

EFFICIENT BRIDGING-BASED DESTINATION INFERENCE IN OBJECT TRACKING

Tohid Ardehshiri, Bashar I. Ahmad, Patrick M. Langdon, Simon J. Godsill

Engineering Department, University of Cambridge, Trumpington Street, Cambridge, UK, CB2 1PZ

ABSTRACT

This paper proposes a probabilistic intent inference approach that is significantly more computationally efficient than other existing bridging-distributions-based predictors. It sequentially determines the probabilities of all possible destinations of a tracked object, whose motion is modelled by a Markov chain with the distribution of its terminal state equal to that of a nominal endpoint. This encapsulates the long term dependencies in the object trajectory as dictated by intent. Simulations using real data show that the notable reductions in computations achieved by the introduced bridging-based predictor does not impact the quality of the overall inference results.

Index Terms— Intent inference, Kalman filter, bridging distributions, human computer interactions.

1. INTRODUCTION

In several application areas, such as human computer interaction (HCI), surveillance and robotics, knowing the destination of a tracked object (e.g. a pointing apparatus or vessel) can offer crucial information on intent, potential conflict or opportunities, thereby, enabling smart predictive system functionalities and automation. For example, unveiling the icon the user intends to select on an in-vehicle touchscreen, early in the free hand pointing gesture, can effectively simplify and expedite accomplishing the selection task. Hence, it can substantially reduce the effort (distractions) associated with interacting with an in-car display [1, 2]. In maritime surveillance, determining the destination of a vessel, which drives its motion (including the vessel future trajectory), in a given geographical area permits the timely identification of possible threats or favorable circumstances [3, 4, 5]. It is noted that a wide range of other applications can benefit from knowing the intent of an object of interest, for instance intelligent robot navigation in the presence of other moving agents [6, 7] such as pedestrians, and advanced driver assistance systems [8], to name a few.

In this paper, we propose an efficient solution to the problem of predicting the intended destination of a tracked object from a finite set of N nominal endpoints, given its available observed partial trajectory. The introduced technique, which is based on the bridging distributions framework, is shown to

be significantly more computationally efficient than the predictors in [9, 10]. The adopted bridging formulation capitalises on the premise that the track of the object or target, albeit random, must end at the intended destination, which is assumed be unknown. A bridge for each possible endpoint is constructed to capture the long term dependencies in the object trajectory due to premeditated actions guided by the intent. Whilst several well-established conventional *sensor-level* tracking algorithms [11, 12, 13] focus on inferring the posterior of the latent *state* x_t (e.g. the tracked object position, velocity, acceleration, etc.) at the current time t given the available noisy observations $y_{1:t}$, the objective here is to estimate at t the likelihood of each of the possible destinations being the endpoint, i.e. $p(D|y_{1:t})$. In this paper, the inference routine implementation entails running one Kalman filter to attain $p(D|y_{1:t})$, in lieu of N such filters as in [9, 10].

Utilising predictive information on the target endpoint to improve the accuracy of the state estimation results, i.e. destination-aware tracker, was proposed in [3, 14], assuming known time of arrival T at destination. New destination-aware tracking approaches that use a discrete stochastic reciprocal or context-free grammar process and incorporate an additional mechanism to determine the intended endpoint are introduced in [15, 16, 17]; the state space is discretised within predefined regions. Whilst discretisation can be a burdensome task in certain scenarios (e.g. tracking in 3D or in large geographical areas), the target can consequently pass through only a finite number of those grids/zones. In this paper, the objective is to estimate $p(D|y_{1:t})$ without *a priori* known T and continuous state space models are applied, thus, no restrictions on the target path to its destination are imposed. Within this formulation, noisy asynchronous observations can be easily handled and a Kalman-filter-type implementation of the inference routine is utilised. It is a simple effective solution to the intent prediction problem.

Various prediction techniques that rely on a dynamical model learnt from previously recorded tracks (complete data sets) exist, e.g. [8, 6]. Whilst such methods typically require a substantial parameters training and have high computational cost, here a state-space modelling approach is applied. It uses known dynamical and sensor models, with a few unknown parameters, as is common in the tracking area [11, 12, 13]. The solution below requires minimal training and is computationally efficient, yet delivers a competitive performance sim-

ilar/identical to the bridging-based algorithms in [9, 10]. For further information on the related work, e.g. in the HCI or anomaly detection areas, the reader is referred to [10].

2. PROBLEM STATEMENT

Consider a random state $x_t \in \mathbb{R}^s$ at time t with the initial prior distribution $\pi(x_1)$. A random time in the future denoted by T has the *a priori* distribution $\pi(T)$; T is the time instant when x_t reaches its terminal state (e.g. a tracked object reaches its destination). The *predicted* terminal state at time T is then given by x_T . The objective here is to establish (within a probabilistic framework) which of the finite possible destinations will be assumed by this random state. Let $D \in \mathbb{D} = \{1, \dots, N\}$ be a categorical random variable denoting the labels of N possible terminal state distributions, also known as destinations. These terminal state distributions are notated by $\pi(x_T|D)$. In the adopted Bayesian formulation, the prior distribution $\pi(D)$ is assigned to the destinations and the sought posterior $p(D|y_{1:t})$ is computed using

$$p(D|y_{1:t}) = \frac{\pi(D)p(y_{1:t}|D)}{p(y_{1:t})}, \quad (1)$$

where $y_{1:t}$ is the available observations up to time t . From (1), the intended destination can be determined based on a decision criterion, e.g. the intuitive *Maximum a Posteriori* (MAP) estimate where the most probable destination at t is the intended one [9, 10]. The noisy observation of the state variable x_t at t are available via the linear and Gaussian model

$$g(y_t|x_t) = \mathcal{N}(y_t; Cx_t, R). \quad (2)$$

Additionally, the evolution of the state variable, which can be continuous, is described by $dx_t = \Gamma x_t dt + \Sigma d\omega_t$ such that ω_t is a standard n -dimensional Wiener process. The continuous-time state evolution can be integrated and discretised to obtain a discrete-time linear and Gaussian state evolution model between two arbitrary times t and τ as in

$$x_t = F_{t\tau}x_\tau + w_{t\tau} \quad (3)$$

where the noise $w_{t\tau}$ is zero mean and its covariance is $Q_{t\tau}$. The class in (3) includes many models used widely in tracking applications, for example the (near) constant velocity or acceleration models [13]. The state evolution model (3) is denoted by $f(x_t|x_\tau)$ in the rest of this paper. This state evolution model does not depend on the destination D . However, when the conditional probability distribution of x_T is available *a priori* or is assumed to be $\pi(x_T|D)$, bridging distributions can be utilised. Bridging distributions are used in a Bayesian setting to incorporate the prior knowledge about the conditional terminal state into the state evolution model in [9, 10]. The bridging state evolution model between two time instances is then expressed by

$$p(x_t|x_\tau, x_T, D, T) \propto p(x_t|x_\tau, D, T)p(x_T|x_t, D, T). \quad (4)$$

It is noted that the tackled intent prediction problem entails estimating the posterior $p(D|y_{1:t})$, and the latent state variables are not sought. In fact, in order to compute the likelihood $p(y_{1:t}|D)$, these latent variables should be integrated out as in

$$p(y_{1:t}|D) = \int p(x_{1:t}, x_T, y_{1:t}|D, T)\pi(T|D)dTdx_{1:t}dx_T.$$

In the next section, we show that the likelihood $p(y_{1:t}|D)$ in (1) can be calculated via a Kalman-filtering-type inference.

3. PROPOSED LOW COMPLEXITY SOLUTION

In [9, 10], the state is augmented by the terminal state variables. A Kalman filter on the new state variable with dimension $2s$ is subsequently applied for each of the N possible destinations. Since the state is augmented by x_T and the state evolution is now conditioned on the categorical variable D , then N Kalman filters should run at the same time in an on-line application to compute the likelihood $p(y_{1:t}|D, T)$. Furthermore, in order to integrate out the scalar random variable T , which may have a non-conjugate distribution, numerical methods are applied. When these numerical techniques are used, the N Kalman filters should be run for each point of the deterministic or stochastic grid point on the support of $\pi(T|D)$. If M points are utilised, then the computational complexity of the bridging-based intent prediction in [9, 10] is of order $\mathcal{O}(4NM s^2)$. In this paper, we propose an efficient algorithm targeting $p(y_{1:t}|D)$, which employs a single filter, and exploits a sound approximation to achieve significantly lower computational cost of the intent inference routine; it is of order less than $\mathcal{O}(NM s^2)$, see Table 1. The introduced method here can also be used to compute the smoothing posterior of the initial state. For example, in the HCI application of pointing at a touchscreen, the posterior of the initial state can be utilised to infer the posterior distribution on the time of arrivals T using a probabilistic version of Fitt's law [18].

Assuming a known prior $\pi(D)$ in (1), next we aim to estimate the likelihood $p(y_{1:t}|D)$ for the state-space model¹,

$$x_1 \sim \pi(x_1), \quad (5a)$$

$$x_k|x_j \sim f(x_k|x_j), \quad \text{for } k \neq T \quad (5b)$$

$$x_k|x_j, T \sim f(x_T|x_j, T), \quad \text{for } k = T \quad (5c)$$

$$y_k|x_k \sim g(y_k|x_k), \quad (5d)$$

$$x_T|D \sim \pi(x_T|D), \quad (5e)$$

$$T|D \sim \pi(T|D). \quad (5f)$$

The following equality holds for the proposed bridging state

¹In order to have a consistent model, the prior distribution on the terminal state in (5e) should be consistent with the motion model (5b) and the prior on the initial state in (5a) because $\int \pi(x_T|D)p(D)dD = \pi(x_T) = \int \pi(x_1)f(x_T|x_1)dx_1$. However, in this work $\pi(x_T|D)$ is assumed to be closely approximated by a normal distribution.

evolution model in (4)

$$p(x_t|x_{t-1}, x_T, T) = \frac{p(x_t, x_T|x_{t-1}, T)}{p(x_T|x_{t-1}, T)} \quad (6a)$$

$$= \frac{p(x_t|x_{t-1})p(x_T|x_t, x_{t-1}, T)}{p(x_T|x_{t-1}, T)} \quad (6b)$$

$$= f(x_t|x_{t-1}) \frac{f(x_T|x_t, T)}{f(x_T|x_{t-1}, T)}, \quad (6c)$$

where the Markov property of the process is used in the last equality. Thus, the bridging evolution model can be expressed by two factors; the first factor is the state model without bridging and the second factor is the rational function $\frac{f(x_T|x_t, T)}{f(x_T|x_{t-1}, T)}$, which has a sequential nature. Below, this sequential characteristic will be exploited to reduce the computational complexity of the algorithm targeting the likelihood $p(y_{1:t}|D)$. Using the conditional prior $\pi(x_1, x_T|D) \triangleq \pi(x_1)\pi(x_T|D)$, the conditional likelihood $p(y_{1:t}|D, T)$ can be expressed by

$$\begin{aligned} p(y_{1:t}|D, T) &= \int p(x_{1:t}, x_T, y_{1:t}|D, T) dx_1 \cdots dx_t dx_T \\ &= \int \pi(x_T, x_1|D) \prod_{k=2}^t p(x_k|x_{k-1}, x_T, T) \\ &\quad \times \prod_{k=1}^t g(y_k|x_k) dx_1 \cdots dx_t dx_T. \end{aligned} \quad (7)$$

Utilising the sequential property of the second factor in (6c), the predictive likelihood simplifies to

$$\begin{aligned} p(y_{1:t}|D, T) &= \int \pi(x_T, x_1|D) \frac{f(x_T|x_t, T)}{f(x_T|x_1, T)} \prod_{k=2}^t f(x_k|x_{k-1}) \\ &\quad \times \prod_{k=1}^t g(y_k|x_k) dx_1 \cdots dx_t dx_T. \end{aligned} \quad (8)$$

Consider the joint smoothing density of a state space model with initial prior, dynamical model and likelihood function according to (5a), (5b) and (5d), respectively, as per

$$q(x_{1:t}, y_{1:t}) = \pi(x_1) \prod_{k=2}^t f(x_k|x_{k-1}) \prod_{k=1}^t g(y_k|x_k) \quad (9)$$

$$= q(x_{1:t}|y_{1:t})q(y_{1:t}) \quad (10)$$

where $q(y_{1:t})$ is equal to the predictive likelihood of a Kalman filter with initial prior, dynamical model and likelihood function according to (5a), (5b) and (5d), respectively. Furthermore, $q(x_{1:t}|y_{1:t})$ is the joint smoothing posterior of the state. Hence, the conditional likelihood can be written as

$$\begin{aligned} p(y_{1:t}|D, T) &= \int \pi(x_T|D) \frac{f(x_T|x_t, T)}{f(x_T|x_1, T)} q(x_{1:t}|y_{1:t}) \\ &\quad \times q(y_{1:t}) dx_1 \cdots dx_t dx_T. \end{aligned} \quad (11)$$

Now by marginalizing x_2 to x_t in (11), we obtain

$$p(y_{1:t}|T, D) = q(y_{1:t}) \int \pi(x_T|D) \frac{q(x_T, x_1|T)}{f(x_T|x_1, T)} dx_1 dx_T, \quad (12)$$

where

$$q(x_1, x_T|T) \triangleq \int f(x_T|x_t, T) q(x_{1:t}|y_{1:t}) dx_2 \cdots dx_t \quad (13)$$

Let $q(x_1, x_t) = \mathcal{N}\left(\begin{bmatrix} x_1 \\ x_t \end{bmatrix}; \begin{bmatrix} x_{1|t} \\ x_{t|t} \end{bmatrix}, \begin{bmatrix} P_{1|t} & P_{1,t|t} \\ P_{t,1|t} & P_{t|t} \end{bmatrix}\right)$, then for $f(x_T|x_t, T) = \mathcal{N}(x_T; F_{Tt}x_t, Q_{Tt})$, we have

$$\begin{aligned} q(x_1, x_T|T) &= \mathcal{N}\left(\begin{bmatrix} x_1 \\ x_T \end{bmatrix}; \begin{bmatrix} x_{1|t} \\ F_{Tt}x_{t|t} \end{bmatrix}, \begin{bmatrix} P_{1|t} & & & \\ & P_{t,1|t} & & \\ & & F_{Tt}P_{t|t} & \\ & & & F_{Tt}P_{t|t}F_{Tt}^T + Q_{T,t} \end{bmatrix}\right). \end{aligned} \quad (14)$$

It is noted that $q(x_T|T)$ is readily available from propagation of the filtering posterior of x_t using the dynamical model $f(x_T|x_t, T)$ without smoothing, while computing the smoothing posterior for x_1 as well as the conditional density $q(x_T|x_1, T)$ requires a fixed point smoother [19]. We apply the parametrizations $\pi(x_T|D) = \mathcal{N}(x_T; \mu_D, \Sigma_D)$ and $f(x_T|x_1, T) = \mathcal{N}(x_T, F_{T1}x_1, Q_{T1})$ for the conditional terminal state distribution and the state evolution model from the initial time to time T , respectively. After some measurement updates and when the expected time delay to the destination is large, Q_{T1} becomes much larger than the covariances of the distributions in the numerator of (12). Thus, when $Q_{T1} \gg Q_{Tt}$ and $Q_{T1} \gg \Sigma_D$, the normal distribution in the denominator can be approximated by a uniform distribution and, consequently, replaced by a constant factor α which later on will be cancelled via normalization of the posterior $p(D|y_{1:t})$. This well motivated approximation leads to

$$\begin{aligned} p(y_{1:t}|T, D) &\approx \alpha q(y_{1:t}) \int \pi(x_T|D) q(x_T, x_1|T) dx_1 dx_T \\ &= \alpha q(y_{1:t}) \int \pi(x_T|D) q(x_T) dx_T \end{aligned} \quad (15a)$$

$$= \alpha q(y_{1:t}) \mathcal{N}(\mu_D; F_{Tt}x_{t|t}, F_{Tt}P_{t|t}F_{Tt}^T + Q_{T,t} + \Sigma_D) \quad (15b)$$

where the following Gaussian identity is used,

$$\begin{aligned} \mathcal{N}(z; \mu_1, \Sigma_1) \mathcal{N}(z; \mu_2, \Sigma_2) \\ = \mathcal{N}(z; \mu, \Sigma) \mathcal{N}(\mu_1; \mu_2, \Sigma_1 + \Sigma_2) \end{aligned} \quad (16)$$

such that $\mu = \Sigma(\Sigma_1^{-1}\mu_1 + \Sigma_2^{-1}\mu_2)$ and $\Sigma = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$. The Likelihood can be computed via the marginalization

$$p(y_{1:t}|D) = \int p(y_{1:t}|D, T) \pi(T|D) dT. \quad (17)$$

The above integral can be computed numerically, for example by a Monte Carlo (MC) method as well as a numerical quadrature technique [9, 10]. These approaches are particularly

Table 1. Efficient likelihood computation for bridging predictors.

- 1: **Inputs:** μ_d, Σ_d for $d = 1, \dots, N, C, R, x_{1|0}, P_{1|0}, y_{1:t}$ and T_1, \dots, T_M .
- 2: $\eta \leftarrow 1$
- 3: **for** $k=1$ to t **do** ▷ for each observation y_k
- 4: **if** $k > 1$ **then**
- 5: Compute $F_{k(k-1)}$ and $Q_{k(k-1)}$
- 6: $x_{k|k-1} \leftarrow F_{k(k-1)}x_{k-1|k-1}$ ▷ filtering time update
- 7: $P_{k|k-1} \leftarrow F_{k(k-1)}P_{k-1|k-1}F_{k(k-1)}' + Q_{k(k-1)}$
- 8: **end if**
- 9: $S_k \leftarrow CP_{k|k-1}C' + R,$ ▷ measurement update
- 10: $K \leftarrow P_{k|k-1}C'S_k^{-1}$
- 11: $\tilde{y}_k \leftarrow Cx_{k|k-1},$ $x_{k|k} \leftarrow x_{k|k-1} + K(y_k - \tilde{y}_k)$
- 12: $P_{k|k} \leftarrow (I - K)P_{k|k-1}$
- 13: $\eta \leftarrow \eta \times \mathcal{N}(y_k; \tilde{y}_k, S_k)$
- 14: **for** $T = T_1$ to T_M **do**
- 15: Compute F_{Tk} and Q_{Tk}
- 16: **for** $d=1$ to N **do**
- 17: $m(T, d) \leftarrow \mathcal{N}(\mu_d; F_{Tk}x_{k|k}, F_{Tk}P_{k|k}F_{Tk}' + Q_{Tk} + \Sigma_d)$
- 18: **end for**
- 19: **end for**
- 20: **for** $d=1$ to N **do**
- 21: $l_{d,k} \leftarrow \eta \times \sum m(T_1 : T_M, d)$
- 22: **end for**
- 23: **end for**
- 24: **Outputs:** $l_{d,k}, x_{k|k}$ and $P_{k|k}$ **for** $k = 1 \dots t$

attractive since $\pi(T|D)$ can have a non-conjugated distribution to the normal distributions where T appears. The pseudocode of the proposed algorithm is listed in Table 1 assuming a uniform prior $\pi(T|D)$ for simplicity; the approximate likelihood based on (12) is denoted by $l_{d,k}$ for $d = 1, \dots, N$ and $k = 1, \dots, t$.

4. SIMULATION RESULTS

Here, we compare the destination prediction performance of the proposed computationally efficient bridging distributions (CE-BD) with the original bridging formulation in [9, 10], i.e. BD. Fifty full trajectories of free hand pointing gestures collected in an instrumented car are examined. They pertain to four users acquiring (i.e. pointing and selecting) GUI icons ($N = 21$) on a touchscreen mounted to the car dashboard. Observation y_t at time t is the 3D cartesian coordinates of the pointing finger, as provided by a gesture tracker. Success is considered to be the ability of the MAP estimate $\hat{D}(t) = \arg \max_{d \in \mathbb{D}} p(D = d | y_{1:t})$ to correctly identify the intended endpoint D^+ , i.e. classification success $S(t) = 1$ if $\hat{D}(t) = D^+$ and zero otherwise, for observations at $t \in \{t_1, t_2, \dots, T\}$, uniform priors on T and D are assumed.

Simulation results from a bridged (near) constant velocity (CV) model of the pointing movements in 3D, i.e. $s = 6$, showed that the BD-CV and CE-BD-CV produced identical intent prediction (classification) results compared to that in

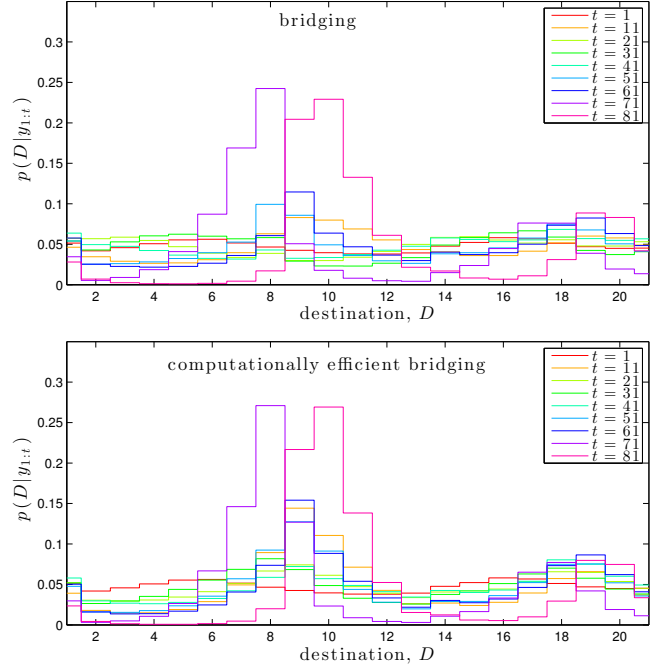


Fig. 1. The posterior distribution of destinations $p(D|y_{1:t})$.

[9, 10] for all observations in the 50 pointing trajectories. This demonstrates that the reductions in the computational cost of the inference routine achieved by the proposed CE-BD does not have any noticeable impact on the overall quality of the results of predicting the on-screen destination of a pointing task. It is noted that endpoint prediction using the bridged CV model is superior to other benchmark competitors for the same data set as reported in [9], hence it is assessed here. On the other, Figure 1 depicts the posterior distribution of destinations $p(D|y_{1:t})$ in a selected pointing trajectory in 3D, for an increasing number of observations over time for BD-CV and CE-BD-CV. The top plot shows the posteriors obtained via exact formulation of bridging while the bottom figure illustrates the same posteriors using the proposed CE-BD. The distributions at each time step marginally differ, specifically when the probability of a given D is significant (meaningful) at a given time instant t .

5. CONCLUSION

A computationally efficient Kalman-filtering-type implementation of the destination inference routine based on the effective bridging distributions framework is introduced in this paper. The substantial reductions in the computational complexity of the predictor, which leverage an intuitive approximation in (16), are shown not to have noticeable negative impact on the quality of the overall endpoint prediction results. Whilst destination prediction is relevant to several application areas, this paper serves to motivate further work on simple and efficient state-space-modelling-based intent inference predictors.

6. REFERENCES

- [1] Bashar I Ahmad, Patrick M Langdon, Simon J Godsill, Richard Donkor, Rebecca Wilde, and Lee Skrypchuk, "You do not have to touch to select: A study on predictive in-car touchscreen with mid-air selection," in *Proc. of the Int. Conf. on Automotive User Interfaces and Interactive Veh. Apps. (AutomotiveUI '16)*, 2016.
- [2] İlhan Aslan, Alina Krischkowsky, Alexander Meschtscherjakov, Martin Wuchse, and Manfred Tscheligi, "A leap for touch: proximity sensitive touch targets in cars," in *Proc. of the Int. Conf. on Automotive User Interfaces and Interactive Vehicular Applications*. ACM, 2015, pp. 39–46.
- [3] David A Castanon, Bernard C Levy, and Alan S Willsky, "Algorithms for the incorporation of predictive information in surveillance theory," *International Journal of Systems Science*, vol. 16, no. 3, pp. 367–382, 1985.
- [4] Branko Ristic, Barbara La Scala, Mark Morelande, and Neil Gordon, "Statistical analysis of motion patterns in AIS data: Anomaly detection and motion prediction," in *11 Int. Conf. on Info. Fusion (FUSION)*, 2008, pp. 1–7.
- [5] Giuliana Pallotta, Steven Horn, Paolo Braca, and Karna Bryan, "Context-enhanced vessel prediction based on Ornstein-Uhlenbeck processes using historical AIS traffic patterns: Real-world experimental results," in *17th Int. Conf. on Information Fusion (FUSION '14)*, 2014, pp. 1–7.
- [6] Kris Kitani, Brian Ziebart, James Bagnell, and Martial Hebert, "Activity forecasting," in *Proc. of 12th European Conference on Computer Vision (ECCV '12)*, 2012, pp. 201–214.
- [7] Graeme Best and Robert Fitch, "Bayesian intention inference for trajectory prediction with an unknown goal destination," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2015, pp. 5817–5823.
- [8] Takashi Bando, Kana Takenaka, Shogo Nagasaka, and Takafumi Taniguchi, "Unsupervised drive topic finding from driving behavioral data," in *IEEE Intelligent Vehicles Symposium (IV)*, 2013, pp. 177–182.
- [9] Bashar I. Ahmad, James K. Murphy, Patrick M. Langdon, Simon J. Godsill, and Robert Hardy, "Destination inference using bridging distributions," in *Proc. of IEEE ICASSP*, Brisbane, 2015, pp. 5585–5589.
- [10] Bashar I Ahmad, James K Murphy, Patrick M Langdon, and Simon J Godsill, "Bayesian intent prediction in object tracking using bridging distributions," *IEEE Transactions on Cybernetics*, vol. PP, no. 99, pp. 1–13, 2016.
- [11] X Rong Li and Vesselin P Jilkov, "Survey of maneuvering target tracking. Part I. Dynamic models," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1333–1364, 2003.
- [12] X Rong Li and Vesselin P Jilkov, "Survey of maneuvering target tracking. part v. multiple-model methods," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 4, pp. 1255–1321, 2005.
- [13] Yaakov Bar-Shalom, Peter Willett, and Xin Tian, *Tracking and Data Fusion: A Handbook of Algorithms*, YBS Publishing, 2011.
- [14] Enzo Baccarelli and Roberto Cusani, "Recursive filtering and smoothing for reciprocal gaussian processes with Dirichlet boundary conditions," *IEEE Trans. on Signal Processing*, vol. 46, no. 3, pp. 790–795, 1998.
- [15] Alex Wang, Vikram Krishnamurthy, and Bhashyam Balaji, "Intent inference and syntactic tracking with GMTI measurements," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 47, no. 4, pp. 2824–2843, 2011.
- [16] Mustafa Fanaswala and Vikram Krishnamurthy, "Detection of anomalous trajectory patterns in target tracking via stochastic context-free grammars and reciprocal process models," *IEEE Journal of Selected Topics in Signal Processing*, vol. 7, no. 1, pp. 76–90, 2013.
- [17] Aaron F Bobick, Yuri Ivanov, et al., "Action recognition using probabilistic parsing," in *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*. IEEE, 1998, pp. 196–202.
- [18] Paul M Fitts, "The information capacity of the human motor system in controlling the amplitude of movement.," *Journal of Experimental Psychology*, vol. 47, no. 6, pp. 381, 1954.
- [19] Simo Särkkä, *Bayesian filtering and smoothing*, vol. 3, Cambridge University Press, 2013.