

## EXAM IN MODELING AND SIMULATION (TSRT62)

SAL: ISY:s datorsalar

TID: Friday 5th January 2018, kl. 8.00–12.00

KURS: TSRT62 Modeling and Simulation

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5

ANTAL BLAD (inkl försättsblad): 10

ANSVARIG LÄRARE: Claudio Altafini, 013-281373, 073-9931092

BESÖKER SALEN: cirka kl. 9 och kl. 10

KURSADMINISTRATÖR: Ninna Stensgård 013-282225, [ninna.stensgard@liu.se](mailto:ninna.stensgard@liu.se)

### TILLÅTNA HJÄLPMEDEL:

1. *L. Ljung & T. Glad* "Modellbygge och Simulering"  
(English title "Modeling and Identification of Dynamical Systems")
2. *T. Glad & L. Ljung*: "Reglerteknik. Grundläggande teori"
3. Tabeller (t ex *L. Råde & B. Westergren*: "Mathematics handbook",  
*C. Nordling & J. Österman*: "Physics handbook",  
*S. Söderkvist*: "Formler & tabeller")
4. Miniräknare

Normala inläsningsanteckningar i läroböckerna är tillåtet. Notera att kommunikation med andra personer och informationshämtning via nätverket eller Internet *inte* är tillåtet under tentamen.

LANGUAGE: you can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: Finns på kursens websida efter skrivningens slut.

VISNING av tentan äger rum 2018-01-19 kl 12.30-13:00 i Ljungeln, B-huset, ingång 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER:   betyg 3   23 poäng  
  betyg 4   33 poäng  
  betyg 5   43 poäng

OBS! Lösningar till samtliga uppgifter ska presenteras så att alla steg (utom triviala beräkningar) kan följas. Bristande motiveringar ger poängavdrag.

*Lycka till!*

## COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose **open terminal**)
  - type

```
module add prog/matlab
matlab &
```
- Print out the model description and the plots requested
- Remember to write your AID number on each printed page you include
- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click **Present**. The model description appears then on the matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose **File** → **Copy figure**, which gives an ordinary matlab plot you can print.
- Printing in Linux:
  - A file called **file.pdf** can be printed out for instance typing in a terminal

```
lp -d printername file.pdf
```

(replace **printername** with the name of the printer in the room you sit in).
  - It is possible to print using **File** → **Print** in a matlab plot, but one must select the printer name writing **-Pprintername** in the **Device option** (again **printername** is the name of your printer).

1. (a) Explain briefly what a stiff system of ODEs is and why it is difficult to simulate it. [2p]
- (b) Is it easier to compute the parameters of an ARX model or of an OE model? Why? [2p]
- (c) The following system

$$\begin{aligned}\dot{x}_1 &= -x_1 + 0.4x_2 \\ \dot{x}_2 &= x_1 - 4096x_2 + 512u\end{aligned}$$

is simulated from the initial condition  $x(0) = [0 \ 0]^T$  and is driven by a high frequency input  $u$ . Compute a simplified version of the system which approximates its behavior. [2p]

- (d) What is the stability region of the Euler backwards method

$$x_n = x_{n-1} + hf_n$$

What does it mean for the system

$$\dot{x} = -10x$$

And for the following one?

$$\dot{x} = 10x$$

[4p]

2. A physical system is described by

$$y(t) = G(q)u(t) + w(t) \tag{1}$$

where  $w$  is a white noise of variance  $\lambda_w = 2$ .

- (a) Assume

$$G(q) = 0.5q^{-1} + 0.3q^{-2} \tag{2}$$

and that the model used for the estimation is the following

$$y(t) = b_1u(t-1) + b_2u(t-2) + e(t) \tag{3}$$

with  $e(t)$  a white noise. Compute the value of the parameters  $\theta = [b_1 \ b_2]$  that for  $N \rightarrow \infty$  minimizes the prediction error square

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^N (y(t) - \hat{y}(t|\theta))^2 \quad (4)$$

when  $u(t)$  is a white noise of variance  $\lambda_u = 3$  uncorrelated with  $w$  and  $e$ . Compute also the variance of the parameter estimation. [3p]

- (b) Consider the same  $G(q)$  in (2) and the model (3). Describe what happens when minimizing (4) if the input  $u(t)$  is a signal uncorrelated with  $w$  and  $e$  and with the following covariance

$$R_u(\tau) = \begin{cases} 1 & \text{if } \tau = 0 \\ 1 & \text{if } |\tau| = 1 \\ 0 & \text{if } |\tau| > 1 \end{cases}$$

[3p]

- (c) Assume the true system is

$$G(q) = \frac{0.5 + 0.3q^{-1}}{1 + 0.2q^{-1} + 0.4q^{-2}} \quad (5)$$

$u(t)$  is a white noise (uncorrelated with  $w$ ) and the OE model for the estimation is

$$y(t) = \frac{b_1 + b_2q^{-1}}{1 + f_1q^{-1} + f_2q^{-2}}u(t) + e(t)$$

where  $e(t)$  is a white noise (uncorrelated with  $u$ ). Estimate the value of the parameter vector  $\theta = [b_1 \ b_2 \ f_1 \ f_2]$  when  $N \rightarrow \infty$ . [Hint: in this case it is easier to use eq. 12:46 (Swedish book; eq. 11.60 on the English book)] [2p]

- (d) Consider the true system (5), and the static model

$$y(t) = bu + e(t)$$

where  $u$  is a constant signal and  $e(t)$  is a white noise. Compute the value of the parameter  $b$  when  $N \rightarrow \infty$ . [Same hint as above]

[2p]

3. The data for this exercise are in a file called `sysid_data_20180105.mat` located in the directory `/site/edu/rt/tsrt62/exam/`. To load it into your Matlab workspace use any of the following:

- type in the Matlab window

```
load /site/edu/rt/tsrt62/exam/sysid_data_20180105.mat
```

- copy the file to your current directory and then load it into your Matlab workspace (typing `load sysid_data_20180105.mat` at the Matlab prompt).

Inside `sysid_data_20180105.mat` you will find the sampled signals  $u$  and  $y$  (the sample time is  $T_s = 0.1$ ).

Assume that the transfer function from noise to output has the Bode plot shown in Fig. 1.

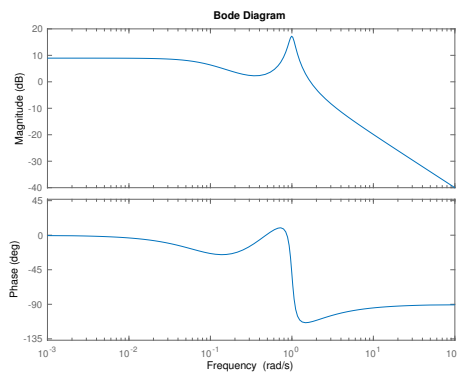


Figure 1: Exercise 3. Transfer function from noise to output.

Construct one or more appropriate black-box models fitting the data and compatible with this transfer function, reporting

- plot of the fitted model vs. validation data
- parameter values and uncertainty
- residual plot
- Bode plots
- poles and zeros placement

Discuss and comment your choices and results. *[Hint: since the noise-output transfer function is available, also the “noise spectrum” function of the SysId GUI main panel (button below “zeros and poles”) can be used to evaluate the models.]* [10p]

4. Consider the hydromechanical system of Fig. 2, where  $A_i$  are the sections of the corresponding pipes/tanks,  $m_i$  the masses,  $k_i$  the spring constants,  $b_i$  the friction coefficients, and  $F$  is the input (external force). Assume pistons 1 and 2 are massless (piston 3 has mass  $m_3$ ). Let  $\rho$  be the density of the fluid and  $h$  the height of the fluid in the tank of section  $A_4$ .

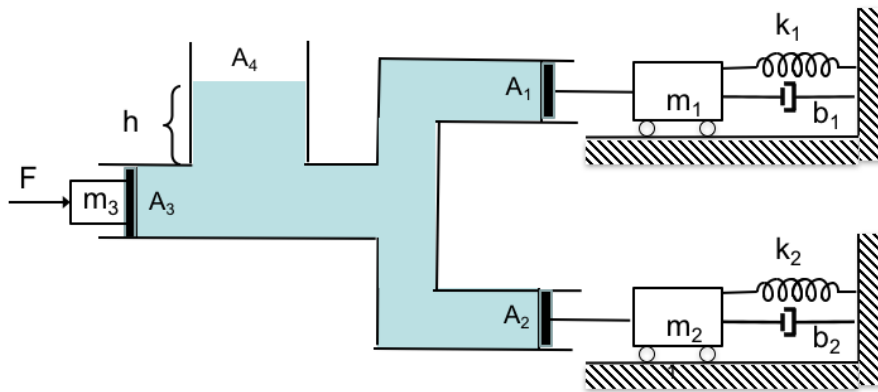


Figure 2: Exercise 4

- (a) Set up a bond graph of the system and mark its causality. [5p]  
 (b) Translate the bond graph into state space equations, taking the height  $h$  as output of the system. [5p]

5. Consider the circuit in Fig. 3 where the two generators are current generators.

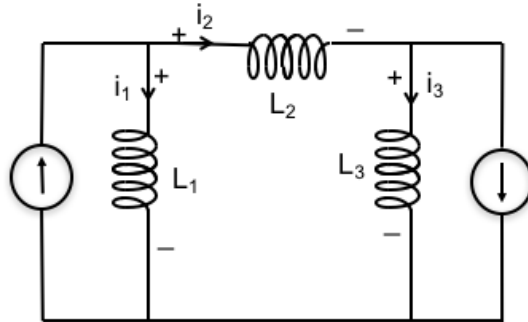


Figure 3: Exercise 5

- (a) Show that the bond graph has a causality conflict. [2p]
- (b) Show that the system can be represented as a DAE. Compute the index of the DAE. [5p]
- (c) What happens if one current generator is replaced by a voltage generator? Is the circuit still representable with a DAE of the same index? [3p]