EXAM IN MODELING AND SIMULATION (TSRT62)

SAL: ISY:s datorsalar

TID: Monday 28th October 2019, kl. 14.00–18.00

KURS: TSRT62 Modeling and Simulation

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5

ANTAL BLAD (inkl försättsblad): 10

ANSVARIG LÄRARE: Claudio Altafini, 013-281373, 073-9931092

BESÖKER SALEN: cirka kl. 15 och kl. 16

KURSADMINISTRATÖR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL:

1. *L. Ljung & T. Glad* ”Modellbygge och Simulering”
   (English title ”Modeling and Identification of Dynamical Systems”)
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller (t ex *L. Råde & B. Westergren*: ”Mathematics handbook”,
   C. Nordling & J. Österman: ”Physics handbook”,
   S. Söderkvist: ”Formler & tabeller”)
4. Miniräknare

Normala inläsningsanteckningar i läroböckerna är tillåtet. Notera att kommunikation med andra personer och informationshämtning via nätverket eller Internet *inte* är tillåtet under tentamen.

LANGUAGE: you can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: Finns på kursens websida efter skrivningens slut.

VISNING av tentan äger rum 2019-11-13 kl 12.30-13:00 i Ljungeln, B-huset, ingång 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER:		
	betyg 3	23 poäng
	betyg 4	33 poäng
	betyg 5	43 poäng

OBS! Lösningar till samtliga uppgifter ska presenteras så att alla steg (utom triviala beräkningar) kan följas. Bristande motiveringar ger poängavdrag.

Lycka till!

COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose `open terminal`)
  - type

    ```
    module add prog/matlab
    matlab &
    ```

- Print out the model description and the plots requested
- Remember to write your AID number on each printed page you include
- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click `Present`. The model description appears then on the matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose `File` → `Copy figure`, which gives an ordinary matlab plot you can print.
- Printing in Linux:
  - A file called `file.pdf` can be printed out for instance typing in a terminal

    ```
    lp -d printername file.pdf
    ```

    (replace `printername` with the name of the printer in the room you sit in).
  - It is possible to print using `File` → `Print` in a matlab plot, but one must select the printer name writing `-Pprintername` in the `Device option` (again `printername` is the name of your printer).

1. (a) Transform the equation

$$\ddot{y} + \dot{y}^2 y + y^3 = (1 + y^2)u$$

into state space form. [2p]

(b) Consider the model

$$y(t) = \frac{q^{-3}(b_1 + b_2 q^{-1})}{1 + a_1 q^{-1} + a_2 q^{-2}} u(t) + w(t)$$

where the disturbance w is given by

$$w(t) = H(q)e(t)$$

with e a white noise. For what choice of $H(q)$ do you get a ARX model? [2p]

(c) Consider the signal

$$y(t) = b_1 e(t-1) + b_2 e(t-2) + \ldots + b_N e(t-N)$$

where e is a zero-mean white noise, $b_k \neq 0$, $k = 1, \ldots, N$. What is the least value of the integer M such that, for all values of b_k, it is $R_y(\tau) = 0$ for all $|\tau| > M$? [3p]

(d) An Adams method for numerically solving differential equations is given by the following expression:

$$x_n = x_{n-1} + h\frac{f(x_n) + f(x_{n-1})}{2}$$

What is the stability region of the method? *[Hint: enough to check using a linear test function, i.e., $\dot{x} = f(x) = \lambda x$]* [3p]

2. In an identification problem, consider the following model

$$y(t) = b_1 u(t-1) + b_2 u(t-2) + e(t)$$

where e is a white noise and b_1 and b_2 are the parameters to be identified.

(a) Assume the true system is

$$y(t) = 0.2u(t-1) + 0.4u(t-2) + 0.5u(t-3) + w(t)$$

where w is a white noise. An identification experiment is performed with an input u which is a white noise (uncorrelated with w). Compute the value of the parameters that minimizes the prediction error $V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}(y(t) - \hat{y}(t|\theta))^2$ when $N \to \infty$. Is the model unbiased? [4p]

(b) Assume that the true system is instead

$$y(t) = 0.2u(t-1) + 0.4u(t-2) + w(t)$$

with w a white noise of variance 1. The identification experiment is this time performed with an input u uncorrelated with w but whose autocovariance is

$$R_u(\tau) = \begin{cases} 1, & \tau = 0 \\ 0.5, & |\tau| = 1 \\ 0, & |\tau| > 1 \end{cases}$$

What is the asymptotic value of the estimates $\hat{b}_i$ when $N \to \infty$? What is the variance of these estimates for finite N? [4p]

(c) For the true system in (b), find an input signal that gives a lower variance of the parameter estimates than the one obtained at point (b). [2p]

3. The data for this exercise are in a file called `sysid_data_20191028.mat` located in the directory `/courses/TSRT62/exam/`. To load it into your Matlab workspace use any of the following:

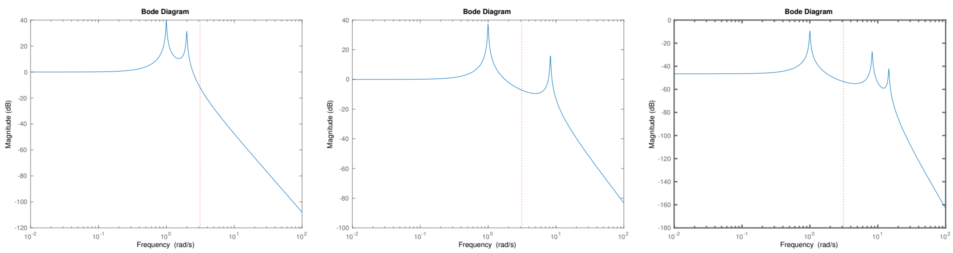
   - type in the Matlab window

     `load /courses/TSRT62/exam/sysid_data_20191028.mat`

   - copy the file to your current directory and then load it into your Matlab workspace (typing `load sysid_data_20191028.mat` at the Matlab prompt).

   Inside `sysid_data_20191028.mat` you will find the sampled signals u and y (the sample time is $T_s = 1$).

   (a) Which of the 3 continuous-time transfer functions shown in Fig. 1 can have generated the data you are studying? Motivate your answer. (In Fig. 1, the dotted line corresponds to the Nyquist frequency $\omega_N = \pi$) [2p]



(a) Res. peak: $\omega_1 = 1\,\text{rad/s}$, and $\omega_2 = 2\,\text{rad/s}$

(b) Res. peaks: $\omega_1 = 1\,\text{rad/s}$, and $\omega_2 = 8.28\,\text{rad/s}$

(c) Res. peaks: $\omega_1 = 1\,\text{rad/s}$, $\omega_2 = 8.28\,\text{rad/s}$, and $\omega_3 = 14.56\,\text{rad/s}$

Figure 1: Exercise 3.

   (b) Use the data to construct one or more appropriate black-box models. For your best model report:
   - plot of the fitted model vs. validation data
   - parameter values and uncertainty
   - residual plot
   - Bode plot
   - poles and zeros placement

   Discuss and comment your choices and results. [8p]

4. Consider the mechanical system shown in Fig. 2, where F_i are external forces, m_i are masses, b is a friction coefficient and k is a spring constant. Assume the mass m_3 slides on the mass m_1 with a friction law

$$F_a = \phi(\Delta v),$$

where Δv is the velocity difference between the two bodies. An analogous law holds also for m_4 and m_2.

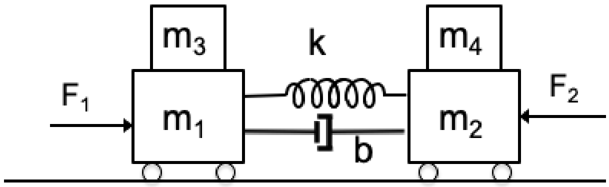


Figure 2: System of Exercise 4.

(a) Compute the bond graph of the system and check its causality. Does it make any difference if $\phi(\cdot)$ is not invertible? [4p]

(b) Assume now that the friction between m_1 and m_3 (as well as that between m_2 and m_4) is so big that the two bodies m_1 and m_3 (as well as m_2 and m_4) can be considered as rigidly connected (no more sliding). How does the bond graph modifies? Show that for the new graph the causality is not conflict-free. Is the conflict eliminable though some simplification? [2p]

(c) For the bond graph obtained in item (b) after the simplifications, compute a state space model. [4p]

5. Consider the electrical circuit in Fig 5, driven by a current source of input current i (and v is a voltage).

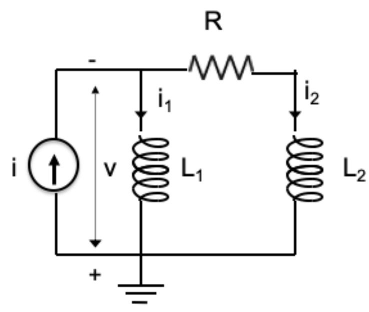


Figure 3: Circuit of Exercise 5.

(a) Write a DAE in the variables v, i_1 and i_2, with i as input. [3p]

(b) What is the index of the DAE? [3p]

(c) Let $w = L_1 i_1 - L_2 i_2$ and $y = v$. Show that the model can be written in the form

$$\dot{w} = Aw + Bi$$
$$y = Cw + D_0 i + \ldots + D_{k-1} i^{(k-1)}$$

where $i^{(k-1)}$ is the $(k-1)$-derivative of i, and k is the differentiability index. [3p]

(d) If the current source is replaced by a voltage source (with voltage v as input), is it possible to write the system in state space form? Motivate your answer. [1p]