}_{k|k}^{(j)}, \tilde{V}_{k|k}^{(j)}\right)$

Table 5: Pseudo-code for GIW-PHD filter target extraction

input: GIW components $\left\{w_{k|k}^{(j)}, \xi_{k|k}^{(j)}\right\}_{j=1}^{J_{k|k}}$.

$\hat{\mathbf{X}}_k = \emptyset$

for $j = 1, \ldots, J_{k|k}$ **do**

$\quad$ **if** $w_{k|k}^{(j)} > 0.5$ **then**

$\quad\quad$ Compute $\hat{X}_{k|k}^{(j)}$, see [1].

$\quad\quad \hat{\mathbf{X}}_k \leftarrow \hat{\mathbf{X}}_k \cup \left(m_{k|k}^{(j)}, \hat{X}_{k|k}^{(j)}\right)$

$\quad$ **end if**

end for

output: Estimated set of targets $\hat{\mathbf{X}}_k$

3 Implementation issues

When computing the corrected weight (Table 3, Line 17), the likelihood often includes ratios of large numbers, leading to numerical overflow. Because of this, computing the log-likelihood is recommended, and then updating the weight with the exponential of the log likelihood. Similarly, the quantities $d_{W_w^p}$ (Table 3, Line 20) are often large, leading to numerical overflow when ω_{p_p} (Table 3, Line 22) is computed. A remedy is to store the log partition weights

$$\tilde{\omega}_{\mathrm{p}_p} = \log\left(\omega_{\mathrm{p}_p}\right) = \sum_{w=1}^{|\mathrm{p}_p|} \log\left(d_{W_w^p}\right), \tag{1}$$

and to normalize the log partition weights (Table 3, Line 26) as follows,

$$\hat{\omega}_{\mathrm{p}_p} = \tilde{\omega}_{\mathrm{p}_p} - \log\left(\sum_{p'=1}^{P} \omega_{\mathrm{p}_{p'}}\right) \tag{2a}$$

$$= \tilde{\omega}_{\mathrm{p}_p} - \left[\tilde{\omega}_{\mathrm{p}_1} + \log\left(1 + \sum_{p'=2}^{P} e^{\tilde{\omega}_{\mathrm{p}_{p'}} - \tilde{\omega}_{\mathrm{p}_1}}\right)\right] \tag{2b}$$

for $p = 1, \ldots, P$. The partition weights are then given as $\omega_{\mathrm{p}_p} = e^{\hat{\omega}_{\mathrm{p}_p}}$ for $p = 1, \ldots, P$.

4 Computational complexity analysis

4.1 Complexity of common operations

The complexity of some common matrix and vector operations is given in Table 6. In addition, computing the inversion and the determinant of a $d \times d$ matrix V both have approximate computational complexity $\mathcal{O}(d^3)$.

4.2 Assumptions and approximations

To simplify the notation, let $J = J_{k-1|k-1}$, $J_b = J_{b,k}$ and $J_+ = J_{k|k-1} = J_{k-1|k-1} + J_{b,k}$. To simplify the analysis, the following assumptions are made;

Table 6: Complexity of common operations

Input	Operation	Multiplications	Summations	Complexity
$A(m \times n)$, $B(n \times p)$	AB	mnp	$m(n-1)p$	$m(2n-1)p$
$A(m \times n)$, $B(p \times q)$	$A \otimes B$	$mnpq$	0	$mnpq$
$A(m \times n)$, $B(p \times q)$ $C(nq \times t)$	$(A \otimes B)C$	$mnpq(t+1)$	$mp(nq-1)t$	$mp((2nq-1)t + nq)$
$A_i(p \times q)$	$\sum_{i=1}^{n} A_i$	0	$(n-1)pq$	$(n-1)pq$
$x_i(d \times 1)$, $y_i(d \times 1)$	$x_i - y_i$	0	d	d
$z_i(d \times 1)$	$\sum_{i=1}^{n} z_i z_i^{\mathrm{T}}$	nd^2	$(n-1)d^2$	$(2n-1)d^2$
$F(n \times n)$, $P(n \times n)$	FPF^{T}	$2n^3$	$2(n^3 - n^2)$	$4n^3 - 2n^2$

1. The estimated target cardinality is approximately correct, i.e. $\hat{N}_{x,k} \approx N_{x,k}$.
2. The number of GIW components are approximately equal to the number of targets, i.e. $J \approx N_{x,k}$.
3. The true target Poisson rate γ is equal for all targets. The number of measurements is $N_{z,k} \approx \gamma N_{x,k} + \lambda_k$, where λ_k is the mean number of clutter measurements.
4. After partitioning, each cell W with target generated measurements has cardinality approximately equal to the Poisson rate γ, i.e. $|W| \approx \gamma$.
5. After the correction step, there are approximately $N_{x,k}$ clusters with GIW components, and each cluster contains approximately J_+ GIW components. Thus, in the pruning and merging step, we have $|L| \approx J_+$, where L is the set of GIW components that are merged into one component.

4.3 Prediction and construction of correction components

The complexity of prediction and construction of correction components is given in Table 7. Predicting each of the J components has approximate complexity

$$\mathcal{O}(J\left(3n_x^2 + 4s^3 + d^2 - n_x - s^2 + 7\right)) \approx \mathcal{O}(Jn_x^2). \tag{3}$$

Constructing the centroid measurements and scatter matrices has approximate complexity

$$\mathcal{O}\left(\left(\sum_{p=1}^{P} |\mathrm{p}_p|\right)\left(\gamma(2d^2 + d) - d^2 + d + 2\right)\right) \approx \mathcal{O}\left(\gamma d^2 \sum_{p=1}^{P} |\mathrm{p}_p|\right). \tag{4}$$

Constructing the gain matrices, innovation factors and innovation vectors has approximate complexity

$$\mathcal{O}\left((J + J_b)(3n_x d + 2s^2 + s - d - 1)\right) \approx \mathcal{O}\left((J + J_b)n_x^2\right). \tag{5}$$

Table 7: Complexity of prediction and correction components

Operation	Multiplications	Summations	Complexity
$p_S w$	1	0	1
$(F \otimes \mathbf{I}_d)m$	$s^2d^2 + s^2d^2$	$sd(sd-1)$	$3n_x^2 - n_x$
$FPF^{\mathrm{T}} + Q$	$2s^3$	$2s^2(s-1) + s^2$	$4s^3 - s^2$
$e^{-T_s/\tau}\nu$	–	–	$\mathcal{O}(3)$
$\frac{\nu-d-1}{\nu-d-1}V$	$d^2 + 1$	4	$d^2 + 4$
$\frac{1}{\lvert W\rvert}\sum \mathbf{z}$	2	$(\gamma-1)d$	$(\gamma-1)d + 2$
$\sum(\mathbf{z}-\mathbf{z})(\mathbf{z}-\mathbf{z})^{\mathrm{T}}$	γd^2	$2d + (\gamma-1)d^2$	$(2\gamma-1)d^2 + 2d$
PH	s^2	$s(s-1)$	$2s^2 - s$
HK	s	$s-1$	$2s-1$
$(H \otimes \mathbf{I}_d)m$	$sd^2 + sd^2$	$d(sd-1)$	$(3n_x - 1)d$

Thus, the overall complexity of prediction and construction of correction components is approximately

$$\mathcal{O}\left(J_+ n_x^2 + \gamma d^2 \sum_{p=1}^{P} |\mathrm{p}_p|\right). \tag{6}$$

4.4 Correction

The complexity of correction is given in Table 8. The correction update of the GIW components has approximate complexity

$$\mathcal{O}\left(\left(J_+ \sum_{p=1}^{P} |\mathrm{p}_p|\right)(3n_x d + d^3 + 3d^2 + d + s + 6)\right) \approx \mathcal{O}\left(n_x^2 J_+ \sum_{p=1}^{P} |\mathrm{p}_p|\right). \tag{7}$$

Computing the cell weights δ_W and partition weights ω_{p} has approximate complexity

$$\mathcal{O}\left((J_+ + 1)\sum_{p=1}^{P} |\mathrm{p}_p| + \sum_{p=1}^{P} |\mathrm{p}_p|\right) \approx \mathcal{O}\left(J_+ \sum_{p=1}^{P} |\mathrm{p}_p|\right). \tag{8}$$

Normalizing the partition weights and updating the GIW components weights has approximate complexity $\mathcal{O}(P + 2 + 3P) \approx \mathcal{O}(P)$. Thus, the overall complexity of the correction is approximately

$$\mathcal{O}\left(n_x^2 J_+ \sum_{p=1}^{P} |\mathrm{p}_p|\right). \tag{9}$$

Table 8: Complexity of correction

Operation	Multiplications	Summations	Complexity
$S + 1/\lvert W\rvert$	1	1	2
KS^{-1}	$1+s$	0	$s+1$
$\mathbf{z} - \mathbf{z}$	0	d	d
$S^{-1}\varepsilon\varepsilon^{\mathrm{T}}$	$1 + d^2 + 1$	0	$d^2 + 1$
$m + (K \otimes \mathbf{I}_d)\varepsilon$	$sd^2 + sd^2$	$sd(d-1) + n_x$	$3n_x d$
$P - KSK^{\mathrm{T}}$	$s^2 + 1$	s^2	$2s^2 + 1$
$\nu + \lvert W\rvert$	0	1	1
$V + N + Z$	0	$2d^2$	$2d^2$
weight update	–	–	$\mathcal{O}(d^3)$
$\delta + \sum w$	0	$J_+ + 1$	$J_+ + 1$
$\prod d_W$	$\lvert\mathrm{p}_p\rvert - 1$	0	$\lvert\mathrm{p}_p\rvert - 1$
$J(\ell + 1)$	1	1	2
$\frac{\omega_p}{\sum \omega_p}$	1	$P - 1$	P
$w\omega$	1	0	1
$J + J\lvert\mathrm{p}\rvert$	1	1	2

4.5 Pruning and merging

The complexity of merging is given in Table 9. Determining whether or not components i and j should be merged has approximate complexity $\mathcal{O}\left(n_x^3\right)$. There are approximately $N_{x,k}J_+$ components remaining after the correction step. In the *worst case*, each component has to be compared to all other components, i.e. $\mathcal{O}\left(N_{x,k}^2 J_+^2\right)$ comparisons. Thus, the *worst case* complexity of the merging is approximately

$$\mathcal{O}\left(N_{x,k}^2 J_+^2 n_x^3\right). \tag{10}$$

4.6 Target extraction

Computing the extension estimate $\hat{X}$ has approximate complexity $\mathcal{O}(d^2)$. Under the assumption that the target cardinality estimate is approximately correct, the complexity of target extraction is approximately

$$\mathcal{O}\left(N_{x,k}d^2\right). \tag{11}$$

4.7 Partitioning the measurement set

For Distance Partitioning, creating the distance matrix requires $3N_{z,k}(N_{z,k}-1)$ multiplications and summations. The distance matrix must then, in the *worst case*, be queried for each measurement for each of the P partitions, i.e. $N_{z,k}P$ times. Note that it is difficult to give an estimate of how many partitions are created in Distance Partitioning, because P depends on the particular measurement set that is being partitioned.

However, using Distance Partitioning gives at most $N_{z,k}$ unique partitions, thus a *worst case* upper limit for P is $N_{z,k}$. The *worst case* complexity of Distance Partitioning is thus approximately $\mathcal{O}(N_{z,k}^4)$. The *worst case* $P = N_{z,k}$ gives

$$\sum_{p=1}^{P} |\mathrm{p}_p| = \sum_{p=1}^{N_{z,k}} p = \frac{N_{z,k}(N_{z,k}+1)}{2} \approx \mathcal{O}(N_{z,k}^2). \tag{12}$$

The complexity of the EM algorithm for Gaussians is given in Table 10. The computational complexity of one iteration of the EM algorithm for Gaussian mixtures is approximately $\mathcal{O}(N_{x,k}N_{z,k}d^2 + n_x^3)$. On average, in our simulations and experiments, convergence is reached in 4 iterations.

Table 9: Complexity of pruning and merging

Operation	Multiplications	Summations	Complexity
$\hat{P}$	s^2d^2+2	3	n_x^2+5
Merge i and j?	–	–	$\mathcal{O}(n_x^3)$
$\sum w$	0	J_+-1	J_+-1
$\frac{1}{w}\sum wm$	$J_+n_x+n_x+1$	$(J_+-1)n_x$	$2J_+n_x+1$
$\frac{1}{w}\sum wP$	$J_+s^2+s^2+1$	$(J_+-1)s^2$	$2J_+s^2+1$
$\frac{1}{w}\sum w\nu$	J_++2	J_+-1	$2J_++1$
$\frac{1}{w}\sum wV$	$J_+d^2+d^2+1$	$(J_+-1)d^2$	$2J_+d^2+1$

Thus, the *worst case* complexity of partitioning the measurement set is approximately

$$\mathcal{O}(N_{z,k}(N_{x,k}d^2 + N_{z,k}^3) + n_x^3). \tag{13}$$

4.8 Overall complexity

The *worst case* overall complexity of one time step is approximately

$$\mathcal{O}\Bigg(J_+ n_x^2 + \gamma d^2 \sum_p^P |\mathrm{p}_p| + J_+ n_x^2 \sum_p^P |\mathrm{p}_p| + N_{x,k}^2 J_+^2 n_x^3 + N_{x,k}d^2 + N_{z,k}N_{x,k}d^2 + N_{z,k}^4 + n_x^3 \Bigg) \tag{14}$$

$$\approx \mathcal{O}\Bigg(\left(J_+ n_x^2 + \gamma d^2\right) \sum_p^P |\mathrm{p}_p| + N_{x,k}N_{z,k}d^2 + J_+^2 N_{x,k}^2 n_x^3 + N_{z,k}^4 \Bigg). \tag{15}$$

Inserting $J_+ = N_{x,k} + J_b$ and $N_{z,k} = \gamma N_{x,k} + \lambda_k$ into (15) gives the *worst case* overall complexity

$$\mathcal{O}\Bigg(\left(\left(N_{x,k} + J_b\right) n_x^2 + \gamma d^2 \right) \sum_{p=1}^P |\mathrm{p}_p| + \left(\gamma N_{x,k} + \lambda_k\right) N_{x,k} d^2 + \left(N_{x,k} + J_b\right)^2 N_{x,k}^2 n_x^3 + P \left(\gamma N_{x,k} + \lambda_k\right)^3 \Bigg). \tag{16}$$

References

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Table 10: Complexity of the EM algorithm

Operation	Multiplications	Summations	Complexity
$p(\mathbf{z}_j \mid m_i P_i)$	–	–	$\mathcal{O}(d^3)$
$\gamma_i(\mathbf{z}_j)$	$N_{x,k}+3$	$N_{x,k}-1$	$2N_{x,k}+2$
m_i	$N_{x,k}(N_{z,k}+2)$	$N_{x,k}N_{z,k}$	$2N_{x,k}N_{z,k}+2N_{x,k}$
P_i	$N_{x,k}(N_{z,k}(d^2+1)+2)$	$N_{x,k}(2d+d^2(N_{z,k}-1))$	$2N_{x,k}N_{z,k}d^2$ $+N_{x,k}(N_{z,k}-d^2+2d+2)$
π_i	$N_{x,k}$	$N_{x,k}(N_{z,k}-1)$	$N_{x,k}N_{z,k}$
log-lik	–	–	$\mathcal{O}(n_x^3)$

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Title

Implementation of the GIW-PHD filter

Författare
Author

Karl Granström, Umut Orguner

Sammanfattning
Abstract

This report contains pseudo-code for, and a computational complexity analysis of, the Gaussian inverse Wishart Probability Hypothesis Density filter.

Nyckelord
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Extended target, probability hypothesis density, random matrix, Gaussian, inverse Wishart, computational complexity.