Institutionen för Reglerteknik
Tekniska Högskolan i Lund

Linear Systems I

Exam Oct 23 – Oct 24, 1996

The time is 36 hours or less. Computers may be used and books may be consulted. You are encouraged to ask me if anything is questionable or difficult to understand, but you may not help each other.

Participants:
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Good Luck!
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1. Determine a minimum realisation for

$$G(s) = \begin{bmatrix} \frac{s+2}{(s+1)(s+3)} & \frac{1}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+2} \end{bmatrix}$$

Find a transfer function $G_1(s)$ such that $u_2(s) = -u_1(s) + G_1(s)u_1(s)$ gives $y(s) = G(s)u(s) = 0$ for any $u_1(s)$. This means that $G(s)$ is rank one.

(10 p)

2. Consider the system

$$A = \begin{bmatrix} -3 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ -2 & 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. Evaluate $H(q) = C(qI - A)^{-1}$, where q is the forward shift operator, $qu(t) = u(t+1)$. (2 p)

b. Determine a full order dead-beat state observer with minimal nilpotency index. (3 p)

c. Determine a reduced order dead-beat state observer with minimal nilpotency index. (5 p)

3. Consider the continuous stirred tank reactor with immobilized yeast cells. Sucrose is converted into ethanol. The following equations describe the sucrose (S) and ethanol (E) concentrations

$$\frac{dS}{dt} = -k_s S + (S_{in} - S)F/V$$
$$\frac{dE}{dt} = k_e S - EF/V$$

The experiments are performed in a $V = 5\ell$ reactor around a timevarying nominal flow $F^0(t) = 0.050 + 0.030 * \text{sign}(\sin(\pi t/T))\,\ell/\text{min}$. Let the concentration of the inlet flow be $S_{in} = 0.5\text{M}$, and $k_s = 0.015\ \text{min}^{-1}$, $k_e = 0.045\ \text{min}^{-1}$.

a. Calculate the stationary periodic solution, and linearize the system around this nominal trajectory. Hint: Find $S(t)$ such that $S(0) = S(2T)$ and similarly for $E(t)$. Outline the calculations, but wait with Maple until you have solved the rest of the exam. To simplify a little you may choose $T = 30$ min. (5 p)

b. The resulting $A(t)$ and $B(t)$ are periodic with period $2T$. Determine the transfer matrix $\Phi(t, \tau)$ and the reachability Gramian $W_r(0, 2T)$ for the linearized system. Outline the calculations, but wait with Maple until you have solved the rest of the exam. (5 p)

c. Show that W_r satisfies the equation

$$W_r(0, k2T + 2T) = W_r(0, 2T) + \Phi(2T, 0)W_r(0, k2T)\Phi^T(2T, 0)$$

Evaluate the limit $\lim_{k\to\infty} W(0, k2T)$. (5 p)

4. Consider the second order system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= -2x_2 + u \\ y &= x_1\end{aligned}$$

a. Determine the observability Gramian $M_o(t_0, t_1)$ and the reconstructability Gramian $M_r(t_0, t_1) = \Phi^{-T}(t_1, t_0) M_o(t_0, t_1) \Phi^{-1}(t_1, t_0)$. (5 p)

b. Calculate the limit of $M_o(t_0, t_1)$ for $t_1 \to \infty$ and the best estimate of $x(t_0)$ given $y(t)$, $t \in [t_0, \infty)$. First assume $u = 0$, then include $u \neq 0$. (5 p)

c. What is the best estimate of $x(t_1)$ given $y(t)$, $t \in (-\infty, t_1]$? (5 p)

5. Consider the system

$$\begin{aligned}y_1(t+1) &= y_2(t) + b_1 u(t) \\ y_2(t+1) &= y_3(t) + b_2 u(t) \\ y_4(t+1) &= y_1(t) + b_3 u(t) \\ y_4(t) &= b_4 u(t)\end{aligned}$$

a. Introduce the pencil formulation $(qE - F)y(t) = Bu(t)$. What transformation is needed to obtain the Weierstrass form? (5 p)

b. If the transfer operator $H(q)$ from u to y is proper, what does that imply about the coefficients $b_1, \ldots, b_4$? (3 p)

c. Express the solution $y(t)$, $t = 0, 1, \ldots$ in $y(0)$ and $u(t)$, $t = 0, 1, \ldots$? Any difficulty? (2 p)