EXAM IN MODELING AND SIMULATION (TSRT62)

ROOM: Distance mode exam

TIME: Monday August 24th, 2020, kl. 8.00–12.00

COURSE: TSRT62 Modeling and Simulation

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page):5

EXAMINER: Anders Hansson, 070-3004401

VISITS: not applicaple for distance mode exam

COURSE ADMINISTRATOR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

HANDING IN THE SOLUTION: The solution should be sent as a pdf-document to the course administrator using the e-mail address above. It should be time-stamped no later than 15 minutes after the end of the examination time above. In the subject line you should write TSRT62.

APPROVED TOOLS:

1. *L. Ljung & T. Glad* ”Modellbygge och Simulering”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Mathematical tables and formulas
4. Calculator and Matlab

Notes in the above books are allowed. Communication with other people and search for information using the Internet or by any other means, except by using the approved tools, is prohibited during the exam.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2020-09-17, 13.00-13:30 in Ljungeln, B-building, entrance 25, A-corridore, room 2A:514.

PRELIMINARY GRADING:	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean. Every page should be marked in such a way that your identity can be clearly identified. Scanning of pages should be done in such a way that it is easy to read the pages.

Good Luck!

1. (a) Describe briefly how a NARX model based on neural network looks like. How does it differ from an ARX model? (2p)

   (b) How can one from a set of step response experiment see that a system is nonlinear? (2p)

   (c) The system

$$\begin{aligned}\dot{x}_1(t) &= -350x_1(t) + 70u(t)\\ \dot{x}_2(t) &= 10x_1(t) - 3x_2(t) + u(t)\\ y(t) &= 5x_1(t) + 4x_2(t)\end{aligned}$$

   is stimulated with a slowly varying input signal $u(t)$. Obtain a simpler first-order model that has approximately the same output signal. (2p)

   (d) Assume that the step response of the system

$$\ddot{y}(t) + 4\dot{y}(t) + 3y(t) = u(t)$$

   has been simulated, and that now we want to study the step response of this other system

$$\ddot{y}(t) + 8\dot{y}(t) + 12y(t) = u(t)$$

   Is it possible to use the results from the first simulation by means of scaling? (2p)

   (e) Consider

$$y(t) = G(p)u(t) + e(t), \qquad G(p) = \frac{1}{p+1}$$

   where both $u(t)$ and $e(t)$ are white noise with variance 1 and independent one from the other. Calculate the spectrum of y. (2p)

2. (a) A set of $N = 100$ input/output measurements is identified with 4 different ARX models

$$A(q)y(t) = B(q)u(t)$$

   where

$$\begin{aligned}A(q) &= 1 + a_1q^{-1} + \ldots + a_{n_a}q^{-n_a}\\ B(q) &= b_1q^{-1} + \ldots + b_{n_b}q^{-n_b}\end{aligned}$$

   As shown in Table 1, when 4 different pairs of $(n_a,\, n_b)$ are used, they give different values of the cost function

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\epsilon^2(t,\theta),$$

   where $\epsilon(t,\theta) = y(t) - \hat{y}(t|\theta)$ is the prediction error.

n_a	n_b	$V_N(\theta)$
2	1	1.13
3	3	0.95
4	4	0.8
6	5	0.79

Table 1:

- Explain why the value of $V_N(\theta)$ decreases when the number of parameters increases (1p)
- Use Akaike Information Criterion (AIC) to select one of the models from the Table (2p)
- Is your choice the same if you use instead the Final Prediction Error (FPE) criterion? (2p)

(b) Assume that the true system

$$y(t) = \frac{1}{1 - \theta q^{-1}} u(t) + e(t)$$

is driven by $u(t)$ zero-mean white noise and that $e(t)$ is also a zero-mean white noise independent of $u(t)$.

- What is the optimal one-step ahead ARX predictor? (1p)
- What is the corresponding optimal one-step ahead OE predictor? (2p)
- Show that the two predictors are not identical. Explain the difference. (2p)

3. Consider the mechanical-electrical system of Fig. 1. An electrical cir-

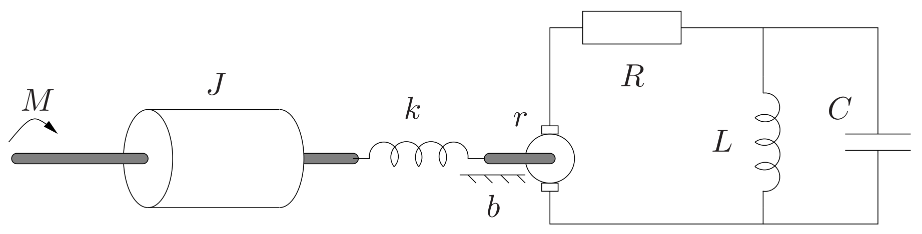

Figure 1:

cuit is driven by a generator with input torque M applied as in Fig. 1. The torque is transmitted through a shaft with moment of inertia J and torsional spring of coefficient k. The connection of the shaft with the generator gives rise to a linear friction of coefficient b. The coupling of the generator with the electrical circuit is given by the following laws: $M_m = ri$ and $u = r\omega$, where M_m is the torque at the generator, ω the angular velocity, u and i are voltage and current induced by the generator on the circuit.

(a) Draw the bond graph of the system and check its causality. (4p)

(b) Compute the state space equations. (4p)

(c) Assume that the torque M is created through a pedal of length ℓ on which a vertical force F is applied. Since the shaft rotates, the effect of F depends on the angle θ, as shown in Fig. 2. (The grey "shaft" of Fig. 2 is the same grey shaft of Fig. 1). Modify the bond graph to include this new part of the system. For simplicity consider only the values of θ for which $-\pi/2 < \theta < \pi/2$. (Suggestion: use controlled elements). (2p)

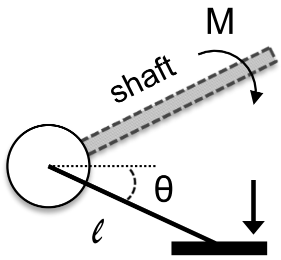

Figure 2:

4. Consider the circuit in Fig. 3.

(a) Write a DAE in the variables v_1, v_2, and i, with v as input signal. (4p)

(b) What is the index of the DAE? (3p)

(c) Let $w_1 = C_1 v_1 - C_2 v_2$ and $y = i$. Show that the model can be written in the form

$$\begin{aligned} \dot{w}_1 &= A w_1 + B v \\ y &= C w_1 + D_0 v + \ldots + D_{k-1} v^{(k-1)} \end{aligned}$$

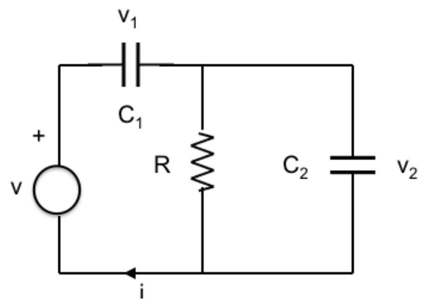

Figure 3:

where $v^{(k-1)}$ is the $(k-1)$-derivative of v, and k is the index of the system. (3p)