

Estimation of the Free Space in Front of a Moving Vehicle

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ABSTRACT

There are more and more systems emerging making use of measurements from a forward looking radar and a forward looking camera. It is by now well known how to exploit this data in order to compute estimates of the road geometry, tracking leading vehicles, etc. However, there is valuable information present in the radar concerning stationary objects, that is typically not used. The present work shows how radar measurements of stationary objects can be used to obtain a reliable estimate of the free space in front of a moving vehicle. The approach has been evaluated on real data from highways and rural roads in Sweden.

INTRODUCTION

For a collision avoidance system it is imperative to have a reliable *map* of the environment surrounding the host vehicle. This map, consisting of both stationary and moving objects, has to be built in real time using measurements from the sensors present in the host vehicle. This is currently a very active research topic within the automotive industry and many other areas as well. Great progress has been made, but much remains to be done. Current state-of-the-art when it comes to the problem of building maps for autonomous vehicles can be found in the recent special issues [3–5] on the 2007 DARPA Urban Challenge. In these contributions measurements from expensive and highly accurate sensors are used, while we in the present paper utilize measurements from off-the-shelf automotive

radars.

In this contribution we consider the problem of estimating the *free space* in front of the vehicle, making use of radar measurements originating from stationary objects. The free space is defined as the space where a ground vehicle can manoeuvre without colliding with other objects. Another name for the free space is the drivable space.

The present solution makes use of an already existing sensor fusion framework [19], which among other things provided a good road geometry estimate. This framework improves the raw vision estimate of the road geometry by fusing it with radar measurements of the leading vehicles. The idea is that the motion of the leading vehicles reveals information about the road geometry [9, 10, 32]. Hence, if the leading vehicles can be accurately tracked, their motion can be used to improve the road geometry estimates. Furthermore, we used a solid dynamic model of the host vehicle allowing us to further refine the estimates by incorporating several additional proprioceptive sensor measurements readily available on the CAN bus. The resulting, rather simple, yet useful *map* of the environment surrounding the host vehicle consists in

- Road geometry, typically parameterized using road curvature and curvature rate.
- Position and velocity of the leading vehicles.
- Host vehicle position, orientation and velocity.

This information can and has indeed been used to design simpler collision avoidance systems. However, in order

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to devise more advanced systems, more information about the environment surrounding the host vehicle is needed. The purpose of this paper is to exploit information already delivered by the radar sensor in order to compute a more complete map. Hence, there is no need to introduce any new sensors, it is just a matter of *making better use of the sensor information that is already present* in a modern premium car. To be more precise, it is the radar echoes from stationary objects that are used to estimate the road borders, which determines the free space in front of the host vehicle. The radar measurements used originate from for instance, guard rails and concrete walls. Obviously these stationary radar measurements are not enough to fully explain the road borders. However, as we will see, there is surprisingly much information present in these measurements.

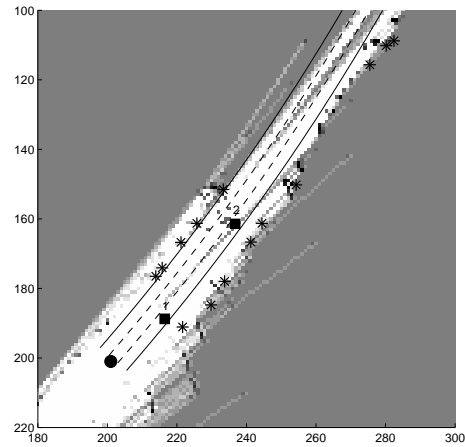
The *key* in our approach is to make use of the road curvature estimate from the sensor fusion framework [19] mentioned above to sort the stationary radar measurements according to which side of the road they originate from. These measurements are then used together with the estimates from the sensor fusion to dynamically form a suitable constrained quadratic program (QP) for estimating the free space in front of the vehicle. This QP models the temporal correlation that exists in roads and the fact that the road shape cannot change arbitrarily fast.

The approach has been evaluated on real data from highways and rural roads in Sweden. The test vehicle is a Volvo S80 equipped with a forward looking 77 GHz mechanically scanned FMCW radar and a forward looking vision sensor (camera).

RELATED WORK

We have also investigated a completely different approach to represent the map of the free space in front of the host vehicle based on so call occupancy grid maps (OGM). This is a commonly used method for tackling the problem of generating consistent maps from uncertain measurements of stationary object under the assumption that the host vehicle pose is known. Occupance grid maps are very popular in the robot community, especially for all sorts of autonomous vehicles equipped with laser scanners, indeed several of the DARPA vehicles [3–5] used OGM’s. The OGM was introduced by Elfes [7] and a solid treatment can be found in the recent textbook [28].

The map is discretized into a number of cells with an associated probability of occupancy. The map is represented by a matrix, with each element corresponding to a map-cell. Figure 1a shows an OGM computed for the highway situation given in the host vehicle’s camera view in Figure 1b. The host vehicle is positioned at (200, 200),



(a)



(b)

Figure 1: The filled circle at position (200, 200) in the occupancy grid map in Figure (a) is the host vehicle, the stars are the radar observations obtained at this time sample, the black squares with numbers 1 and 2 are the two leading vehicles that are currently tracked. The gray-level in the figure indicates the probability of occupancy, the darker the grid cell, the more likely it is to be occupied. The shape of the road is given as solid and dashed lines, calculated as described in [19]. The camera view from the host vehicle is shown in Figure (b), the concrete walls, the guardrail and the pillar of the bridge are interesting landmarks. Furthermore, the two tracked leading vehicles are clearly visible in the right lane.

indicated by the filled circle. The gray-level in the occupancy map indicates the probability of occupancy, the darker the grid cell, the more likely it is to be occupied. As can be seen in Figure 1a, the OGM generates a good-looking *overview* of the traffic situation. However, since the measurements are obtained from a standard automotive radar the results are not very informative for a collision avoidance system, better accuracy is needed. For a more complete description of the application of the OGM to the present problem we refer to [18].

The work presented here is clearly related to lane tra-

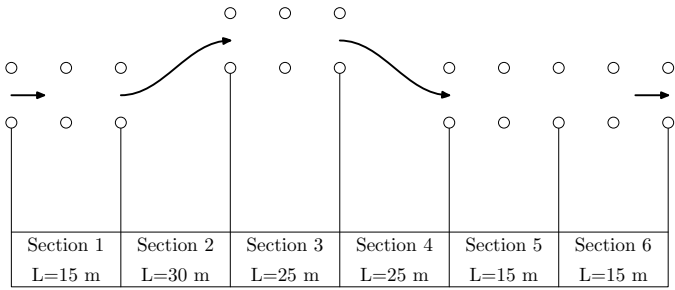


Figure 2: ISO 3888 double lane change maneuver [13].

cking, which by now is a very well-studied problem, see e.g. [23] for a recent survey using cameras. In fact the required sensor fusion framework [19] makes use of the estimates from a visual lane tracker. The recent book [6] contains a lot of interesting information about detecting and tracking lanes using cameras. Lane tracking has also been tackled using radar sensors, see e.g., [14, 17, 21, 24] and laser sensors, see e.g. [31]. Using laser scanners there have been several approaches making use of reflections from the road boundary, such as crash barriers and reflection posts, to compute information about the free space, see e.g. [15, 16, 26]. Furthermore, the use of a side looking radar to measure the lateral distance to a sidewall is described in various papers, e.g., [8, 22, 27]. The intended application in these papers [8, 22, 27] were automatic lateral control. Here, we have no specific application in mind, we just try to obtain the best possible map based on the available sensor information. This map can then be used by any control system.

In [30] the authors presents an algorithm for free space estimation, capable of handling non-planar roads, using a stereo camera system. Similar to the present paper the authors make use of a parametric model of the road ahead. An interesting avenue for future work is to combine the idea presented in this paper with the ideas of [30] within a sensor fusion framework.

PROBLEM FORMULATION

An important question is how the information of the free space should be represented and for which distances ahead of the vehicle that it is needed. We will start by addressing the latter through an example, the standard double lane change manoeuvre according to ISO 3888 [13]. In this maneuver a vehicle has to overtake an obstacle and come back to its original lane as shown in Figure 2. Assume that the host vehicle is entering section 1 at a velocity of 100 km/h and that there is an obstacle straight ahead in section 3. The free space, i.e. the distance to the left and right road borders has to be known in order to autonomously overtake the obstacle as shown in the fi-

gure. This means that an automatic collision avoidance system needs to have information about the free space at least three sections ahead in order to make a decision on where to steer the vehicle. From this simple, yet informative, calculation we conclude that the road must be well estimated for at least 60 m ahead when driving at approximately 100 km/h.

In this paper we will use the planar coordinate transformation matrix

$$A^{RL} = \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix} \quad (1)$$

to transform a vector, represented in the vehicle's coordinate system L , into a vector, represented in the reference coordinate system R , where ψ_{LR} is the angle of rotation from R to L . We will refer to this angle as the yaw angle of the vehicle, and in order to simplify the notation we will use $\psi \triangleq \psi_{LR}$. The point O is the origin of R and P is the origin of L situated in the vehicles center of gravity. The geometric displacement vector r_{PO}^R is the direct straight line from O to P represented with respect to the frame R . The angles and distances are shown in Figure 3.

A stationary object i will be referred to as an observation in the point S_i . The radar in the host vehicle measures the azimuth angle $\psi_{S_i P}$ and the range $r = \|r_{S_i P}^L\|_2$ to the

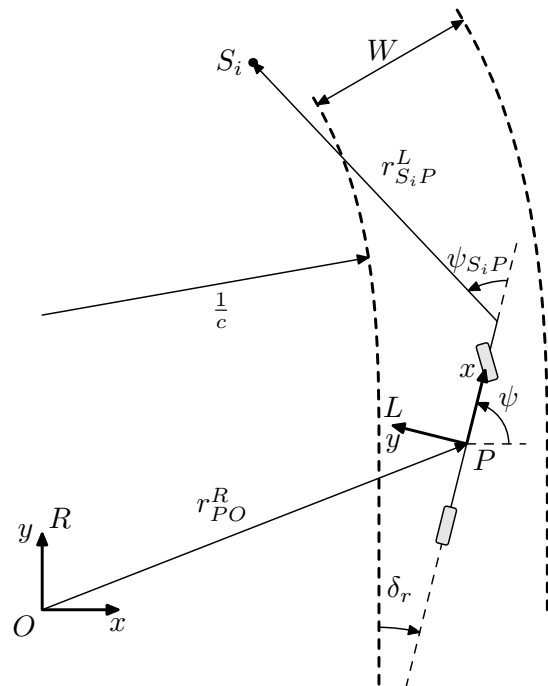


Figure 3: The host vehicle's coordinate frame L has its origin P situated in the vehicle's center of gravity. A stationary object S_i is observed at a distance $\|r_{S_i P}^L\|_2$ and an angle $\psi_{S_i P}$ with respect to the vehicles radar, which is mounted in the radiator cowling. The lane width is W , the heading angle is δ_r and the road curvature is c .

stationary object. These are transformed into Cartesian coordinates $(x_{S_iO} \ y_{S_iO})^T$ in any coordinate frame.

All the observations of stationary objects $S = \{S_i\}_{i=1}^{N_s}$ from the radar are sorted into two ordered sets, one for the left side S_l and one for the right side S_r of the road. In order to be able to perform this sorting we need some information about the road geometry, otherwise it is of course impossible. In [19] we provide a sensor fusion framework for sequentially estimating the parameters l, δ_r, c_0 in the following model of the road's white lane markings,

$$y^L = l + \delta_r x^L + \frac{c_0}{2} (x^L)^2, \quad (2)$$

where x^L and y^L are expressed in the host vehicle's coordinate frame L . The angle between the longitudinal axis of the vehicle and the road lane is δ_r , see Figure 3. It is assumed that this angle is small and hence the approximation $\sin \delta_r \approx \delta_r$ is used. The curvature parameter is denoted by c_0 and the offset between the host vehicle and the white lane is denoted by l .

The information about the road shape in (2) can now be used to decide if an observation should be sorted into the left set according to

$$S_l = \left\{ S_i \in S \mid y_{S_iP}^L \geq l + \delta_r x_{S_iP}^L + \frac{c_0}{2} (x_{S_iP}^L)^2 \right\} \quad (3)$$

or the right set according to

$$S_r = \left\{ S_i \in S \mid y_{S_iP}^L < l + \delta_r x_{S_iP}^L + \frac{c_0}{2} (x_{S_iP}^L)^2 \right\}. \quad (4)$$

Observations which lay more than 200 m behind the vehicle are removed from the set. The two sets S_l and S_r are resorted at every sample, according to the new curvature estimate.

Given the data in S_l we seek a road border model, provided by a predictor

$$\hat{y}_{S_lP}^L(x_{S_lP}^L, \theta), \quad (5)$$

where θ denotes a parameter vector describing the road boarders. The exact form of this predictor is introduced in the subsequent section, where two different predictors are derived. The data in S_r is treated analogously. The road boarder parameters θ are estimated by solving the following least-square problem

$$\min_{\theta} \sum_{i=1}^N \lambda_i (y_{S_iP}^L - \hat{y}_{S_iP}^L(x_{S_iP}^L, \theta))^2, \quad (6)$$

where N is the number of observations and λ_i is a weighting factor. The problem (6) is formulated as if there is only an error in the y -coordinate. Obviously there are errors present also in the x -coordinate. This can be taken

care of by formulating a so called errors-in-variables problem (within the optimization literature this problem is referred to as a total least squares problem), see e.g., [1]. However, for the sake of simplicity we have chosen to stick to an ordinary least squares formulation in this work.

ROAD BORDER MODEL

In this section we will derive and analyze two different predictor models, one linear and one nonlinear.

An important problem to be solved is to decide which radar measurements that should be used in estimating the parameters. Later in this section we will introduce suitable constraints that must be satisfied. This will allow us to remove non-relevant data, i.e., outliers.

PREDICTOR – The two ordered sets S_l and S_r are handled analogously. Hence, only the processing related to the left set is described here. The observations are expressed in the reference coordinate system R when they are stored in S_l . Obviously it is straightforward to transform them into the vehicle's coordinate system, using the rotation matrix $A^{LR} = (A^{RL})^T$.

As depicted earlier the lanes are modeled using the polynomial (2). Let us assume that the white lane markings are approximately parallel with the road border. In order to allow the number of lanes to change, without simultaneously changing the curvature, we extend the second order model (2) with a fourth element. Hence, a linear predictor is provided by

$$\hat{y}_1^L(x^L, \theta_1) = l_0 + l_1 x^L + l_2 (x^L)^2 + l_3 (x^L)^3, \quad (7)$$

which is a third order polynomial, describing the road's left border, given in the host vehicle coordinate system.

By analyzing road construction standards, such as [29], we assume that the increment and decrement of the number of lanes can be modelled using the arctan function illustrated in Figure 4a. This allows for a continuous, but possible rapid, change in shape. Let us now, as a second approach, extend (2) and form the following nonlinear predictor

$$\hat{y}_2^L(x^L, \theta_2) = l_0 + l_1 x^L + l_2 (x^L)^2 + k \arctan \tau (x^L - b), \quad (8)$$

where the parameter b indicates where arctan crosses zero. The slope τ and magnitude k could be chosen according to typical road construction constants. An example of the complete nonlinear road border model (8) is shown in Figure 4b.

We will start describing the linear model (7) and come back to the nonlinear model (8) later in this section. Given

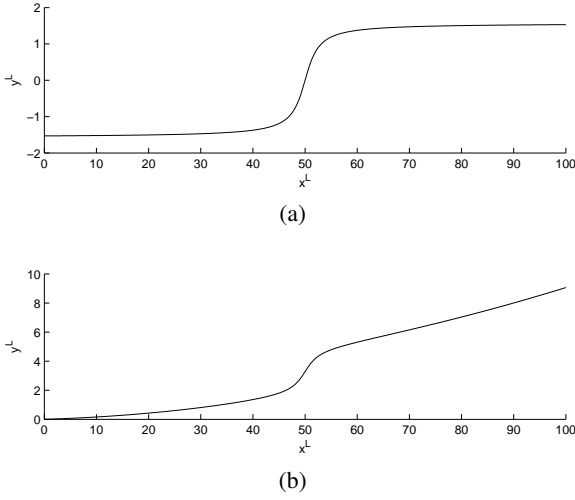


Figure 4: A pure arctan is shown in Figure (a), whereas the complete expression (8) is shown in Figure (b) for a typical example.

the N_l observations in S_l , the parameters

$$\theta_l = (l_0 \quad l_1 \quad l_2 \quad l_3)^T \quad (9)$$

can be approximated by rewriting the linear predictor (7) according to

$$\widehat{Y}_1^L = (\Phi^L)^T \theta_l, \quad (10)$$

where the regressors ($i = 1, \dots, N_l$)

$$\varphi_i^L = (1 \quad x_{S_i P}^L \quad (x_{S_i P}^L)^2 \quad (x_{S_i P}^L)^3)^T. \quad (11)$$

are stacked on top of each other in order to form

$$\Phi^L = (\varphi_1^L, \dots, \varphi_{N_l}^L), \quad (12)$$

The parameters are found by minimizing the weighted least square error (6), here in matrix form

$$\|Y^L - \Phi^L \theta_l\|_\Lambda^2 = (Y^L - \Phi^L \theta_l)^T \Lambda (Y^L - \Phi^L \theta_l), \quad (13)$$

where Λ is a weighting matrix

$$\Lambda = \text{diag}(\lambda_1 \quad \dots \quad \lambda_{N_l}). \quad (14)$$

and the y -coordinates are given by

$$Y^L = (y_1^L, \dots, y_{N_l}^L)^T. \quad (15)$$

The right hand side of the road is modeled analogously, using the following parameter vector,

$$\theta_r = (r_0 \quad r_1 \quad r_2 \quad r_3)^T. \quad (16)$$

The azimuth angle $\psi_{S_i O}$ is measured with lower accuracy than the range r in the radar system. This influences the uncertainty of the measurements, when transformed into

Cartesian coordinates in accordance to the measured distance. Therefore, the elements of the weight matrix Λ in (13) are defined as

$$\lambda_i = \frac{1}{\log r_i}, \quad i = 1, \dots, N_l, \quad (17)$$

modelling the fact that stationary objects close to the vehicle are measured with higher accuracy than distant objects. Hence, the closer the object is, the higher the weight.

The problem of minimizing (13) can be rewritten as a quadratic program [2] according to

$$\min_{\theta_l} \theta_l^T \Phi^T \Lambda \Phi \theta_l - 2(Y^L)^T \Lambda \Phi \theta_l. \quad (18)$$

A straightforward solution of this problem will not work due to the simple fact that not all of the stationary objects detected by the radar stem from relevant objects for our purposes. For example, under some circumstances the radar also detects objects at the opposite side of the highway. These observations could for example stem from a guard rail or the concrete wall of a gateway from e.g. a bridge, see Figure 5b. If the road borders are estimated according to the quadratic program in (18) using these observations the result will inevitably be wrong. In order to illustrate that this is indeed the case the result is provided in Figure 5a. In the subsequent section we will explain how this situation can be avoided by deriving a set of feasibility conditions that the curve parameters θ_l and θ_r have to fulfill.

Let us briefly revisit the nonlinear model (8). Since this predictor is nonlinear, it cannot be factored in the same way as we did for the linear predictor in (10). Instead, we have to keep the nonlinear form, resulting in the following optimization problem to be solved

$$\min_{\theta} \left\| Y^L - \widehat{Y}_2^L(X^L, \theta_2) \right\|_\Lambda^2, \quad (19)$$

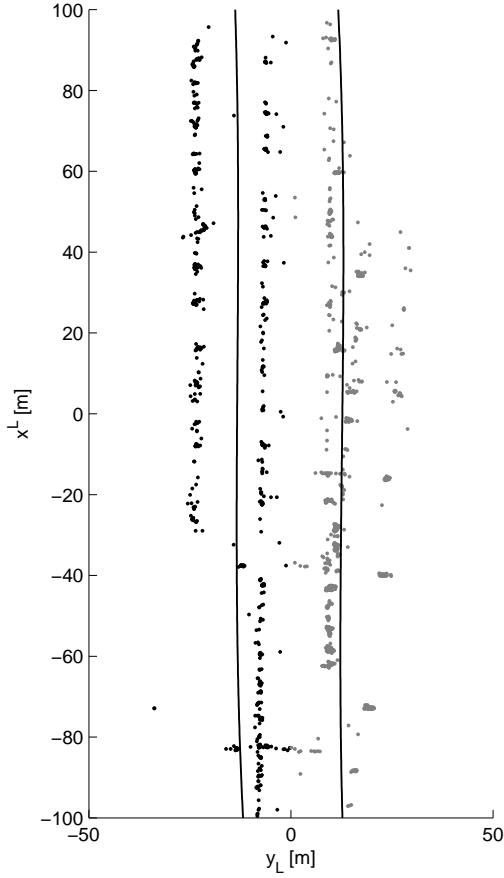
where Y^L was defined in (15) and similarly \widehat{Y}_2^L are the nonlinear predictions

$$\begin{aligned} \widehat{y}_2^L(x^L, \theta_2) &= l_0 + l_1 x^L + l_2 (x^L)^2 \\ &\quad + k \arctan \tau (x^L - b) \end{aligned} \quad (20)$$

stacked on top of each other. Hence, the parameters θ_2 used in (19) are given by

$$\theta_2 = (l_0 \quad l_1 \quad l_2 \quad k \quad \tau \quad b)^T. \quad (21)$$

The resulting problem (19) is a non-convex least-squares problem.



(a)



(b)

Figure 5: The gateway shown on the opposite side of the highway in Figure (b) misleads the road border estimation. The stored observations are shown together with the estimated road borders (lines) in Figure (a). The black points belongs to the left set S_l and the gray points belongs to the right set S_r .

CONSTRAINING THE PREDICTOR – The predictor has to be constrained for the problem formulation to be interesting. More specifically, we will in this section derive constraints forming a convex set, guaranteeing that

the resulting linear optimization problem remains quadratic. This problem can then be efficiently solved using a dual active set method* [11].

As we assume that the white lane markings (2) are approximately parallel with the road border (7), we could use the angle δ_r to constrain the second border parameter l_1 and we could use the curvature c_0 to constrain the third border parameter l_2 according to

$$(1 - \Delta)\delta_r - \epsilon_{\delta_r} \leq l_1 \leq (1 + \Delta)\delta_r + \epsilon_{\delta_r} \quad \text{if } \delta_r \geq 0, \quad (22a)$$

$$(1 + \Delta)\delta_r - \epsilon_{\delta_r} \leq l_1 \leq (1 - \Delta)\delta_r + \epsilon_{\delta_r} \quad \text{if } \delta_r < 0, \quad (22b)$$

$$\frac{(1 - \Delta)c_0 - \epsilon_{c_0}}{2} \leq l_2 \leq \frac{(1 + \Delta)c_0 + \epsilon_{c_0}}{2} \quad \text{if } c_0 \geq 0, \quad (22c)$$

$$\frac{(1 + \Delta)c_0 - \epsilon_{c_0}}{2} \leq l_2 \leq \frac{(1 - \Delta)c_0 + \epsilon_{c_0}}{2} \quad \text{if } c_0 < 0, \quad (22d)$$

where the allowed deviation Δ is chosen as 10%, i.e., $\Delta = 0.1$. A small value ϵ is added to avoid that both the upper and lower bounds are equal to 0 in case δ_r or c_0 is equal to 0. Several different approaches for estimating the road curvature c_0 are described in [20].

The first border parameter l_0 is not constrained, because the number of lanes may change at e.g. a gateway. It should be possible for the border of the road to move in parallel to the host vehicles motion without any conditions.

In order to create a feasibility condition for the fourth parameter l_3 of the linear model, the estimated position of the host vehicle expressed in the reference frame R is saved at each time sample. A data entry is removed from the set if it lays more than 200 m behind the current position. Furthermore, the estimated curvature is used to extrapolate points 200 m ahead of the vehicle. These points together with information about the host vehicle's earlier positions are used to derive a driven path as a third order polynomial

$$y^L = l + \delta_r x^L + \frac{c_0}{2} (x^L)^2 + \frac{c_1}{6} (x^L)^3. \quad (23)$$

Especially the parameter c_1 is of interest and can be used to constrain l_3 . Hence, the final inequality, which will further constrain (18) is given by

$$\frac{(1 - \Delta)c_1 - \epsilon_{c_1}}{6} \leq l_3 \leq \frac{(1 + \Delta)c_1 + \epsilon_{c_1}}{6} \quad \text{if } c_1 \geq 0, \quad (24a)$$

$$\frac{(1 + \Delta)c_1 - \epsilon_{c_1}}{6} \leq l_3 \leq \frac{(1 - \Delta)c_1 + \epsilon_{c_1}}{6} \quad \text{if } c_1 < 0. \quad (24b)$$

To summarize, the constrained optimization problem to be

* The QP code was provided by Dr. Adrian Wills at the University of Newcastle, Australia, see <http://sigpromu.org/quadprog>. This code implements the method described in [12, 25].

solved based on the linear predictor (7) is given by

$$\begin{aligned} \min_{\theta_1} \quad & \|Y^L - \widehat{Y}_1^L(X^L, \theta_1)\|_{\Lambda}^2 \\ \text{s.t.} \quad & (22) \\ & (24) \end{aligned} \quad (25)$$

The parameter b , of the nonlinear model (8) is constrained by the measurement distance and the parameters k and τ are constrained by road construction standards. The resulting nonlinear least-squares problem is finally given by

$$\begin{aligned} \min_{\theta_2} \quad & \|Y^L - \widehat{Y}_2^L(X^L, \theta_2)\|_{\Lambda}^2 \\ \text{s.t.} \quad & (22) \\ & b_{\max} \leq b \leq -b_{\max} \\ & k_{\max} \leq k \leq -k_{\max} \\ & \tau_{\max} \leq \tau \leq \tau_{\min}. \end{aligned} \quad (26)$$

OUTLIER REJECTION – The difference between the observed point and the calculated road border lines is used to separate and remove outliers which lie more than 1.5 lane width (W) from the lines. Subsequently the quadratic program (18) is used a second time and the result is shown in Figure 6. For this case, the two predictor models yields approximately the same result.

An advantage of the nonlinear model is its ability to model changes in the number of lanes, as can be seen in Figure 7a, where the number of lanes changes from two to three. Recall that it is the use of the arctan function that allows us to model changes in the number of lanes. The new lane originates from an access road to the highway. The corresponding camera view is shown in Figure 7b.

COMPUTATIONAL TIME – We have compared the computation time for the two proposed predictors with constraints. The nonlinear least square problem (26) was solved using the function `fmincon` in MATLABS optimization toolbox. Furthermore, we have used two different methods for solving the quadratic problem (25). The first method is the active set method mentioned earlier, where parts are written in C-code, and the second method used is `quadprog` in MATLABS optimization toolbox. The computational time was averaged over a sequence of 1796 samples. The sample time is 0.1 s, implying that the measurements were collected during 179.6 s highway driving. The results are shown in Table 1.

The computation time of the nonlinear predictor is about 38 % higher than it is for the linear predictor proposed in this paper. The MATLAB function `quadprog` needs 149 % more computational time. This indicates that the computational time of the nonlinear predictor can possibly be reduced by utilizing an optimized C-code implementation.

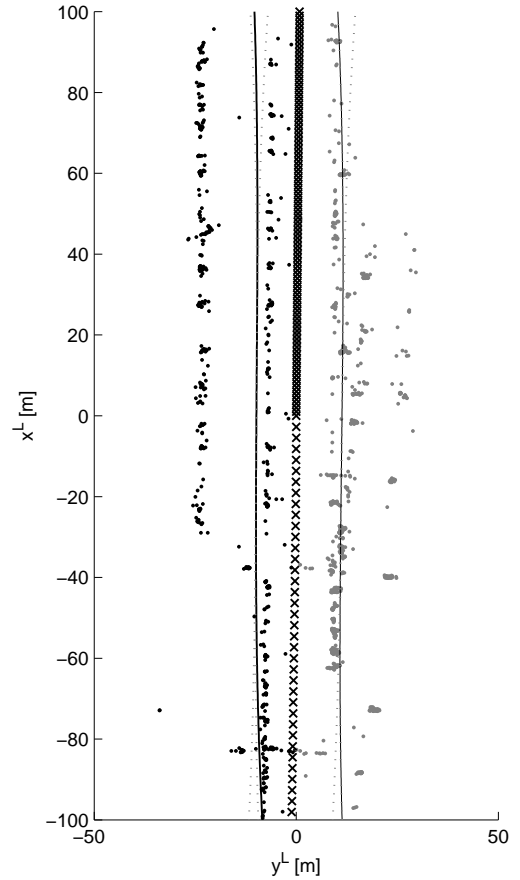


Figure 6: Road border estimation for the same situation as in Figure 5a, but the additional constraints are now used. The feasible set for the parameters l_1 , l_2 and l_3 is between the dashed lines. The crosses shows the driven path (for $x < 0$) and the estimated path (for $x > 0$).

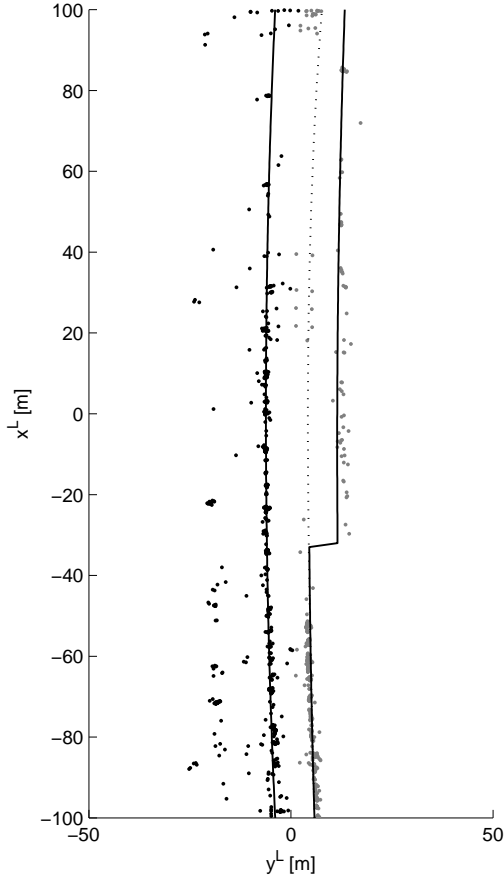
Table 1 Average computational time for one sample.

Method	Time [ms]
Linear Predictor (this paper)	84
Linear Predictor (quadprog)	209
Nonlinear Predictor	116

CALCULATING THE FREE SPACE

The free distance to the left and the right road borders is now easily calculated by considering the first parameters l_0 and r_0 respectively. The number of lanes on the left hand side is given by

$$\max\left(\left\lfloor \frac{l_0 - L}{W} \right\rfloor, 0\right) \quad (27a)$$



(a)



(b)

Figure 7: A change in the number of lanes is modeled accurately using the arctan function in the nonlinear predictor, as shown by the solid line in Figure (a). The dashed line is the result of the linear predictor. The camera view of the traffic situation is shown in Figure (b).

and the number of lanes on the right hand side is given by

$$\max \left(\left\lfloor \frac{-r_0 - R - 2}{W} \right\rfloor, 0 \right). \quad (27b)$$

In the expressions above L and R are the distances from the sensor in the host vehicle to the left and right lane

markings of the currently driven lane. We assume that the emergency lane is 2 m on the right hand side of the road [29].

The number of observed stationary objects depends on the surrounding environment. A guard rail or a concrete wall results in more observations than for example a forest. Hence, the estimated border lines are accompanied by a quality measure which depends on the number of observations and their variance. The variance is calculated before and after the outliers have been removed.

It is still a problem to detect the distance to the road border if there is a noise protection wall some meters to the right of the road. This wall generates many observations with small variance and cannot be distinguished from a guard rail. However, one solution might be to include camera information in a sensor fusion framework.

BORDER LINE VALIDITY – A very thrilling problem with the present curve fitting approach is that there are no gaps to properly leave or enter the road at a gateway. A collision avoidance system would brake the vehicle automatically if leaving the road at a gateway when simultaneously crossing the border line. This leads us to the conclusion that the border lines should only be defined if the number of observations around it lies above a certain limit.

In a first step we calculate the distance between the line and the observations in the set S_1

$$d_{l,i} = \left| y_i^L - \left(\delta_r x_i^L + \frac{c_0}{2} (x_i^L)^2 \right) \right| \quad \text{for } i = 1, \dots, N_l \quad (28)$$

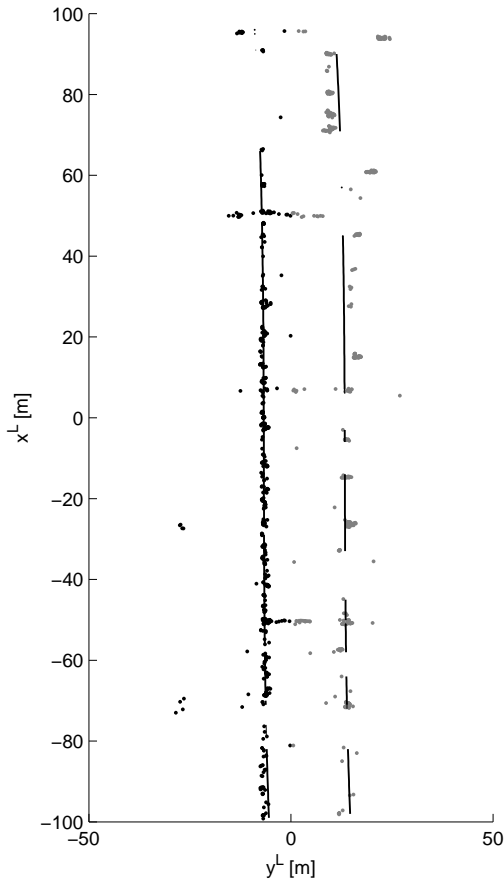
and compare it with a constant or variable, e.g. the lane width W

$$n_i = \begin{cases} 1 & \text{if } d_{l,i} > W \\ 0 & \text{otherwise.} \end{cases} \quad (29)$$

In a second step the border line is segmented in valid and not valid parts. The start and end points of the valid parts are given by identifying the indices of two non equal and adjoined elements in the vector n . By applying the XOR function (\oplus) according to

$$c = n_{2:N_l} \oplus n_{1:N_l-1}, \quad (30)$$

the start and end points of the border line are identified as the indices with $c = 1$. These indices are stored in two additional sets for the left and right border lines, respectively. An example is shown in Figure 8a and the corresponding camera view in Figure 8b. The gateway to the right leads to a gap in the right border line, between 48 – 73 m ahead of the host vehicle. One of the leading vehicles lies between the host vehicle and the guard rail, this is the reason why there are so few stationary object on the left hand side from about 70 m ahead and why no line could be drawn.



(a)



(b)

Figure 8: The gateway to the right in Figure (b) leads to a gap in the right border line, between 48 – 73 m ahead, as shown in Figure (a).

CONCLUSIONS AND FUTURE WORK

In this contribution we have derived a method for estimating the free space in front of a moving vehicle, making use of radar measurements originating from stationary objects along the road side. There is no need to introduce any new sensors, since the radar sensor is already present in modern premium cars. It is just a matter of making bet-

ter use of the sensor information that is already present.

Two different road border models are introduced, one linear model containing four parameters and one nonlinear model containing six parameters. These models do not depend on the fact that a radar sensor is used, implying that it is straightforward to add more sensor information from additional sensors. In other words, the approach introduced here fits well within a future sensor fusion framework, where additional sensors, such as cameras and additional radars, are incorporated.

The present approach has been evaluated on real data from both highways and rural road in Sweden. The results are encouraging and surprisingly good at times. It is of course not always perfect, but it is much more informative than just using the raw measurements. The problems typically occur when there are too few measurements or if the measurements stems from other objects than the road side objects.

Currently there is a lot of activity within the computer vision community to be able to handle non-planar road models, making use of parametric models similar to the ones used in this paper. A very interesting avenue for future work is to combine the idea presented in this paper with information from a camera about the height differences on the road side within a sensor fusion framework. This would probably improve the estimates, especially in situations when there are too few radar measurements available.

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