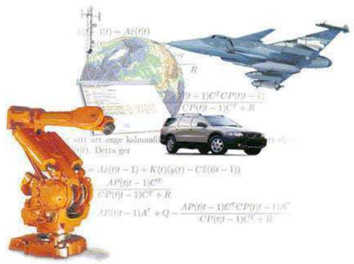
Sysid Course VT1 2016
Experiment Design.



Lennart Ljung

Automatic Control, ISY, Linköpings Universitet

Outline

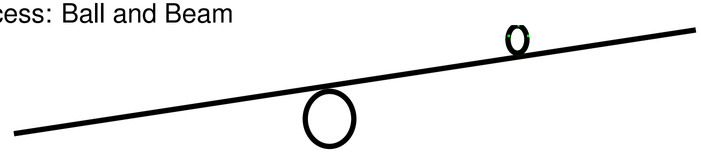
Experiment Design

To make sure that the experimental data are (maximally) informative with respect to the model we want to build.

- What to measure?
- When to measure?
- What to manipulate?
- How to manipulate?

Think about this:

Process: Ball and Beam



Estimated model:

$$y(t) - 1.8y(t-1) + 0.91y(t-2) = 0.5(1.1u(t-1) + 0.9u(t-2))$$

Theoretically a double integrator:

$$y(t) - 2y(t-1) + y(t-2) = 0.5(u(t-1) + u(t-2))$$

Actually worse at low frequencies than one would expect for a long experiment with low noise level. Why?

Outline

Informative Experiments

An experiment is INFORMATIVE if it allows you to distinguish between two different models (in the sets that you might consider).
Example 1: $u(t) = \sin \omega t$
For models of higher order than 1, two models can give the same response if their bode plots coincide for frequency ω. Not informative in models sets of order > 1!

Example 2: $u(t) = -f * y(t)$
Try simple model structure $y(t) + ay(t-1) = bu(t-1) + e(t)$.
Inserting the feedback we get $y(t) + (a + bf)y(t-1) = e(t)$.
So all models with the same value on $a + bf$ give identically the same y and hence u. So unless either a or b is fixed the experiment is not informative even in this simple model class.

Basic Idea For Informative Experiments (Linear Models)

Model 1: $\hat{y}_1(t|t-1) = H_1^{-1}(q)[G_1(q)u(t) + (1 - H_1^{-1}(q))y(t)]$
Model 2: $\hat{y}_2(t|t-1) = H_2^{-1}(q)[G_2(q)u(t) + (1 - H_2^{-1}(q))y(t)]$

Experiment not informative
$\Longleftrightarrow \hat{y}_1(t|t-1) \equiv \hat{y}_2(t|t-1) \qquad [G_1, H_1] \neq [G_2, H_2]$

$$\Longleftrightarrow$$

($) $$M(q)u(t) \equiv L(q)y(t)$$

(orders of M & L $\approx$ 2*model orders and not both zero)
Hence if ($) holds (BUT ONLY THEN!) we are in trouble.

Basic Criteria for Informative Experiments

Open Loop: $\rightarrow L = 0$
Require

$$M(q)u(t) \equiv 0 \Rightarrow u(t) \equiv 0$$

If $M(q)$ is of order n, we the say that $u(t)$ is Persistently Exciting of order n, p.e.(n) This is the same as requiring more than $n/2$ different sinusoids in the input

Closed Loop:
If there is no linear, time-invariant, noise/reference signal -free feedback from y to u we are OK.
.

Formal Calculations 1/3

True system (or second order LTI invariant) G_0: Then

$$\varepsilon(t,\theta) = H_\theta^{-1}(y(t) - G_\theta u(t)) = H_\theta^{-1}[(G_0 - G_\theta)u(t) + H_0 e(t)]$$
$$= H_\theta^{-1}[\Delta G_\theta u(t) + \Delta H_\theta e(t)] + e(t)$$

$$(\hat{G}, \hat{H}) \to \arg\min \int_{-\pi}^{\pi} |H_\theta|^{-2} \begin{bmatrix}\Delta G_\theta & \Delta H_\theta\end{bmatrix} \Phi_\zeta \begin{bmatrix}\Delta G_\theta^* \\ \Delta H_\theta^*\end{bmatrix} d\omega$$
$$\Phi_\zeta(\omega) = \begin{bmatrix}\Phi_u(\omega) & \Phi_{ue}(\omega) \\ \Phi_{eu}(\omega) & \Lambda_0\end{bmatrix}$$

No assumption about feedback etc, just that the spectrum exists.

Note also that any pre-filter L, $\varepsilon_F(t) = L(q)\varepsilon(t)$ can be included in the noise model, $\tilde{H}_\theta = H_\theta / L$.

Formal Calculations 2/3

Factorize!

$$\begin{bmatrix}\Phi_u & \Phi_{ue} \\ \Phi_{eu} & \Lambda_0\end{bmatrix} = \begin{bmatrix}I & \Phi_{ue}\Lambda_0^{-1} \\ 0 & I\end{bmatrix} \begin{bmatrix}\Phi_u^r & 0 \\ 0 & \Lambda_0\end{bmatrix} \begin{bmatrix}I & 0 \\ \Lambda_0^{-1}\Phi_{eu} & I\end{bmatrix}$$

$$\Phi_u^r = \Phi_u - \Phi_{ue}\Lambda_0^{-1}\Phi_{eu}, \quad \Phi_u = \Phi_u^r + \Phi_u^e$$
$$\Phi_e^r = \Lambda_0 - \Phi_{eu}\Phi_u^{-1}\Phi_{ue}$$

Φ_u^r="That part of u that cannot be estimated from e by a LTI filter"

Formal Calculations 3/3

Basic Idea For Informative Experiments

$$\int_{-\pi}^{\pi} \begin{bmatrix}\Delta G_\theta & \Delta H_\theta\end{bmatrix} \Phi_\zeta \begin{bmatrix}\Delta G_\theta^* \\ \Delta H_\theta^*\end{bmatrix} d\omega = 0 \Rightarrow \Delta H_\theta = 0,\ \Delta G_\theta = 0$$

Recall

$$\Phi_\zeta = \begin{bmatrix}\Phi_u & \Phi_{ue} \\ \Phi_{eu} & \Lambda_0\end{bmatrix} = \begin{bmatrix}I & \Phi_{ue}\Lambda_0^{-1} \\ 0 & I\end{bmatrix} \begin{bmatrix}\Phi_u^r & 0 \\ 0 & \Lambda_0\end{bmatrix} \begin{bmatrix}I & 0 \\ \Lambda_0^{-1}\Phi_{eu} & I\end{bmatrix}$$

So the question is

$$\int |\Delta G_\theta(e^{i\omega})|^2 \Phi_u^r(\omega) d\omega = 0 \Rightarrow \Delta G_\theta = 0?$$

The signal u^r should be persistently exciting of the same order as the model/system.

Outline

- Introduction
- Informative experiments
- Identification of closed loop systems
- Good designs
- The input waveform
- Sampling interval and practical aspects
- Pretreatment of measured data

Identification of Closed Loop Systems

- Direct Approach:
  - Forget about feedback! Just apply the estimation procedure as usual.
  - OK if experiment informative nda PEM is used with a correct noise model
- Any Problems?
  - Typically less information in data
  - Be careful with spectral and correlation analysis
  - Be careful with IV- and subspace-methods
  - Be careful with Output-Error methods. The noise needs to be modeled
- Other approaches?
  - Yes, there are many ...

Outline

Good Designs – Basic Principles

Recall slide I-22.
$\mathcal{X}$: The design variables

$$\hat{\theta}_N \to \theta^*(\mathcal{X}) \qquad \text{Cov } \hat{\theta}_N \approx \frac{\lambda}{N} P_\theta(\mathcal{X})$$

- The model $\mathcal{M}(\theta^*(\mathcal{X}))$ is the best approximation of the system under $\mathcal{X}$
-

$$P_\theta(\mathcal{X}) \approx \frac{1}{N}[\mathrm{E}\psi(t)\psi^T(t)]^{-1} \qquad \psi(t) = \frac{d}{d\theta}\hat{y}(t|\theta)$$

Consequences

- Let the experimental conditions resemble those under which the model is to be used.
  Recall (slide I-25)

$$\theta^* \approx \arg\min \int_{-\pi}^{\pi} |\, G_0(e^{i\omega}) - G(e^{i\omega}, \theta)\,|^2 \cdot \frac{\Phi_u(\omega) \cdot |\, L(e^{i\omega})\,|^2}{|\, H(e^{i\omega}, \theta^*)\,|^2} d\omega$$

- Choose experimental conditions and inputs, so that the predictor $\hat{y}(t|\theta)$ becomes sensitive to interesting and important parameters.
  Recall (slide I-25)

$$\text{Cov } \hat{G}_N(e^{i\omega}) \approx \frac{n}{N} \cdot \frac{\Phi_v(\omega)}{\Phi_u(\omega)}$$

Optimal Designs

Typical problem formulation:

$$\min_{\mathcal{X}\in X} \alpha(P_\theta(\mathcal{X}))$$

X :Constrained input variance

Model properties depend only on the input spectrum $\Phi_u(\omega)$, the "color" of the input. It does not depend on the actual wave-form of the input.

.

General Bottom Line

Use your input energy in frequency bands where you need a good model and/or where the disturbances are significant.

$$"\Phi_u^{\text{opt}}(\omega) = \alpha\sqrt{C(\omega)\Phi_v(\omega)}"$$

$$\min_{\mathcal{X}} \mathrm{E} \int_{-\pi}^{\pi} |\hat{G}(e^{i\omega}) - G_0(e^{i\omega})|^2 C(\omega) d\omega$$

Formal Calculations:

Choose all design variables so that the criterion

$$J(\mathcal{D}) = \int \mathrm{Var}|\hat{G}(e^{i\omega})|^2 C(\omega) d\omega$$

is minimized. Suppose that the design variables are:

- Reference signal spectrum
- Output feedback law
- Pre-filter L

under the constraints

- $$\alpha E u^2 + \beta E y^2 \leq 1$$

Formal Calculations 2/4

Then the solution is

- regulator $u(t) = -F_y(q)y(t)$ that solves the standard LQG problem

$$F_y^{\text{opt}} = \arg\min_{F_y} [\alpha E u^2 + \beta E y^2], \quad y = G_0 u + H_0 e$$

- Reference signal spectrum

$$\Phi_r^{\text{opt}}(\omega) = \mu\sqrt{\Phi_v(\omega)C(\omega)} \frac{|1 + G_0(e^{i\omega})F_y^{\text{opt}}(e^{i\omega})|^2}{\sqrt{\alpha + \beta|G_0(e^{i\omega})|^2}}$$

Note the special case $\beta = 0$ and stable system $\Rightarrow F_y = 0$

MSE minimization
Choose all design variables so that the criterion

$$J(\mathcal{D}) = \int \mathrm{E}|\hat{G}(e^{i\omega}) - G_0(e^{i\omega})|^2 C(\omega) d\omega$$

is minimized Suppose that the design variables are:

- Reference signal spectrum
- Output feedback law
- Pre-filter L

under the constraints

- $$Eu^2 \leq 1/\alpha$$

Then the solution is

- Open loop
- Input spectrum $\sim \sqrt{C \cdot \Phi_v}$
- Pre-filter $\sim \sqrt{\frac{\Phi_v}{C}}$

We want to

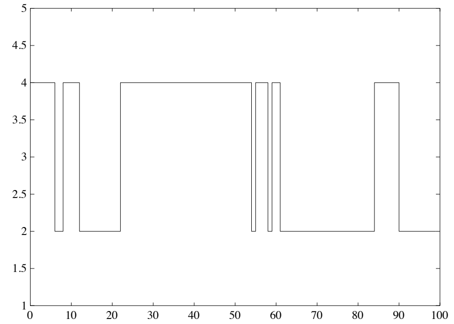
- Control the input spectrum
- Have small maximum amplitude for given power (crest factor)
- Utilize periodicity

Choices:

- Random Gaussian Noise
- (Pseudo) Random Binary Noise
- Sum of sinusoids, including swept sinusoids.

Binary Signals

$u(t) = \begin{cases} \overline{u} \\ \underline{u} \end{cases}$ shifting in a certain fashion, giving a certain spectrum $\Phi_u(\omega)$.



Time domain thinking: Occasionally, let a step response almost settle. No use to let the input shift so quickly that the system's response is hardly visible.

Periodic Inputs

When allowed, periodic inputs have certain advantages:

- Independent noise estimation (Compare the responses to the same input over different periods)
- Reduction of data sets, by averaging over the periods
- No leakage if frequency domain methods are applied

Some Typical Periodic Inputs

- PRBS (Pseudo Random Binary Signal)
- Sum of sinusoids with tailored phases
- Swept sinusoid, (chirp signal)

Outline

- Introduction
- Informative experiments
- Identification of closed loop systems
- Good designs
- The input waveform
- Sampling interval and practical aspects
- Pretreatment of measured data

Choice of T

- Variance increases rapidly when sampling slower than dominating time constants
- Poor return for extra work with fast sampling
- Sample $\approx$ 10 - 20 times the system bandwidth.
- Check step response: Put 3–5 measurements during the rise time.

Some Practical Aspects

- Always use Anti-alias filters!
  They provide noise reduction and avoid confusion with alias.
- With cheap data acquisition, sample fast at source.
  Postpone decision about T to software phase.
  [Digital anti alias filtering + decimation]
  SPTB command `resample`

Outline

- Introduction
- Informative experiments
- Identification of closed loop systems
- Good designs
- The input waveform
- Sampling interval and practical aspects
- Pretreatment of measured data

Pretreatment of Data

ALWAYS FIRST PLOT THE DATA! Possible problems with measured data:

- Drift, offset, low frequency disturbances
- Occasional bursts and outliers
- High frequency disturbances
- Select good/interesting frequency range for model fit

SELECT "NICE" PORTIONS OF DATA FOR ESTIMATION AND VALIDATION!

Off-Set

The measured $y(t)$ and $u(t)$ may not have zero mean.
Dynamics: $A(q)y(t) = B(q)u(t) + e(t)$
Static: $A(1)y(t) = B(1)u(t)$
May be conflicting

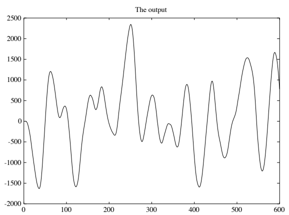
How to Deal with Off-Sets and Trends

- Let y and u be deviations from physical equilibrium.
- Subtract means (possibly time-varying) from data. (*)
- Use ARIMAX-models.
- Increase order
- Estimate off-set level
- Difference data
- Use High-pass filtering.(*)

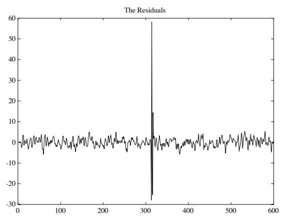
(*): Best

Outliers and Bad Data

Always plot and check data for “bad points”!
Best visible in residuals!



This data set contains one bad value. Can you find it?



Residuals for a 4th order ARX model.

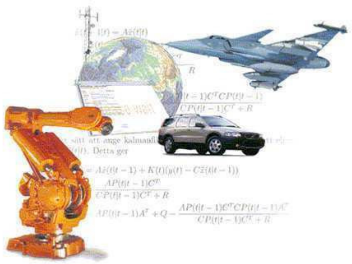
Measures for outliers

- Cut out data pieces without outliers
- Use "robustified" criteria (increasing slower than quadratically). This is done by the option `ErrorThreshold` in SITB.
- Replace outlier by smoothed value

High frequency disturbances above the frequency range of interest to the dynamics show that the choices of sampling interval and pre-sampling filters were not thoughtful enough.
Can be removed by low-pass filtering or decimation.

- Let the system be excited!
- Open loop inputs: Periodic signals with full control of spectral properties. Binary inputs good for linear systems!
- It is possible to identify systems in closed loop, but it requires some caution
- Let the predictor be sensitive to important parameters!
  "$\text{Cov } \hat{G}_N(e^{i\omega}) \approx \frac{n}{N} \cdot \frac{\Phi_v(\omega)}{\Phi_u(\omega)}$"
- Sample 10-20 times bandwidth!
- Look at measured data before you start the estimation. Typically remove trends.

VUB Course on System Identification. Summary



Lennart Ljung

Automatic Control, ISY, Linköpings Universitet