Linear Systems I

Course notes and exercises based on a course developed at the Department of Automatic Control, Lund Institutet of Technology, Lund by **Per Hagander** and **Anders Rantzer**.

Lecture 1

Example time-varying systems, two tanks

Linearization around trajectory

Integral operators

Induced norms

Lecture 2

$\exists$ solution to $\dot{x}(t) = A(t)x(t)$ – Peano-Baker

uniqueness, Grnwall

$\Phi(t,\tau)$ in general hard

scalar case and

$A(t)\int_\tau^t A(\sigma)d\sigma = \int_\tau^t A(\sigma)d\sigma A(t)$ gives $\Phi(t,\tau) = \exp\left\{\int_\tau^t A(\sigma)d\sigma\right\}$

Properties: Composition, inversion, detΦ

Least squares, scalar products, adjoints

$$< y, Mx >_Y = < M^* y, x >_X$$
$$M : X \to Y \quad M^* : Y \to X$$

Lecture 3

Special cases,

Time invariant e^{At}, Discrete time,

Periodic

$$\Phi(t,\tau) = P(t)e^{R(t-\tau)}P^{-1}(\tau), \quad e^{RT} = \Phi(T,0)$$

Periodic solution – "resonance"

Uniformly as stab, Lyapunov transformation

Lecture 4

Reachable (Controllable), $x_f = Lu_{[t_0,t_f]}$, $R(L)$

Gramian, LL^*, matrix, degree of reach

timeinv PBH, reach subspace transf

Unobservable (Reconstr), $y_{[t_0,t_f]} = Lx_0$, $N(L)$

Gramian L^*L matrix, degree of obs

Controllability index, controller form

Observability index, observer form

Balanced realisation

Lecture 5

Realisation: Weighting pattern (impulse response) to state space

$$G(t,\sigma) = H(t)F(\sigma) = C(t)\Phi(t,\sigma)B(\sigma)$$

Minimality, controllable-observable

LTI, Laplace, $d(s)G(s) = N(s)$, Gilbert

Markov, CA^kB, $\mathcal{O}\mathcal{C}$, Hankel matrix

minimal, unique $z = Px$

Lecture 6

Linear feedback – stabilization (also $A(t)$), eigenvalue assignment via controller form

Decoupling, relative degree, Markov

Observers, reduced order with direct feed-through

Youla

Lecture 7

Intro to other descriptions, polynomial-matrix fraction, pencils

Examples - OO-modeling

Weierstrass form

Exercises

Maple, Matlab